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Hypersurfaces of Kenmotsu manifolds endowed with a quarter-symmetric non-metric connection

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ABSTRACT

The object of the present paper is to define a quarter-symmetric non-metric connection in a Kenmotsu manifold and consider non-invariant and anti-invariant hypersurfaces of Kenmotsu manifold endowed with a quarter-symmetric non-metric connection. Finally, we obtain the Gauss and Codazzi equations with respect to a quarter-symmetric non-metric connection.

Keywords: Codazzi equations; Gauss equations; hypersurfaces; Kenmotsu manifold; quarter-symmetric non-metric connection.

AMS Subject Classification: 53D12; 53C05

INTRODUCTION

Let ∇ be a linear connection in a $(2n+1)$-dimensional differentiable manifold M. The torsion tensor T and the curvature tensor R of ∇ are given, respectively, by

$$T(X, Y) = \nabla_X Y - \nabla_Y X - [X, Y]$$

and

$$R(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z.$$

The connection ∇ is symmetric if its torsion tensor T vanishes, otherwise it is non-symmetric. The connection ∇ is a metric connection if there is a Riemannian metric g in M such that $\nabla g = 0$, otherwise it is non-metric. It is well known that a linear connection is symmetric and metric if and only if it is the Levi-Civita connection. In 1975, Golab (1975) introduced the notion of a quarter-symmetric linear connection in a differentiable manifold. A linear connection is said to be a quarter-symmetric connection if its torsion tensor T is of the form

$$T(X, Y) = u(Y)\varphi X - u(X)\varphi Y,$$

where u is a 1-form and φ is a (1, 1)-tensor field. Later Mishra & Pandey (1980) deduced some properties of the Riemannian, Kaehlerian and Sasakian manifolds that admits quarter-symmetric metric connection. In addition, Gupta et al. (2004) studied slant submanifolds of a Kenmotsu manifold. In a recent paper Khan et al.(2007) studied slant and semi-slant submanifolds of Kenmotsu manifolds. Also Gupta & Pandey (2008) studied structure on slant submanifold of a Kenmotsu manifold. Recently, Sular & Ozgur (2009) studied some submanifolds of Kenmotsu manifolds. Motivated by these circumtances, in this paper we study non-invariant and anti-invariant hypersurfaces of Kenmotsu manifolds endowed with a quarter-symmetric non-metric connection.

The paper is organized as follows: After preliminaries in section 3, we prove that the connection induced on a hypersurface of a Kenmotsu manifold with a quarter-symmetric non-metric connection with respect to the normal vector is also a quarter-symmetric non-metric connection and establish some basic properties with respect to quarter-symmetric non-metric connection. In section 4, we find the characteristic properties of non-invariant and anti-invariant hypersurfaces of a Kenmotsu manifold with a quarter-symmetric non-metric connection and prove some results. Finally, we obtain Gauss and Codazzi equations associated with quarter-symmetric non-metric connection and prove an interesting result.

PRELIMINARIES

Let M be a $(2n+1)$-dimensional connected differentiable manifold endowed with an almost contact metric structure (φ, ξ, η, g), where φ, ξ, η are tensor fields on M of types (1, 1), (1, 0), (0, 1), respectively, such that (Blair (1976); Blair (2002); Yano & Kon (1984)),

$$\varphi^2 = -I + \eta \otimes \xi, \quad \varphi\xi = 0, \quad \eta(\xi) = 1, \quad \eta \circ \varphi = 0, \tag{1}$$

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2}$$

$$g(\varphi X, Y) + g(X, \varphi Y) = 0, \tag{3}$$

for all vector fields $X, Y \in T(M)$, where $T(M)$ is the Lie algebra of vector fields of the manifold M.

An almost contact metric manifold M is said to be a Kenmotsu manifold (Kenmotsu (1972)) if the relation

$$(\nabla_X\varphi)Y = g(\varphi X, Y)\xi - \eta(Y)\varphi X \tag{4}$$

holds on M. From the above equation, for a Kenmotsu manifold we also have

$$\nabla_X\xi = X - \eta(X)\xi. \tag{5}$$

A Kenmotsu manifold is normal (that is, the Nijienhuis tensor of φ equals to $-2d\eta \otimes \xi$) but not Sasakian. Moreover, it is also not compact, since from (5) we get $div\xi = 2n$. Kenmotsu (1972) showed: (a) that locally a Kenmotsu manifold is a Warped product $I \times_f N$ of an interval I and a Kaehler manifold N with warping function $f(t) = se^t$, where s is a non-zero constant; (b) that a Kenmotsu manifold of constant φ sectional curvature is a space of constant curvature -1 and so it is locally hyperbolic space.

HYPERSURFACES OF KENMOTSU MANIFOLDS

Let M^{2n+1} be a Kenmotsu manifold with a positive definite metric g and M^{2n} be a hypersurface immersed in M^{2n+1} by immersion $i : M^{2n} \to M^{2n+1}$. If B denotes the differentiable of i, then any vector field $\overline{X} \in T(M^{2n})$ implies $B\overline{X} \in T(M^{2n+1})$. We denote the objects belonging to M^{2n} by $\overline{X}, \overline{\varphi}, \overline{\eta}, \overline{\xi}$ etc.

Let N be the unit normal vector field to M^{2n}. Then the induced metric $\bar{g}$ on M^{2n} is $\overline{g}(\overline{X}, \overline{Y}) = g(\overline{X}, \overline{Y})$. Then we have (Chen, 1973).

$$g(\overline{X}, N) = 0 \quad \text{and} \quad g(N, N) = 1. \tag{6}$$

Let ∇^* be the Riemannian connection in M^{2n+1} and if $\overline{\nabla}^*$ is the induced connection on hypersurface from ∇^* with respect to the unit normal N, then Gauss equation is given by

$$\nabla^*_{\overline{X}}\overline{Y} = \overline{\nabla}^*_{\overline{X}}\overline{Y} + h(\overline{X}, \overline{Y})N, \tag{7}$$

where h is the second fundamental tensor satisfying $h(\overline{X}, \overline{Y}) = h(\overline{Y}, \overline{X}) = g(H(\overline{X}), \overline{Y})$ and H is the shape operator of M^{2n} in M^{2n+1}.

The hypersurface of an almost contact manifold M^{2n+1} is called invariant (resp. anti-invariant) if for each point $p \in M^{2n}$,

$$\varphi T_p(M^{2n}) \subset T_p(M^{2n})(res.\varphi T_p(M^{2n}) \subset T_p^{\perp}(M^{2n})).$$

A non-invariant hypersurface of an almost contact manifold is a hypersurface such that the transform of a tangent vector of the hypersurface by (1, 1) tensor field defining an almost contact structure is never tangent to the hypersurface (Goldberg & Yano, 1970).

The hypersurface of an almost contact manifold M^{2n+1} is said to be quasi-umbilical (Chen, 1973) at a point $\overline{X} \in T(M^{2n})$ if at the point $\overline{X}$ its second fundamental tensor h satisfies the condition $h = ag + b\omega \otimes \omega$, where ω is a 1-form and a and b are scalars on M^{2n}. If $a = 0$ (respectively, $b = 0; or, a = b = 0$) holds at $\overline{X}$ it is called cylindrical (respectively, totally umbilical or totally geodesic) at $\overline{X}$.

We define a quarter-symmetric non-metric connection ∇ on M^{2n+1} by

$$\nabla_X Y = \nabla^*_X Y + \eta(Y)\varphi X. \tag{8}$$

If $\overline{\nabla}$ is the induced connection on the hypersurface from the quarter-symmetric non-metric connection ∇ with respect to the unit normal N, then we have

$$\nabla_{\overline{X}}\overline{Y} = \overline{\nabla}_{\overline{X}}\overline{Y} + m(\overline{X}, \overline{Y})N, \tag{9}$$

where m is a tensor field of type (0, 2) on the hypersurface M^{2n}.

Now every vector field X on M^{2n+1} is decomposed as

$$X = \overline{X} + l(X)N, \tag{10}$$

where l is a 1-form on M^{2n+1} and for any vector field $\overline{X}$ on M^{2n} and normal N, we have

$$\varphi N = \overline{N} + KN, \tag{11}$$

$$\xi = \overline{\xi} + aN, \tag{12}$$

$$\varphi\overline{X} = \overline{\varphi}\overline{X} + b(\overline{X})N, \tag{13}$$

where b is a 1-form and$K$ is a scalar function on M^{2n}.

Using (13) in (8) we have

$$\nabla_{\overline{X}}\overline{Y} = \nabla^*_{\overline{X}}\overline{Y} + \overline{\eta}(\overline{Y})(\overline{\varphi}\overline{X} + b(\overline{X})N). \tag{14}$$

Using (7) and (9) in (13) yields

$$\overline{\nabla}_{\overline{X}}\overline{Y} + m(\overline{X}, \overline{Y})N = \overline{\nabla}^*_{\overline{X}}\overline{Y} + h(\overline{X}, \overline{Y})N + \overline{\eta}(\overline{Y})(\overline{\varphi}\overline{X} + b(\overline{X})N). \tag{15}$$

Now taking tangential and normal part we have

$$\overline{\nabla}_{\overline{X}}\overline{Y} = \overline{\nabla}^*_{\overline{X}}\overline{Y} + \overline{\eta}(\overline{Y})\overline{\varphi}\overline{X} \tag{16}$$

and

$$m(\overline{X}, \overline{Y}) = h(\overline{X}, \overline{Y}) + \overline{\eta}(\overline{Y})b(\overline{X}). \tag{17}$$

Thus we get the following:

Theorem 1. The connection induced on a hypersurface of a Kenmotsu manifold with a quarter-symmetric non-metric connection with respect to the normal vector is also a quarter-symmetric non-metric connection.

Using (17), from (9) we get

$$\nabla_{\overline{X}}\overline{Y} = \overline{\nabla}_{\overline{X}}\overline{Y} + h(\overline{X}, \overline{Y})N + \overline{\eta}(\overline{Y})b(\overline{X})N, \tag{18}$$

which is the Gauss equation for a quarter-symmetric non-metric connection. The equation of the Weingarten with respect to the Riemannian connection ∇^* is given by

$$\nabla^*_{\overline{X}}N = -H\overline{X}, \tag{19}$$

for every vector field $\overline{X}$.

Using (13) in (8) yields

$$\nabla_{\overline{X}}N = \nabla^*_{\overline{X}}N + a(\overline{\varphi}\overline{X} + b(\overline{X})N), \tag{20}$$

where $a = \eta(N)$.

From (19) and (20) we get

$$\nabla_{\overline{X}}N = -H\overline{X} + a(\overline{\varphi}\overline{X} + b(\overline{X})N), \tag{21}$$

which is the Weingarten equation with respect to the quarter-symmetric non-metric connection.

From (16) the torsion tensor with respect to the quarter-symmetric non-metric connection is given by

$$T(\overline{X},\overline{Y}) = \overline{\eta}(\overline{Y})\overline{\varphi}\overline{X} - \overline{\eta}(\overline{X})\overline{\varphi}\overline{Y}. \tag{22}$$

Making use of (10), (11), (12) and (13), from (1)-(5), we obtain

$$\overline{\varphi}^2\overline{X} + b(\overline{X})\overline{N} = -\overline{X} + \overline{\eta}(\overline{X})\overline{\xi}, \tag{23}$$

$$b(\overline{\varphi}\overline{X}) + Kb(\overline{X}) = a\overline{\eta}(\overline{X}), \tag{24}$$

$$\overline{\varphi}\overline{N} + K\overline{N} = a\overline{\xi}, \tag{25}$$

$$b(\overline{N}) + K^2 = a^2 - 1, \tag{26}$$

$$\overline{\varphi}\overline{\xi} + a\overline{N} = 0, \tag{27}$$

$$aK + b(\overline{\xi}) = 0, \tag{28}$$

$$\overline{\eta} \circ \overline{\varphi}(\overline{X}) + ab(\overline{X}) = 0, \tag{29}$$

$$\overline{\eta}(\overline{\xi}) = 1 - a^2, \tag{30}$$

$$\overline{\eta}(\overline{X}) = \overline{g}(\overline{X}, \overline{\xi}), \tag{31}$$

$$\overline{g}(\overline{\varphi}\overline{X}, \overline{\varphi}\overline{Y}) + b(\overline{X})b(\overline{Y}) = \overline{g}(\overline{X}, \overline{Y}) - \overline{\eta}(\overline{X})\eta(\overline{Y}), \tag{32}$$

$$\overline{g}(\overline{\varphi}\overline{X}, \overline{Y}) + \overline{g}(\overline{X}, \overline{\varphi}\overline{Y}) = 0, \tag{33}$$

$$\overline{g}(\overline{X}, \overline{N}) = -b(\overline{X}), \tag{34}$$

$$\begin{aligned}(\nabla_{\overline{Y}}\varphi)\overline{X} &= (\overline{\nabla}_{\overline{Y}}\overline{\varphi})\overline{X} - h(\overline{X},\overline{Y})\overline{N} - \overline{\eta}(\overline{X})b(\overline{Y})\overline{N}\\ &+b(\overline{X})(-H\overline{Y} + a\overline{\varphi}\overline{Y}) + [h(\overline{\varphi}\overline{X},\overline{Y}) + Kh(\overline{X},\overline{Y})\\ &-K\overline{\eta}(\overline{X})b(\overline{Y}) + (\nabla_{\overline{Y}}b)(\overline{X})]N,\end{aligned} \tag{35}$$

$$\begin{aligned}(\nabla_{\overline{Y}}\varphi)N &= \overline{\nabla}_{\overline{Y}}\overline{N} + \overline{\varphi}(H\overline{Y}) + a(\overline{Y} + \overline{\eta}(\overline{Y})\overline{\xi})\\ &+K(-H\overline{Y} + a\overline{\varphi}\overline{Y}) + [h(\overline{Y},\overline{N}) + \overline{\eta}(\overline{N})\\ &+\overline{Y}K + b(H\overline{Y}) - ab(\overline{\varphi}\overline{Y})]N,\end{aligned} \tag{36}$$

$$\begin{aligned}\nabla_{\overline{Y}}\xi &= \overline{\nabla}_{\overline{Y}}\overline{\xi} - aH\overline{Y} + a^2\overline{\varphi}\overline{Y} \\ &+[h(\overline{Y},\overline{\xi}) + \overline{Y}a + b(\overline{Y})]N,\end{aligned} \tag{37}$$

$$(\nabla_{\overline{Y}}\overline{\eta})\overline{X} = (\overline{\nabla}_{\overline{Y}}\overline{\eta})\overline{X} - ah(\overline{X},\overline{Y}) - a\overline{\eta}(\overline{X})b(\overline{Y}). \tag{38}$$

The above results will be used in the next sections.

THE MAIN RESULTS

Now suppose that $\nabla\varphi = 0$ along M^{2n}. Then from (35) we have

$$\begin{aligned}&(\overline{\nabla}_{\overline{Y}}\overline{\varphi})\overline{X} - h(\overline{X},\overline{Y})\overline{N} - \overline{\eta}(\overline{X})b(\overline{Y})\overline{N} \\ &+b(\overline{X})(-H\overline{Y} + a\overline{\varphi}\overline{Y}) = 0\end{aligned} \tag{39}$$

and

$$h(\overline{\varphi}\overline{X},\overline{Y}) + Kh(\overline{X},\overline{Y}) - K\overline{\eta}(\overline{X})b(\overline{Y}) + (\overline{\nabla}_{\overline{Y}}b)(\overline{X}) = 0. \tag{40}$$

If M^{2n} is totally geodesic, then $h = 0$ and $H = 0$, and hence from (39) we get

$$(\overline{\nabla}_{\overline{Y}}\overline{\varphi})\overline{X} - \overline{\eta}(\overline{X})b(\overline{Y})\overline{N} + ab(\overline{X})\overline{\varphi}\overline{Y} = 0. \tag{41}$$

Therefore we have the following:

Theorem 2. If M^{2n} is a totally geodesic non-invariant hypersurface of a Kenmotsu manifold M^{2n+1} endowed with a quarter-symmetric non-metric connection satisfying $\nabla\varphi = 0$ along M^{2n}, then $(\overline{\nabla}_{\overline{Y}}\overline{\varphi})\overline{X} = \overline{\eta}(\overline{X})b(\overline{Y})\overline{N} - ab(\overline{X})\overline{\varphi}\overline{Y}$, where $a = \eta(N)$.

The converse of the above Theorem is not, in general, true. If we consider

$$(\overline{\nabla}_{\overline{Y}}\overline{\varphi})\overline{X} - \overline{\eta}(\overline{X})b(\overline{Y})\overline{N} + ab(\overline{X})\overline{\varphi}\overline{Y} = 0, \tag{42}$$

then (39) yields

$$h(\overline{X},\overline{Y})\overline{N} + b(\overline{X})H\overline{Y} = 0. \tag{43}$$

Using (34) in (43) yields

$$b(\overline{X})h(\overline{Y},\overline{Z}) = b(\overline{Y})h(\overline{X},\overline{Z}). \tag{44}$$

Now putting $\overline{Y} = \overline{Z} = \overline{e}_i$, in (44), where $\{\overline{e}_i\}$, i = 1,2,3....,2n is an orthonormal basis of the tangent space at each point of the hypersurface M^{2n} and i is summed for $1 \leq i \leq 2n$, we get

$$n\mu b(\overline{X}) = b(H\overline{X}), \tag{45}$$

where μ is the mean curvature.

From (44) we obtain

$$\begin{aligned} b(\overline{N})h(\overline{X},\overline{Y}) &= b(\overline{X})h(\overline{N},\overline{Y}) \\ &= b(\overline{X})\overline{g}(H\overline{Y},\overline{N}) \\ &= -b(\overline{X})b(H\overline{Y}). \end{aligned} \tag{46}$$

Since $b(\overline{N}) = -1$, using (45) in (46) we get

$$h(\overline{X},\overline{Y}) = b(\overline{X})b(H\overline{Y}) = n\mu b(\overline{X})b(\overline{Y}) = \delta b(\overline{X})b(\overline{Y}),$$

where $\delta = n\mu$. Therefore the hypersursace is cylindrical.

This leads to the following:

Theorem 3. If M^{2n} is a non-invariant hypersurface of a Kenmotsu manifold M^{2n+1} endowed with a quarter-symmetric non-metric connection satisfying $\nabla\varphi = 0$ along M^{2n} and $(\overline{\nabla}_{\overline{Y}}\overline{\varphi})\overline{X} = \overline{\eta}(\overline{X})b(\overline{Y})\overline{N} - ab(\overline{X})\overline{\varphi}\overline{Y}$, then M^{2n} is cylindrical.

Now we state the following:

Lemma 1. (Bucki, 1989) $\overline{\nabla}_{\overline{X}}(trace\overline{\varphi}) = trace(\overline{\nabla}_{\overline{X}}\overline{\varphi})$.

From (39) we obtain

$$\overline{g}((\overline{\nabla}_{\overline{Y}}\overline{\varphi})\overline{X},\overline{X}) = \overline{\eta}(\overline{X})b(\overline{Y})b(\overline{X}) - ab(\overline{X})\psi(\overline{X},\overline{Y}), \tag{47}$$

where $\psi(\overline{X},\overline{Y}) = \overline{g}(\overline{X},\overline{\varphi}\overline{Y})$.

Now using (11), (25), (31) and Lemma 1 we obtain

$$\begin{aligned} \overline{\nabla}_{\overline{X}}(trace\overline{\varphi}) &= trace(\overline{\nabla}_{\overline{X}}\overline{\varphi}) \\ &= \overline{g}((\overline{\nabla}_{\overline{X}}\overline{\varphi})\overline{e}_i,\overline{e}_i) = -a^2\overline{\eta}(\overline{X}). \end{aligned} \tag{48}$$

So we have the following:

Theorem 4. Let M^{2n} be a non-invariant hypersurface of a Kenmotsu manifold M^{2n+1} endowed with a quarter-symmetric non-metric connection satisfying $\nabla\varphi = 0$ along M^{2n}. Then ξ is tangent to M^{2n} if and only if $trace\overline{\varphi} =$ constant.

Now suppose M^{2n} is anti- invariant, that is, $\varphi T_p(M^{2n}) \subset T_p^{\perp}(M^{2n})$, which implies $\overline{\varphi} = 0$. Using this we get from (27), $\eta(N) = 0$, which implies ξ is tangent to M^{2n}. Also from (25), $K = 0$, implies $\overline{\varphi}(\overline{N}) \subset T(M^{2n})$. From (29) and (25) we have

$$b(\overline{\xi}) = 0 \quad \text{and} \quad b(\overline{N}) = -1. \tag{49}$$

Since $\overline{\varphi} = 0$, from (23) we have

$$b(\overline{X})\overline{N} = -\overline{X} + \eta(\overline{X})\overline{\xi}. \tag{50}$$

If $\{\overline{e}_i\}_{i=1,2,\ldots\ldots 2n.}$ is an orthonormal basis in $T(M^{2n})$, then $\overline{N} = \sum_{i=1}^{2n} b(\overline{e}_i)\overline{e}_i$ and $b(\overline{N}) = -1$ implies

$$b^2(\overline{e}_i) = -1. \tag{51}$$

Also $\overline{\xi} = \sum_{i=1}^{2n} \overline{\eta}(\overline{e}_i)\overline{e}_i$ and $\overline{\eta}(\overline{\xi}) = 1$ implies

$$\overline{\eta}(\overline{e}_i)^2 = 1. \tag{52}$$

Therefore from (50) we get

$$b(\overline{X})^2 = -\overline{g}(\overline{X}, \overline{X}) + \overline{\eta}(\overline{X})^2. \tag{53}$$

Putting $\overline{X} = \overline{e}_i$ in (53), and using (51) and (52), we have n = 1. Hence we have the following:

Proposition 1. If M^{2n} be an anti-invariant hypersurface of a Kenmotsu manifold M^{2n+1} endowed with a quarter-symmetric non-metric connection, then M^{2n+1} reduces to a 3-dimensional Kenmotsu manifold with an orthonormal basis $\{\xi, \overline{N}, N\}$.

If M^{2n} is anti-invariant, then M^{2n+1} is 3-dimensional and $\varphi(N) = \overline{N}$ and $\xi = \overline{\xi}$.

Conversely, assume that M^{2n+1} is a 3-dimensional Kenmotsu manifold with $\varphi(N)$ and ξ are tangent to M^{2n}. From (11) and (12) we obtain $K = 0$ and $a = 0$.

From (25) and (27) we have

$$\overline{\varphi}(\overline{N}) = 0 \qquad \text{and} \qquad \overline{\varphi}\overline{\xi} = 0. \tag{54}$$

From (28) it follows that $b(\overline{\xi}) = 0$; or, $g(\overline{N}, \overline{\xi}) = 0$. So since n = 1, $\dim(T(M^{2n})) = 2$ and $\{\overline{\xi}, \overline{N}\}$ is a basis of $T(M^{2n})$. Hence for any $\overline{X} \in T(M^{2n})$, $\overline{X} = c_1\overline{\xi} + c_2\overline{N}$ and from (54), $\overline{\varphi} = 0$, which means that M^{2n} is anti-invariant.

Thus, we can state the following:

Theorem 5. If M^{2n} be a hypersurface of a Kenmotsu manifold M^{2n+1} endowed with a quarter-symmetric non-metric connection, then M^{2n} is anti-invariant if and only if M^{2n+1} is a 3-dimensional Kenmotsu manifold and $\varphi(N)$ and ξ are tangent to M^{2n}.

Also using (5) in (37) we have

$$\overline{\nabla}_{\overline{Y}}\overline{\xi} = \overline{X} - \overline{\eta}(\overline{X})\overline{\xi} - a^2\overline{\varphi}\overline{X} + aH\overline{X}, \tag{55}$$

and

$$h(\overline{X}, \overline{\xi}) = -\overline{X}a - b(\overline{X}) - a\overline{\eta}(\overline{X}). \tag{56}$$

Now if ξ is tangent to M^{2n} and $h = 0$, then from (55) and (56) we get

$$\overline{\nabla}_{\overline{X}}\overline{\xi} = \overline{X} - \eta(\overline{X})\overline{\xi}$$

and $b(\overline{X}) = 0$. Then we can state the following:

Theorem 6. If M^{2n} is totally geodesic hypersurface of a Kenmotsu manifold M^{2n+1} endowed with a quarter-symmetric non-metric connection and ξ is tangent to M^{2n}, then M^{2n} is invariant.

EQUATIONS OF GAUSS AND CODAZZI

We denote

$$K(X, Y)Z = \nabla^*_X\nabla^*_Y Z - \nabla^*_Y\nabla^*_X Z - \nabla^*_{[X,Y]}Z,$$

the curvature tensor of M^{2n+1} with respect to the Levi-Civita connection ∇^* and by $\overline{K}(\overline{X}, \overline{Y})\overline{Z} = \overline{\nabla}^*_{\overline{X}}\overline{\nabla}^*_{\overline{Y}}\overline{Z} - \overline{\nabla}^*_{\overline{Y}}\overline{\nabla}^*_{\overline{X}}\overline{Z} - \overline{\nabla}^*_{[\overline{X},\overline{Y}]}\overline{Z}$, of M^{2n} with respect to $\overline{\nabla}^*$.

Then the equation of Gauss is given by
$K'(\overline{X}, \overline{Y}, \overline{Z}, \overline{W}) = \overline{K}'(\overline{X}, \overline{Y}, \overline{Z}, \overline{W}) + h(\overline{X}, \overline{W})h(\overline{Y}, \overline{Z}) - h(\overline{Y}, \overline{W})h(\overline{X}, \overline{Z})$,
where $\overline{K}'(\overline{X}, \overline{Y}, \overline{Z}, \overline{W}) = g(K(X, Y)Z, W)$ and $\overline{K}'(\overline{X}, \overline{Y}, \overline{Z}, \overline{W}) = \overline{g}(\overline{K}(\overline{X}, \overline{Y}, \overline{Z}), \overline{W})$;
and the equation of Codazzi is given by $\overline{K}'(X, Y, Z, N) = (\overline{\nabla}^*_{\overline{X}}h)(\overline{Y}, \overline{Z}) - (\overline{\nabla}^*_{\overline{Y}}h)(\overline{X}, \overline{Z})$.

We shall now find the equation of Gauss-Codazzi with respect to the quarter-symmetric non-metric connection. The curvature tensor of the quarter-symmetric non-metric connection ∇ of M^{2n+1} and $\overline{\nabla}$ is by definition,

$$R'(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z \tag{57}$$

and

$$\overline{R}'(\overline{X},\overline{Y})\overline{Z} = \overline{\nabla}_{\overline{X}}\overline{\nabla}_{\overline{Y}}\overline{Z} - \overline{\nabla}_{\overline{Y}}\overline{\nabla}_{\overline{X}}\overline{Z} - \overline{\nabla}_{[\overline{X},\overline{Y}]}\overline{Z}. \tag{58}$$

Putting $X = \overline{X}$, $Y = \overline{Y}$ and $Z = \overline{Z}$ in (57) and using (18) we obtain

$$\begin{aligned} R'(\overline{X},\overline{Y})\overline{Z} &= \overline{\nabla}_{\overline{X}}\nabla_{\overline{Y}}\overline{Z} + (h(\overline{X},\nabla_{\overline{Y}}\overline{Z}) + \overline{\eta}(\nabla_{\overline{Y}}\overline{Z})b(\overline{X}))N \\ &+\nabla_{\overline{X}}(h(\overline{Z},\overline{Y}) + \overline{\eta}(\overline{Z})b(\overline{Y}))N + (h(\overline{Y},\overline{Z}) + \overline{\eta}(\overline{Z})b(\overline{Y}))\nabla_{\overline{X}}N \\ &-\overline{\nabla}_{\overline{Y}}\nabla_{\overline{X}}\overline{Z} - (h(\overline{Y},\nabla_{\overline{X}}\overline{Z}) + \overline{\eta}(\nabla_{\overline{X}}\overline{Z})b(\overline{Y}))N - \nabla_{\overline{Y}}(h(\overline{Z},\overline{X}) \\ &+\overline{\eta}(\overline{Z})b(\overline{Y}))N - (h(\overline{X},\overline{Z}) + \overline{\eta}(\overline{Z})b(\overline{X}))\nabla_{\overline{Y}}N - \overline{\nabla}_{[\overline{X},\overline{Y}]}\overline{Z} \\ &-(h([\overline{X},\overline{Y}],\overline{Z}) - \overline{\eta}(\overline{Z})b([\overline{X},\overline{Y}]))N, \end{aligned} \tag{59}$$

Taking account of (21), we have

$$\begin{aligned} R'(\overline{X},\overline{Y})\overline{Z} &= \overline{R}'(\overline{X},\overline{Y})\overline{Z} + h(\overline{Y},\overline{Z})(-H\overline{X} + a\overline{\varphi}\overline{X} + ab(\overline{X})N) \\ &-h(\overline{X},\overline{Z})(-H\overline{Y} + a\overline{\varphi}\overline{Y} + ab(\overline{Y})N) + \{(\overline{\nabla}_{\overline{X}}h)(\overline{Y},\overline{Z}) - (\overline{\nabla}_{\overline{Y}}h)(\overline{X},\overline{Z}) \\ &+h(\overline{\eta}(\overline{Y})\overline{\varphi}\overline{X} - \overline{\eta}(\overline{X})\overline{\varphi}\overline{Y},\overline{Z})\}N + \{(\overline{\nabla}_{\overline{X}}\overline{\eta})(\overline{Z})b(\overline{Y}) - (\overline{\nabla}_{Y}\overline{\eta})(\overline{Z})b(\overline{X}) \\ &+\overline{\eta}(\overline{Z})(\nabla_{\overline{X}}\overline{b}(\overline{Y}) - \nabla_{\overline{Y}}\overline{b}(\overline{X}) - b([\overline{X},\overline{Y}]))\}N + \overline{\eta}(\overline{Z})\{b(\overline{Y})(-H\overline{X} + a\overline{\varphi}\overline{X}) \\ &-b(\overline{X})(-H\overline{Y} + a\overline{\varphi}\overline{Y})\}. \end{aligned} \tag{60}$$

Now putting $R'(\overline{X},\overline{Y},\overline{Z},\overline{W}) = \overline{g}(R'(\overline{X},\overline{Y},\overline{Z}),\overline{W})$ and $R(\overline{X},\overline{Y},\overline{Z},\overline{W}) = \overline{g}(\overline{R}'(\overline{X},\overline{Y},\overline{Z}),\overline{W})$ eq. (60) can be expressed as

$$\begin{aligned} R'(\overline{X},\overline{Y},\overline{Z},\overline{W}) &= \overline{R}'(\overline{X},\overline{Y},\overline{Z},\overline{W}) - h(\overline{Y},\overline{Z})h(\overline{X},\overline{W}) \\ &+ah(\overline{Y},\overline{Z})g(\overline{\varphi}\overline{X},\overline{W}) + h(\overline{X},\overline{Z})h(\overline{Y},\overline{W}) - ah(\overline{X},\overline{Z})g(\overline{\varphi}\overline{Y},\overline{W}) \\ &-\overline{\eta}(\overline{Z})b(\overline{Y})h(\overline{X},\overline{W}) + a\overline{\eta}(\overline{Z})b(\overline{Y})g(\overline{\varphi}\overline{X},\overline{W}) + \overline{\eta}(\overline{Z})b(\overline{X})h(\overline{Y},\overline{W}) \\ &-a\overline{\eta}(\overline{Z})b(\overline{X})g(\overline{\varphi}\overline{Y},\overline{W}), \end{aligned} \tag{61}$$

and

$$
\begin{aligned}
R'(\overline{X},\overline{Y},\overline{Z},N) &= \overline{R}'(\overline{X},\overline{Y},\overline{Z},N) + ah(\overline{Y},\overline{Z})b(\overline{X}) \\
&-ah(\overline{X},\overline{Z})b(\overline{Y}) + (\overline{\nabla}_{\overline{X}}h)(\overline{Y},\overline{Z}) - (\overline{\nabla}_{Y}h)(\overline{X},\overline{Z}) \\
&+h(\overline{\eta}(\overline{Y})\overline{\varphi}\overline{X} - \overline{\eta}(\overline{X})\overline{\varphi}\overline{Y},\overline{Z}) + (\overline{\nabla}_{\overline{X}}\overline{\eta})(\overline{Z})b(\overline{Y}) \\
&-(\overline{\nabla}_{\overline{Y}}\overline{\eta})(\overline{Z})b(\overline{X}) + \overline{\eta}(\overline{Z})(\overline{\nabla}_{\overline{X}}b(\overline{Y}) - \overline{\nabla}_{\overline{Y}}b(\overline{X}) - b([\overline{X},\overline{Y}])).
\end{aligned} \tag{62}
$$

Equations (61) and (62) are the equations of Gauss and Codazzi with respect to the quarter-symmetric non-metric connection, respectively.

Now if we put $R' = 0$ and $h = 0$ in (61) we get

$$\overline{R}'(\overline{X},\overline{Y},\overline{Z},\overline{W}) = a\overline{\eta}(\overline{Z})\{g(\overline{\phi X},\overline{W})b(\overline{Y}) - g(\overline{\phi Y},\overline{W})b(\overline{X})\}. \tag{63}$$

From (63) we get the following:

Theorem 7. Let M^{2n} be a totally geodesic hypersurface of M^{2n+1} with vanishing curvature tensor with respect to the quarter-symmetric non-metric connection. Then M^{2n} is of vanishing curvature tensor if and only ifξ is tangent to M^{2n}

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فوسطوح لمنطويات كينموتسو مزودة بصلة ربع – متناظرة غير مقاسية

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خلاصة

الغرض من هذا البحث هو أن نعرّف صلة ربع – متناظرة غير مقاسية لمنطوي كينموتسو. ثم ندرس فوسطوح غير لا متغيرة ولا متغيرة متخالفة لمنطوي كينموتسو ومزودة بصلة ربع متناظرة وغير مقاسية. وأخيراً، نحصل على معادلات غاوس وكودازي بالنسبة إلى الصلة ربع – المتناظرة وغير المقاسية.