Kuwait J. Sci. Eng. **39** (1A) pp. 19-41, 2012

On the method of determination of a developable timelike ruled surface

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ABSTRACT

A method of determination of a developable ruled surface is given by Köse. In this paper, first we have obtained the parametric equations of dual hyperbolic and Lorentzian unit spheres (DHUS) $H_0^{2+}$and (DLUS) S_1^2 and then presented a method of determination of a developable time-like ruled surface. We have given an application about two-component Wadati-Konno-Ichikawa (WKI) equation. Inextensible flows of developable surfaces are given. In the final part of this paper, we have discussed inextensible flows of a developable time-like ruled surface.

Keywords: Developable time-like ruled surface; dual hyperbolic and lorentzian unit spheres; linear differential equation.

INTRODUCTION

Ruled surfaces and especially developable surfaces are used in designing cars, ships, manufacturing products and some other areas such as motion analysis, simulation of rigid body and model-based object recognition systems. After 1960, Coons, Ferguson, Gordon, Bezier, and others developed new surface definitions. However, modern surface modeling system still includes ruled surfaces.

Minkowski space is named after the German mathematician Hermann Minkowski, who around 1907 realized that the theory of special relativity (previously developed by Einstein) could be elegantly described using a four-dimensional spacetime, which combines the dimension of time with the three dimensions of space. There are a number of papers dealing with ruled surfaces in Lorentz-Minkowski space (Çöken *et al.*, 2008; Kasap *et al.*, 2005; Kazaz *et al.*, 2008; Köse, 1999 and Turgut & Hacisalihoğlu, 1997). Motion of plane curves in the Minkowski space M^3 was also investigated by Gürses (1998).

Furthermore, motion of curves in the Minkowski 3-space has been considered by several authors (Study, 1903; Köse, 1982 and Yaylı *et al.*, 2002). Considering that the normal speed of the curve can be given by $-\kappa_s$, one finds that u satisfies the celebrated Wadati-Konno-Ichikawa (WKI) equation[1] $u_t = \left[u_{xx}.(1+u_x^2)^{-3/2}\right]_x$ where $u = u(x,t)$, called the one-parameter family of the curve, is the inextensible evolution of the curve. It is shown that two-component WKI equation, i.e. a generalization of the well-known WKI equation obtained from the motion of space curves in Euclidean geometry. Moreover, motions of curves in the four-dimensional Euclidean and Minkowski space are discussed; it has been pointed out in Zhang & Hou (2007) that the three component WKI equation and its hyperbolic type arise from certain curve motion flows.

Dual numbers were introduced by W.K. Clifford (1845-79) as a tool for his geometrical investigations. After him Study (1903) used dual numbers and dual vectors in his research on the geometry of lines and kinematics (Blaschke, 1930). He devoted special attention to the representation of directed lines by dual unit vectors and defined the mapping that is known by his name. There exists a one-to-one correspondence between the vectors of dual unit sphere S^2 and the directed lines of space of lines $\mathbf{R}^3$ (Study, 1903). Therefore, the motion locus of a straight line in $\mathbf{R}^3$ can be described by that of a point on the surface of dual unit sphere S^2 in dual space $\mathbf{D}^3$. Then a ruled surface in $\mathbf{R}^3$ corresponds to a unique dual curve on the surface of S^2 (Guegenheimer, 1977). It was also studied on the dual spherical motions by Köse (1982). If we take the Minkowski 3-space R_1^3 instead of R^3 the E. Study mapping can be stated as follows. The dual time-like and space-like unit vectors of dual hyperbolic and Lorentzian unit spheres H_0^{2+} and S_1^2 at the dual Lorentzian space D_1^3 are in one-to-one correspondence with the directed time-like and space-like lines of the space of Lorentzian lines R_1^3, respectively (Uğurlu & Çalişkan, 1996). Then a differentiable curve on H_0^{2+} corresponds to a time-like ruled surface in R_1^3. Similarly the time-like (resp. space-like) curve on S_1^2 corresponds to any space-like (resp. time-like) ruled surface in R_1^3.

A dual number has the form $x + \varepsilon x^*$ where x and x^* are real numbers and $\varepsilon = (0,\, 1)$ is the dual unit with the property that $\varepsilon^2 = 0$. The set of all dual numbers forms a commutative ring over the real number field and is denoted by D.

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[1] WKI equation obtained from the motion of space curve $\phi\,(s,\, t)$ in Euclidean space, the subscript s is the arc-length parameter, κ and τ are respectively the curvature and torsion of the curve, which satisfy the vector equation $\kappa_t + \kappa_{sss} + \frac{3}{2}\kappa^2\kappa_s = 0$ where $\kappa = (k\cos\theta,\, k\sin\theta)$, $\quad \theta = \int^{s} \tau(t,\, s')ds'$.

The dual vector space D^3 (or D-Module) can be written as

$$\mathbf{D}^3 = \{(A_1,\ A_2,\ A_3)\ :\ A_1,\ A_2,\ A_3 \in \mathbf{D}\}.$$

The inner-product of two dual vectors $A, B \in D^3$ is defined as,

$$\begin{array}{lccl} \langle , \rangle\ : & \mathbf{D}^3 \times \mathbf{D}^3 & \rightarrow & \mathbf{D} \\ & (A,\ B) & \rightarrow & \langle A, B\rangle = \langle x, y\rangle + \varepsilon(\langle x^*, y\rangle + \langle x, y^*\rangle) \end{array}$$

Given a dual vector $A = x + \varepsilon x^*$, the norm of A is

$$\|A\| = (\langle A, A\rangle)^{\frac{1}{2}} = \|x\| + \varepsilon\frac{\langle x,\ x^*\rangle}{\|x\|},\ x \neq 0$$

The cross-product of two dual vectors $A,\ B \in D^3$ is defined as ,

$$\begin{array}{lccl} \wedge : & D^3 \times D^3 & \rightarrow & D^3 \\ & (A,\ B) & \rightarrow & A \wedge B = x \wedge y + \varepsilon(x \wedge y^* + x^* \wedge y) \end{array}$$

where $\wedge$ stands for the Lorentzian cross-product in R^3 given by

$$x \wedge y = \sum_{i=1}^{3} \varepsilon_i \det(e_i,\ x,\ y)e_i,\ \ \varepsilon_i = \langle e_i, e_i\rangle. \tag{1}$$

Detailed information on Lorentzian cross-product on R^3_1 can be found in (Akutagawa & Nishikawa, 1990).

The Lorentzian inner-product of two dual vectors $A = x + \varepsilon x^*$ and $B = y + \varepsilon y^*,\quad x,\ y \in R^3$ is given as

$$\langle A, B\rangle = \langle x, y\rangle + \varepsilon(\langle x^*, y\rangle + \langle x, y^*\rangle)$$

with the signature $(+,\ +,\ -)$ in R^3_1. The D-module D^3 with the Lorentzian inner-product is named as the semi-dual space D^3_1 (Uğurlu & Çalişkan, 1996). Similar to O'Neill (1983), given a dual vector A in D^3_1, if $\langle x, x\rangle < 0$, A is called a time-like vector; if $\langle x, x\rangle > 0$, A is called space-like vector; and if $\langle x, x\rangle = 0$, A is called null vector. A smooth curve on the semi-dual space D^3_1 is said to be time-like, space-like or null if its tangent vectors are time-like, space-like or null, respectively (Uğurlu & Çalişkan, 1996). Observe that, a time-like curve corresponds to the path of an observer moving at less than the speed of light while the space-like curves are faster and the null curves are equal to the speed of light (Inoguchi, 1998).

The dual-semihyperbolic space (the dual hyperbolic unit sphere) H_0^{2+} and the dual-semisphere (or the dual Lorentzian unit sphere) S_1^2 can be given as

$$H_0^{2+} = \left\{ \hat{x}(t) = x(t) + \varepsilon x^*(t) | \, \|\hat{x}\| = (1,0); x, x^* \in \mathbf{R}_1^3, \text{and where } x \text{ timelike vector} \right\}$$

and

$$S_1^2 = \left\{ \hat{x}(t) = x(t) + \varepsilon x^*(t) \Big| \, \| \hat{x}\| = (1,0), x, x^* \in \mathbf{R}_1^3, \text{and where } x \text{ spacelike vector} \right\}.$$

Study (1903) mapping in D_1^3 can be stated as "there is a one-to-one correspondence between dual vectors of dual-hyperquadrics and directed straight lines in R_1^3" (Study, 1903). As a result, we can say that in D_1^3,

(1) The dual timelike unit vectors of dual hyperbolic unit sphere H_0^{2+} are in one-to-one correspondence with directed timelike straight lines in R_1^3 .

(2) The dual spacelike unit vectors of dual-Lorentzian unit sphere S_1^2 are in one-to-one correspondence with directed spacelike straight lines in $\mathbf{R}_1^3$ (Uğurlu & Çalişkan, 1996).

Now, we give some properties of $\wedge$ without proof:

Letting $A, B, C \in D_1^3$, it is straight forward to see the following. $A \wedge B = 0 \Leftrightarrow A$ and B are linearly dependent; $A \wedge B = -B \wedge A$; $\langle A \wedge B, A\rangle = \langle A \wedge B, B\rangle = 0$; $\langle A \wedge B, C\rangle = \langle B \wedge C, A\rangle$; A or B is time-like $\Rightarrow$ $A \wedge B$ is space-like and $\langle A \wedge B, A \wedge B\rangle = \langle A, B\rangle^2 - \langle A, A\rangle\langle B, B\rangle$ (Özdemir & Ergin, 2006).

In this study, using the methods given in Köse (1999), we determine a method of determination of a developable time-like ruled surface on both dual hyperbolic unit spheres and Lorentzian unit spheres and obtain a linear differential equation of the first order, which can also provide an application in physics.

THE RELATION BETWEEN *K* AND Δ

Let L be any line, x be the direction vector of L and p be the position vector of any point on L. It is known that a dual vector representation allows us the Plucker vectors x and $p \wedge x$. If we denote the dual vector by $\hat{x}(t)$, then we can write

$$\hat{x}(t) = x + \varepsilon(p \wedge x) = x + \varepsilon x^*$$

where ε is called the dual unit such that $\varepsilon^2 = 0$. Let t be a parameter and $\hat{x}(t)$ be the dual vector function. We can write

$$\hat{x}(t) = x(t) + \varepsilon(p(t) \wedge x(t)) = x(t) + \varepsilon x^*(t)$$

The dual vector function $\hat{x}(t)$ defines a ruled surface $\mathbf{m}(t, u) = p(t) + ux(t)$. It is known that the dual time-like or space-like unit vector $\hat{x}(t)$ is a differentiable curve on a dual hyperbolic unit sphere H_0^{2+}or dual Lorentzian unit sphere S_1^2, respectively.

By Study map, if $\hat{x}(t)$is time-like, then it corresponds to time-like ruled surface on R_1^3; if $\hat{x}(t)$ is spacelike, it corresponds to either time-like or space-like ruled surface on R_1^3 (Uğurlu & Çalişkan, 1996). The dual arc-length of ruled surfaces $\hat{x}(t)$is defined by

$$\hat{s}(t) = \int_0^t \left\| \frac{d\hat{x}}{dt} \right\| dt$$

The integrant of this equation ($\hat{s}(t)$) is the dual speed, $\hat{\delta}$ of $\hat{x}(t)$and is

$$\hat{\delta} = \left\| \frac{dx}{dt} \right\| \left[1 + \varepsilon \left(\frac{\left\langle \frac{dx}{dt}, \frac{dp}{dt} \wedge x \right\rangle}{\left\| \frac{dx}{dt} \right\|^2} \right) \right] = \left\| \frac{dx}{dt} \right\| (1 + \varepsilon \Delta).$$

The curvature function

$$\Delta = \frac{\left\langle \frac{dx}{dt}, \frac{dp}{dt} \wedge x \right\rangle}{\left\| \frac{dx}{dt} \right\|^2} = \frac{\left\langle \frac{dx}{dt}, \frac{dx^*}{dt} \right\rangle}{\left\| \frac{dx}{dt} \right\|^2} \tag{2}$$

is the well-known distribution parameter of the time-like ruled surface.

Given a time-like ruled surface $\mathbf{m}(t, u)$, if Δ is the distribution parameter and K is the Gaussian curvature of $\mathbf{m}(t, u) = p(t) + ux(t)$, then the relation between Δ and K is as follows:

$$\mathbf{m}_t = p'(t) + ux'(t), \ \mathbf{m}_u \ = x(t) \ \text{and} \ \mathbf{m}_{uu} = 0 \tag{3}$$

and

$$EG - F^2 = \|\delta'\|^2 \left(\Delta^2 + u^2\right) \tag{4}$$

The Gaussian curvature of a surface is given by

$$K = \frac{l.n - m^2}{EG - F^2} \tag{5}$$

where $E =< \mathbf{m}_t, \mathbf{m}_t >$, $F =< \mathbf{m}_t, \mathbf{m_u} >$ and $G =< \mathbf{m}_u, \mathbf{m}_u >$ are the first fundamental coefficients and

$$l = \left\langle \mathbf{m}_{tt}, \frac{\mathbf{m}_t \wedge \mathbf{m}_u}{\|\mathbf{m}_t \wedge \mathbf{m}_u\|} \right\rangle, \; m = \left\langle \mathbf{m}_{tu}, \frac{\mathbf{m}_t \wedge \mathbf{m}_u}{\|\mathbf{m}_t \wedge \mathbf{m}_u\|} \right\rangle \text{ and } n = \left\langle \mathbf{m}_{uu}, \frac{\mathbf{m}_t \wedge \mathbf{m}_u}{\|\mathbf{m}_t \wedge \mathbf{m}_u\|} \right\rangle$$

are the second fundamental coefficients. Then the relation between the Gaussian curvature K and the distribution parameter Δ of a time-like ruled surface $\mathbf{m}(t, u)$ is given by (Kasap *et al.*, 2005):

$$K = \frac{l.n - m^2}{EG - F^2} = -\frac{\Delta^2}{(\Delta^2 + u^2)^2} \quad . \tag{6}$$

If K is zero everywhere, then Δ is zero everywhere. In this case, the time-like ruled surface is called developable. If $\hat{x}$ is a space-like unit vector, then

$$K = -\frac{\Delta^2}{(\Delta^2 - u^2)^2}. \tag{7}$$

A METHOD OF THE DETERMINATION OF DEVELOPABLE TIMELIKE RULED SURFACE ON H_0^{2+}

Given any point $\hat{x}$ on the dual hyperbolic unit sphere H_0^{2+}, the dual coordinates of $\hat{x}(t)$can be given as

$$\hat{x}_i = x_i + \varepsilon x_i^*, \; i = 1, 2, 3.$$

It is known that, the dual hyperbolic radian angle between two dual time-like vectors in the Lorentzian plane L^2 is $\hat{\phi} = \phi + \varepsilon \phi^*$ (Birman & Nomizu, 1984). Let $\hat{x}(t)$be a time-like ruled surface and $\hat{a}_1$, $\hat{a}_2$, $\hat{a}_3$ be the Blashcke, (1930) trihedron of striction line of $\hat{x}(t)$. Here $\hat{x} = \hat{a}_1$ is time-like, $\hat{a}_2 = \frac{\hat{a}_1}{q}$, $q = \sqrt{|< \hat{a}_1', \hat{a}_1' >|}$ and $\hat{a}_3 = -\hat{a}_1 \wedge \hat{a}_2$ are space-like vectors. Given two unit dual vectors $\hat{a}_1$ and $\hat{b}$, as in Fig. 3.1,

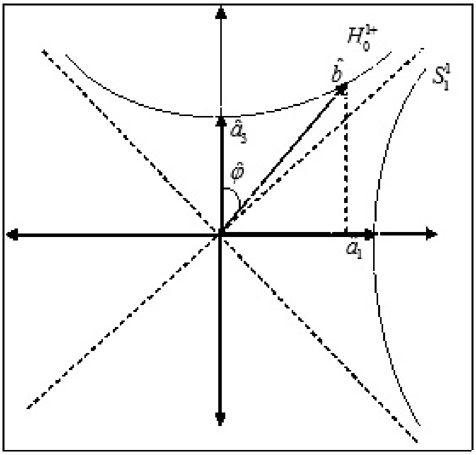

Fig. 3.1

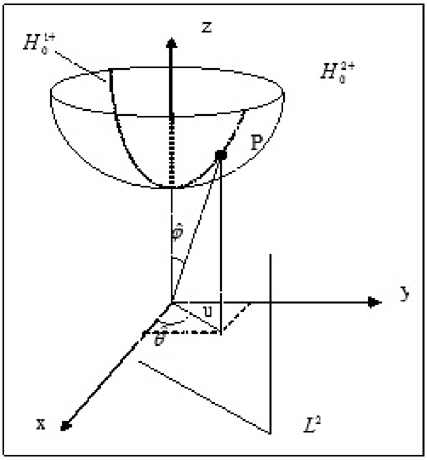

Fig. 3.2

if we denote the dual hyperbolic angle between $\hat{a}_1$ and $\hat{b}$ by $\hat{\phi} = \phi + \varepsilon\phi^*$, we can write $\hat{b}(t) = \sinh\hat{\phi}\,\hat{a}_1 + \cosh\hat{\phi}\,\hat{a}_3$ (Birman & Nomizu, 1984). The line $\hat{b}(t)$ for the time-like ruled surface corresponds to the spherical curvature center on the dual hyperbolic unit sphere (Uğurlu & Çalişkan, 1996).

Let $P = (\hat{x}_1, \hat{x}_2, \hat{x}_3)$ be a point on DHUS H_0^{2+}. The intersection of H_0^{2+} and the L^2 plane, passing through the point P and the origin, is the Lorentzian unit circle in L^2(Fig. 3.2). Let's denote the point on the unit circle as P_1. Clearly, $P_1 = \left(\sinh\hat{\phi},\ \cosh\hat{\phi}\right)$. Thus, we can write

$$\begin{aligned} \hat{x}_1 &= u\cos\hat{\theta} \\ \hat{x}_2 &= u\sin\hat{\theta} \end{aligned} \tag{8}$$

where $\hat{\theta}$ is the angle of real unit sphere. On the other hand, $\hat{x}_3$ can be given in terms of $\hat{\phi}$ as $\hat{x}_3 = \cosh\hat{\phi}$. Since $u = \sinh\hat{\phi}$ in L^2, we can finalize that

$$\begin{aligned} \hat{x}_1 &= \cos\hat{\theta}\sinh\hat{\phi} \\ \hat{x}_2 &= \sin\hat{\theta}\sinh\hat{\phi} \\ \hat{x}_3 &= \cosh\hat{\phi} \end{aligned} \tag{9}$$

where $\hat{\theta} = \theta + \varepsilon\theta^*$ and $\hat{\phi} = \hat{\phi} + \varepsilon\phi^*$ are dual angles.

As a result, we obtain the parametric equation of time-like ruled surface on H_0^{2+} by using the Study map. Now let f be a differentiable function. Then the Taylor series generated by f is

$$f(\hat{x}) = f(x + \varepsilon x^*) = f(x) + \varepsilon x^* f'(x),$$

where $f'(x)$ is the derivative of f (Hacısalihoğlu, 1983).

Now, let's write $\hat{x}_1$, $\hat{x}_2$ and $\hat{x}_3$ in terms of the real and dual parts. Using the Taylor series expansion ($\varepsilon^2 = \varepsilon^3 = ... = 0$) from (9), we obtain

$$\begin{aligned} \hat{x}_1 &= \cos(\theta + \varepsilon\theta^*) \sinh(\phi + \varepsilon\phi^*) \\ &= \cos\theta \sinh\phi + \varepsilon[\phi^* \cos\theta \cosh\phi - \theta^* \sin\theta \sinh\phi] \end{aligned} \tag{10}$$

and

$$\begin{aligned} \hat{x}_2 &= \sin(\theta + \varepsilon\theta^*) \sinh(\phi + \varepsilon\phi^*) \\ &= \sin\theta \sinh\phi + \varepsilon[\phi^* \sin\theta \cosh\phi + \theta^* \cos\theta \sinh\phi] \end{aligned} \tag{11}$$

and

$$\hat{x}_3 = \cosh(\phi + \varepsilon\phi^*) = \cosh\phi + \varepsilon\phi^* \sinh\phi \tag{12}$$

From (10), (11) and (12), we obtain the real parts as

$$\begin{aligned} x_1 &= \cos\theta \sinh\phi \\ x_2 &= \sin\theta \sinh\phi \\ x_3 &= \cosh\phi \end{aligned} \tag{13}$$

and the dual parts as

$$\begin{aligned} x_1^* &= \phi^* \cos\theta \cosh\phi - \theta^* \sin\theta \sinh\phi \\ x_2^* &= \phi^* \sin\theta \cosh\phi + \theta^* \cos\theta \sinh\phi \\ x_3^* &= \phi^* \sinh\phi \end{aligned} \tag{14}$$

Hence, a dual curve may be represented by

$$\begin{aligned} \hat{x}(t) =\ & (\cos\theta(t) \sinh\phi(t),\ \sin\theta(t) \sinh\phi(t),\ \cosh\phi(t)) \\ & +\varepsilon[\phi^*(t) \cos\theta(t) \cosh\phi(t) - \theta^*(t) \sin\theta(t) \sinh\phi(t), \\ & \phi^*(t) \sin\theta(t) \cosh\phi(t) + \theta^*(t) \cos\theta(t) \sinh\phi(t),\ \phi^*(t) \sinh\phi(t)] \end{aligned} \tag{15}$$

In accordance with the Study map, the dual curve on a DHUS corresponds to the time-like ruled surface

$$\mathbf{m}(t, u) = p(t) + ux(t) \tag{16}$$

Since $x^*(t) = p(t) \wedge x(t)$, we have

$$x^*(t) = p(t) \wedge x(t) = -\begin{vmatrix} e_1 & e_2 & -e_3 \\ P_1 & P_2 & P_3 \\ \cos\theta(t)\sinh\phi(t) & \sin\theta(t)\sinh\phi(t) & \cosh\phi(t) \end{vmatrix}$$

where $P_1,\ P_2$ and P_3 are the coordinates of $p(t)$. Thus,

$$\begin{aligned} x^*(t) \quad &= (-P_2\cosh\phi(t) + P_3\sin\theta(t)\sinh\phi(t), \\ &\quad P_1\cosh\phi(t) - P_3\cos\theta(t)\sinh\phi(t), \\ &\quad P_1\sin\theta(t)\sinh\phi(t) - P_2\cos\theta(t)\sinh\phi(t)) \end{aligned} \tag{17}$$

Using this in (15) and (17), we get

$$\hat{x}(t) = x(t) + \varepsilon(p(t) \wedge x(t))$$

and

$$\begin{aligned} -P_2\cosh\phi + P_3\sin\theta\sinh\phi &= \phi^*\cos\theta\cosh\phi - \theta^*\sin\theta\sinh\phi \\ P_1\ \cosh\phi - P_3\ \cos\theta\ \sinh\phi &= \phi^*\sin\theta\cosh\phi + \theta^*\cos\theta\sinh\phi \\ P_1\sin\theta\sinh\phi - P_2\cos\theta\sinh\phi &= \varphi^*\sinh\phi \end{aligned} \tag{18}$$

The coefficient matrix with respect to $P_1,\ P_2,\ P_3$ of (18) is as follows

$$A = \begin{pmatrix} 0 & -\cosh\phi(t) & \sin\theta(t)\sinh\phi(t) \\ \cosh\phi(t) & 0 & -\cos\theta(t)\sinh\phi(t) \\ \sin\theta(t)\sinh\phi(t) & -\cos\theta(t)\sinh\phi(t) & 0 \end{pmatrix}$$

The rank of matrix A is 2. Hence the solution of (18) depends on a parameter. That is,

$$\begin{aligned} P_1 &= (P_3 + \theta^*)\ \cos\theta\tanh\phi + \phi^*\sin\theta \\ P_2 &= (P_3 + \theta^*)\ \sin\theta\tanh\phi - \phi^*\cos\theta \\ P_3 &= P_3 \end{aligned} \tag{19}$$

Since $P_3(t)$ can be chosen arbitrarily, then we may take $P_3(t) = -\theta^*(t)$. In this case, (19) becomes

$$
\begin{aligned}
P_1(t) &= \phi^*(t)\sin\theta(t) \\
P_2(t) &= -\phi^*(t)\cos\theta(t) \\
P_3(t) &= -\theta^*(t)
\end{aligned} \tag{20}
$$

The distribution parameter of time-like ruled surface given by (15) is

$$
\Delta = \frac{\left(\frac{d\theta}{dt}\right)^2 \phi^* \sinh\phi\cosh\phi + \frac{d\theta}{dt}\frac{d\theta^*}{dt}\sinh^2\phi + \frac{d\phi}{dt}\frac{d\phi^*}{dt}}{\sinh^2\phi\left(\frac{d\theta}{dt}\right)^2 + \left(\frac{d\phi}{dt}\right)^2} \tag{21}
$$

If this timelike ruled surface is developable, then $\Delta = 0$ and so (21) becomes

$$
\left(\frac{d\theta}{dt}\right)^2 \phi^* \sinh\phi\cosh\phi + \frac{d\theta}{dt}\frac{d\theta^*}{dt}\sinh^2\phi + \frac{d\phi}{dt}\frac{d\phi^*}{dt} = 0 \tag{22}
$$

If (22) is divided by both $-\sinh^2\phi$ and $\frac{d\phi^*}{dt}$, then we obtain

$$
\frac{d}{dt}(\coth\phi) - \frac{\left(\frac{d\theta}{dt}\right)^2 \phi^*}{\frac{d\phi^*}{dt}}(\coth\phi) - \frac{\frac{d\theta}{dt}\frac{d\theta^*}{dt}}{\frac{d\phi^*}{dt}} = 0 \tag{23}
$$

Setting

$$
y(t) = \coth\phi(t),\ A(t) = \frac{-\left(\frac{d\theta}{dt}\right)^2 \phi^*}{\frac{d\phi^*}{dt}} \text{ and } B(t) = -\frac{\frac{d\theta}{dt}\frac{d\theta^*}{dt}}{\frac{d\phi^*}{dt}}
$$

we are led to a linear homogeneous differential equation of the first order

$$
\frac{d}{dt}y(t) + A(t)y(t) + B(t) = 0. \tag{24}
$$

If we assume that $\theta(t)$ and $\phi(t)$ are both constant, (24) is identically zero, in other words, the developable timelike ruled surface $\hat{x}(t)$ is a cylinder.

Now we can ask a question:

"When a curve $p(t)$ is given, can we find a time-like developable ruled surface such that its base curve is the curve $p(t)$?".

The answer is YES. In fact from (20), we have

$$\frac{P_1}{P_2} = -\frac{\phi^* \sin\theta}{\phi^* \cos\theta} = -\tan\theta, \quad \phi^* = \pm\sqrt{P_1^2 + P_2^2} \ \text{ and } \ \theta^* = -P_3$$

Now we need to determine $\phi(t)$. The solution of the linear homogeneous differential equation (24) gives $\coth\phi(t)$. This solution includes an integral constant, therefore we have infinitely many developable ruled surfaces such that their base curve is $p(t)$.

It is to be noted that $\phi^*(t)$ has two values; when we use the minus sign, we obtain the reciprocal of the ruled surface $\hat{x}(t)$ obtained by using the plus sign for a given integral constant.

Example 1:

Consider the timelike curve

$$p(t) = (3t, \ 2t+1, \ t+1)$$

Then,

$$\tan\theta = -\frac{3t}{2t+1}, \quad \phi^* = \pm\sqrt{13t^2+4t+1} \ \text{ and } \ \theta^* = -t-1$$

and

$$\frac{d\theta^*}{dt} = -1, \ \frac{d\theta}{dt} = \frac{3}{13t^2+4t+1} \ \text{ and } \ \frac{d\phi^*}{dt} = \frac{13t+2}{\sqrt{13t^2+4t+1}}$$

Substituting these values into (24), we obtain

$$\frac{dy(t)}{dt} + \frac{\frac{18}{(2t+1)^4}(13t^2+4t+1)}{\left(1+\frac{9t^2}{(2t+1)^2}\right)^2(26t+4)}y(t) - \frac{\frac{18}{(2t+1)^4}(-t-1)\sqrt{13t^2+4t+1}}{\left(1+\frac{(t+1)^2}{(2t+1)^2}\right)(26t+4)} = 0$$

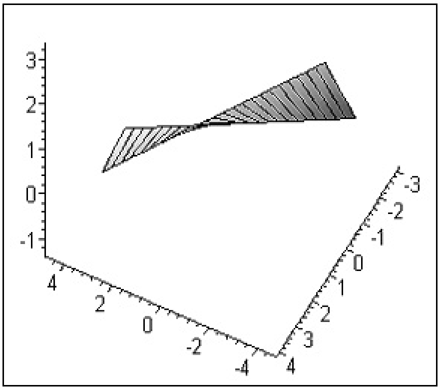

Fig. 3.3

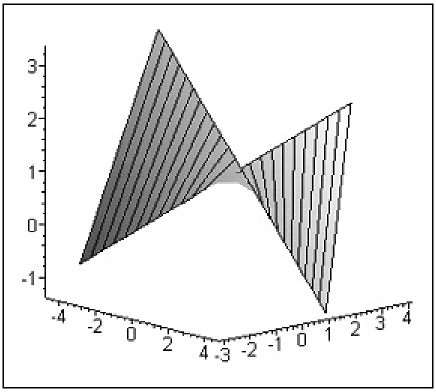

Fig. 3.4

The solution of this differential equation (using the mathematical program MAPLE) is

$$\begin{aligned} y(t) &= \coth\phi \\ &= \frac{\sqrt{13t^2+4t+1}}{13(13t+2)}\left(3\ln(13t^2+4t+1)+11\arctan(\frac{13t+2}{3})\right)+C \end{aligned} \tag{25}$$

Hence, the family of the developable time-like ruled surfaces is given by

$$\begin{aligned} \mathbf{m}(t,u) &= p(t)+ux(t) \\ &= (3t,\ 2t+1,\ t+1)+u\left(-\frac{P_2}{\phi^*}\sinh\phi,\ \frac{P_1}{\phi^*}\sinh\phi,\ \cosh\phi\right) \end{aligned}$$

where $x(t)$ is a timelike vector and $x(t)=\left(-\frac{P_2}{\phi^*}\sinh\phi,\ \frac{P_1}{\phi^*}\sinh\phi,\ \cosh\phi\right)$.

The graph of the developable timelike ruled surface, given by (25), for $C=0$ in domain $D\ :\ \{-3\le t\le 3,\ -4\le u\le 4\}$ is given in Fig. 3.3 and Fig. 3.4.

A METHOD OF THE DETERMINATION OF DEVELOPABLE TIME-LIKE RULED SURFACE ON S_1^2

Let $\hat{x}=x+\varepsilon x^*$ be a dual unit spacelike vector. Dual Lorentzian spherical geometry (constructed by dual spacelike unit vectors) is similar to Real Lorentzian spherical geometry (constructed by real spacelike unit vectors). We denote the Dual Lorentzian Unit Sphere (DLUS) as

$$S_1^2=\left\{\hat{x}(t)=x(t)+\varepsilon x^*(t)\,\middle|\,\|\hat{x}\|=(1,0),\ x,\ x^*\in\mathbf{R}^3,\ x \text{ spacelike}\right\}$$

where $ds^2=dx^2+dy^2-dz^2,\quad x_1^2+x_2^2-x_3^2=1$.

If dual unit spacelike vector $\hat{x} = x + \varepsilon x^*$ is connected to a parameter t, it draws a curve on DLUS S_1^2. $\hat{x}(t) = x(t) + \varepsilon x^*(t)$ dual curve corresponds to a ruled surface in the Lorentzian line spaces $\mathbf{R}_1^3$. Let this be a time-like ruled surface. This ruled surface corresponds to a timelike surface in the Lorentzian line spaces $\mathbf{R}_1^3$.

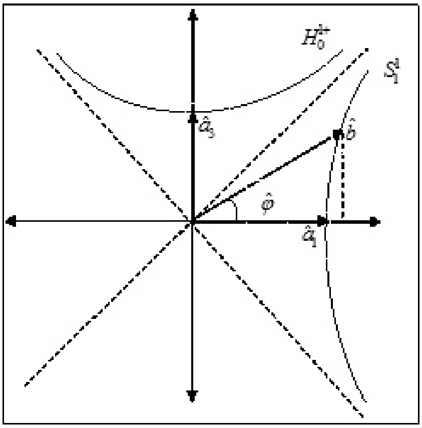

Fig. 4.1

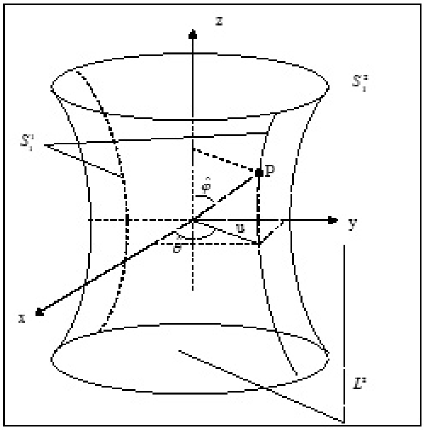

Fig. 4.2

Assume that the ruling curve is time-like. Let the dual hyperbolic central angle between $\hat{a}_1$ and $\hat{b}$ be $\hat{\phi} = \phi + \varepsilon\phi^*$. We can write

$$\hat{b}(t) = \cosh\hat{\phi}\,\hat{a}_1 + \sinh\hat{\phi}\,\hat{a}_3$$

Let $P = (\hat{x}_1, \hat{x}_2, \hat{x}_3)$ be a point on S_1^2. The intersection of $\mathbf{L}^2$ plane, passing through the origin and the point P and S_1^2 is a unit Lorentzian circle on $\mathbf{L}^2$ (Fig. 4.2). Denote the corresponding point of P on $\mathbf{L}^2$ plane as P_1. Then $P_1 = \left(\cosh\hat{\phi},\ \sinh\hat{\phi}\right)$. Hence, we can write

$$\begin{aligned} \hat{x}_1 &= u\cos\hat{\theta} \\ \hat{x}_2 &= u\sin\hat{\theta} \end{aligned} \tag{26}$$

where $\hat{\theta}$ is the angle of real unit sphere.

In this case, the first component of P_1 is $u = \cosh\hat{\phi}$ and the second component of P_1 is $\hat{x}_3 = \sinh\hat{\phi}$. Therefore, we have the parametric equation of S_1^2 as

$$\begin{aligned} \hat{x}_1 &= \cos\hat{\theta}\cosh\hat{\phi} \\ \hat{x}_2 &= \sin\hat{\theta}\cosh\hat{\phi} \\ \hat{x}_3 &= \sinh\hat{\phi} \end{aligned} \tag{27}$$

where $\hat{\theta} = \theta + \varepsilon\theta^*$ and $\hat{\phi} = \phi + \varepsilon\phi^*$ are dual angles. Thus, we constructed the parametric equation of a timelike ruled surface by using Study mapping of dual points on DLUS.

Now, let us write $\hat{x}_1$, $\hat{x}_2$ and $\hat{x}_3$ as dual and real parts. Let $\varepsilon^2 = \varepsilon^3 = ... = 0$. Using the Taylor series expansion, we have

$$\begin{aligned} \hat{x}_1 &= \cos(\theta + \varepsilon\theta^*)\ \cosh(\phi + \varepsilon\phi^*) \\ &= \cos\theta\cosh\phi + \varepsilon[\phi^*\cos\theta\sinh\phi - \theta^*\sin\theta\cosh\phi] \end{aligned} \tag{28}$$

and

$$\begin{aligned} \hat{x}_2 &= \sin(\theta + \varepsilon\theta^*)\ \cosh(\phi + \varepsilon\phi^*) \\ &= \sin\theta\cosh\phi + \varepsilon[\theta^*\cos\theta\cosh\phi + \phi^*\sin\theta\sinh\phi] \end{aligned} \tag{29}$$

and

$$\hat{x}_3 = \sinh(\phi + \varepsilon\phi^*) = \sinh\phi + \varepsilon\phi^*\cosh\phi \tag{30}$$

From the equations (28), (29) and (30), we obtain the real parts

$$\begin{aligned} x_1 &= \cos\theta\cosh\phi \\ x_2 &= \sin\theta\cosh\phi \\ x_3 &= \sinh\phi \end{aligned} \tag{31}$$

and dual parts

$$\begin{aligned} x_1^* &= \phi^*\cos\theta\sinh\phi - \theta^*\sin\theta\cosh\phi \\ x_2^* &= \phi^*\sin\theta\sinh\phi + \theta^*\cos\theta\cosh\phi \\ x_3^* &= \phi^*\cosh\phi \end{aligned} \tag{32}$$

Hence, a dual curve may be represented by

$$\begin{aligned} \hat{x}(t) = \ & (\cos\theta(t)\cosh\phi(t),\ \sin\theta(t)\cosh\phi(t),\ \sinh\phi(t)) \\ & +\varepsilon(\phi^*(t)\cos\theta(t)\sinh\phi(t) - \theta^*(t)\sin\theta(t)\cosh\phi(t), \\ & \theta^*(t)\cos\theta(t)\cosh\phi(t) + \phi^*(t)\sin\theta(t)\sinh\phi(t), \\ & \phi^*(t)\cosh\phi(t)) \end{aligned} \tag{33}$$

In accordance with Study map, the dual curve on a DLUS corresponds to the ruled surface

$$\mathbf{m}(t,\, u) = p(t) + ux(t) \tag{34}$$

Since $x^*(t) = p(t) \wedge x(t)$, we have

$$x^*(t) = p(t) \wedge x(t) = -\begin{vmatrix} e_1 & e_2 & -e_3 \\ P_1 & P_2 & P_3 \\ \cos\theta(t)\cosh\phi(t) & \sin\theta(t)\cosh\phi(t) & \sinh\phi(t) \end{vmatrix}$$

where $P_1,\ P_2,\ P_3$ are the coordinates of $p(t)$. Then

$$\begin{aligned} x^*(t) = \quad & (-P_2\sinh\phi(t) + P_3\sin\theta(t)\cosh\phi(t), \\ & P_1\sinh\phi(t) - P_3\cos\theta(t)\cosh\phi(t), \\ & P_1\sin\theta(t)\cosh\phi(t) - P_2\cos\theta(t)\cosh\phi(t)) \end{aligned} \tag{35}$$

Using this result in (33) and (35), we get

$$\begin{aligned} -P_2\sinh\phi + P_3\sin\theta\cosh\phi \quad &= \phi^*\cos\theta\sinh\phi - \theta^*\sin\theta\cosh\phi \\ P_1\sinh\phi - P_3\cos\theta\cosh\phi \quad &= \phi^*\sin\theta\sinh\phi + \theta^*\cos\theta\cosh\phi \\ P_1\sin\theta\cosh\phi - P_2\cos\theta\cosh\phi \quad &= \phi^*\cosh\phi \end{aligned} \tag{36}$$

The coefficient matrix with respect to $P_1,\ P_2,\ P_3$ of (36) is as follows

$$A = \begin{pmatrix} 0 & -\sinh\phi(t) & \sin\theta(t)\cosh\phi(t) \\ \sinh\phi(t) & 0 & -\cos\theta(t)\cosh\phi(t) \\ \sin\theta(t)\cosh\phi(t) & -\cos\theta(t)\cosh\phi(t) & 0 \end{pmatrix}$$

The rank of matrix A is 2. Hence the solution of (36) depends on a parameter. That is,

$$\begin{aligned} P_1 &= (P_3 + \theta^*)\ \cos\theta\coth\phi + \phi^*\sin\theta \\ P_2 &= (P_3 + \theta^*)\ \sin\theta\coth\phi - \phi^*\cos\theta \\ P_3 &= P_3 \end{aligned} \tag{37}$$

Since $P_3(t)$ can be chosen arbitrary, we may take $P_3 = -\theta^*(t)$. In this case, equation (37) reduces to

$$\begin{aligned} P_1 &= \phi^*(t)\sin\theta(t) \\ P_2 &= -\phi^*(t)\cos\theta(t) \\ P_3 &= -\theta^*(t) \end{aligned} \tag{38}$$

The distribution parameter of the timelike ruled surface given by (33) is

$$\Delta = \frac{\left(\dfrac{d\theta}{dt}\right)^2 \phi^* \cosh\phi \sinh\phi + \dfrac{d\theta}{dt}\dfrac{d\theta^*}{dt}\cosh^2\phi - \dfrac{d\phi}{dt}\dfrac{d\phi^*}{dt}}{\cosh^2\phi\left(\dfrac{d\theta}{dt}\right)^2 - \left(\dfrac{d\phi}{dt}\right)^2} \tag{39}$$

If this time-like ruled surface is developable, then $\Delta = 0$. So (39) becomes

$$\left(\frac{d\theta}{dt}\right)^2 \phi^* \cosh\phi \sinh\phi + \frac{d\theta}{dt}\frac{d\theta^*}{dt}\cosh^2\phi - \frac{d\phi}{dt}\frac{d\phi^*}{dt} = 0 \tag{40}$$

If (40) is divided by $-\cosh^2\phi$ and $\dfrac{d\phi^*}{dt}$, then we obtain

$$\frac{d}{dt}(\tanh\phi) - \frac{\left(\dfrac{d\theta}{dt}\right)^2 \phi^*}{\dfrac{d\phi^*}{dt}}(\tanh\phi) - \frac{\dfrac{d\theta}{dt}\dfrac{d\theta^*}{dt}}{\dfrac{d\phi^*}{dt}} = 0 \tag{41}$$

Setting

$$y(t) = \tanh\phi(t), \quad A(t) = -\frac{\left(\dfrac{d\theta}{dt}\right)^2 \phi^*}{\dfrac{d\phi^*}{dt}} \text{ and } B(t) = -\frac{\dfrac{d\theta}{dt}\dfrac{d\theta^*}{dt}}{\dfrac{d\phi^*}{dt}}$$

we are led to a linear homogeneous differential equation of the first order

$$\frac{d}{dt}y(t) + A(t)y(t) + B(t) = 0 \tag{42}$$

If we assume that $\theta(t)$ and $\phi(t)$ are both constant, (42) is identically zero; in other words, the developable timelike ruled surface $\hat{x}(t)$ is a cylinder.

Now we can ask a similar question to the case of DHUS:

"When a curve $p(t)$ is given, can we find a time-like developable ruled surface such that its base curve is the curve $p(t)$?".

The answer is, again, YES. In fact from (38), we have

$$\frac{P_1}{P_2} = -\frac{\phi^* \sin\theta}{\phi^* \cos\theta} = -\tan\theta, \quad \phi^* = \pm\sqrt{P_1^2 + P_2^2} \quad \text{and } \theta^* = -P_3$$

Now we need to determine $\phi(t)$. The solution of the linear homogeneous differential equation (42) gives $\tanh\phi(t)$. This solution includes an integral constant, therefore we have infinitely many developable ruled surfaces such that their base curve is $p(t)$.

It is to be noted that $\phi^*(t)$ has two values; when we use the minus sign, we obtain the reciprocal of the ruled surface $\hat{x}(t)$ obtained by using the plus sign for a given integral constant.

In the following example, we obtain an application about two-component WKI equation and inextensible flows of a developable timelike ruled surface.

Lemma 3.1:

$$\mathbf{m}(t,\ u,\ v) = p(t,\ v) + ux(t,\ v),$$

$|x| \equiv 1, x' \neq 0$ and $(x,\ x',\ p') \equiv 0$, and let t be the arc-length of p. Then the first fundamental form of $m(t,\ u,\ v)$ is

$$\begin{aligned} E &= <\mathbf{m}_t,\ m_t> = 1 + 2u <p',\ x'> + u^2|x'|^2, \\ F &= <\mathbf{m}_t,\ \mathbf{m}_u> = <p',\ x>, \\ G &= <\mathbf{m}_u,\ \mathbf{m}_u> = 1. \end{aligned}$$

We now make precise the notion of an inextensible evolution of a surface by imposing appropriate constraints on its first fundamental form.

A surface evolution $\mathbf{m}(t,\ u,\ v)$ and its flow $\dfrac{\partial \mathbf{m}}{\partial v}$ are said to be inextensible, if its first fundamental form $\{E,\ F,\ G\}$ satisfies

$$\frac{\partial E}{\partial v} = \frac{\partial F}{\partial v} = \frac{\partial G}{\partial v} = 0.$$

The following theorem is an immediate consequence of Lemma 3.1.

Theorem 3.1: Let the flow $\mathbf{m}(t,\ u,\ v)$ be as in Lemma 3.1. Then **m** is inextensible if and only if

$$\frac{\partial}{\partial v}\langle p',\ x'\rangle = \frac{\partial}{\partial v}|x'|^2 = \frac{\partial}{\partial v}\langle p',\ x\rangle.$$

Example 2:

Consider the curve

$$p(t) = (t+1,\ 2t,\ 3t)$$

Then,

$$\tan\theta = -\frac{t+1}{2t},\ \ \phi^* = \pm\sqrt{5t^2+2t+1}\ \ \text{and}\ \ \theta^* = -3t$$

and

$$\frac{d\theta^*}{dt} = -3,\ \frac{d\theta}{dt} = -\frac{2}{5t^2+2t+1}\quad\text{and}\quad \frac{d\phi^*}{dt} = \frac{5t+1}{\sqrt{5t^2+2t+1}}$$

Substituting these values into (42), we obtain

$$\frac{d}{dt}y(t) + \frac{2\left(\frac{1}{2t}-\frac{t+1}{2t^2}\right)^2(5t^2+2t+1)}{\left(1+\frac{(t+1)^2}{4t^2}\right)^2(10t+2)}y(t) - \frac{6t\left(\frac{1}{2t}-\frac{t+1}{2t^2}\right)^2\sqrt{5t^2+2t+1}}{\left(1+\frac{(t+1)^2}{4t^2}\right)(10t+2)} = 0$$

The solution of this differential equation (using the mathematical program MAPLE) is

$$\begin{aligned} y &= \tanh\phi \\ &= \frac{3\sqrt{5t^2+2t+1}}{5(5t+1)}\left(-\ln(5t^2+2t+1) + \arctan(\frac{5t}{2}+\frac{1}{2})\right) + C \end{aligned} \tag{43}$$

Hence, the family of the developable time-like ruled surfaces is given by

$$\begin{aligned} m(t,u) &= p(t) + ux(t) \\ &= (t+1,\ 2t,\ 3t) + u\left(-\frac{P_2}{\phi^*}\cosh\phi,\ \frac{P_1}{\phi^*}\cosh\phi,\ \sinh\phi\right) \end{aligned}$$

where $x(t)$ is a space-like vector and $x(t) = \left(-\frac{P_2}{\phi^*}\cosh\phi,\ \frac{P_1}{\phi^*}\cosh\phi,\ \sinh\phi\right)$.

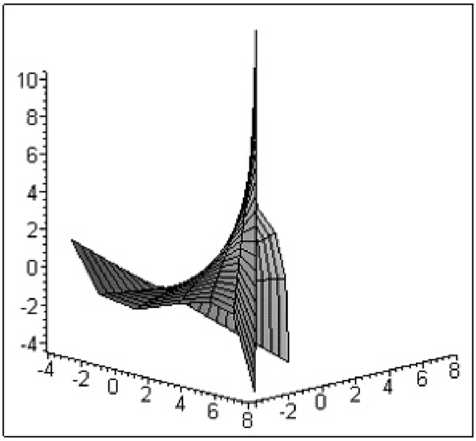

Fig. 4.3

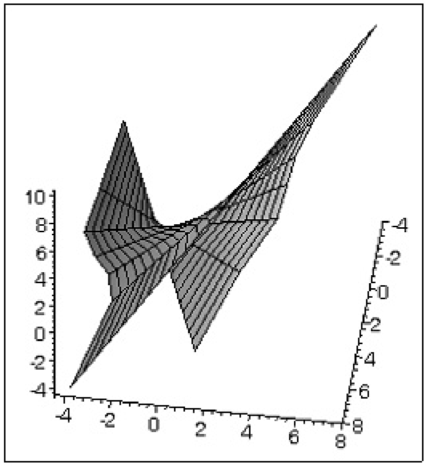

Fig. 4.4

The graph of the developable time-like ruled surface, given by (43), for $C = 0$ in domain $D \ : \ \{-3 \leq t \leq 3, -4 \leq u \leq 4\}$ is given in Fig. 4.3 and Fig. 4.4.

In this section, we obtain the multi-component WKI equation of hyperbolic type by considering motion of curves in the Minkowski space. We denote the three-dimensional Minkowski space by M^3. The Serret-Frenet formula in M^3 is

$$\frac{\partial}{\partial s}\begin{pmatrix} T \\ N \\ B \end{pmatrix} = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & \tau & 0 \end{pmatrix}\begin{pmatrix} T \\ N \\ B \end{pmatrix}$$

and we have $T.T = 1$, $N.N = -1$, $B.B = -1$.

The corresponding geometric quantities in M^3, using the graph of the curve, are given by

$$ds = gdx,$$

$$T = (l, u_x, v_x)/g,$$

$$N = (u_x u_{xx} - v_x v_{xx}, (1 - v_x^2)u_{xx} + u_x v_x v_{xx}, (1 + u_x^2)v_{xx} + v_x u_x u_{xx})/gh$$

$$B = (u_x v_{xx} - v_x u_{xx}, -v_{xx}, -u_{xx})/h$$

$$k = h/g^3,$$

$$\tau = (v_{xx}u_{xxx} - u_{xx}v_{xxx})/h^2$$

where

$$\begin{aligned} g &= \sqrt{(1+u_x^2-v_x^2}, \\ h &= \sqrt{(u_{xx}^2-v_{xx}^2)-(u_xu_{xx}-v_xv_{xx},\ (u_xv_{xx}-v_xu_{xx})^2}. \end{aligned}$$

The curve motion flow is governed by

$$\gamma_t = -\frac{1}{2}\kappa^2 T - \kappa_s N - \kappa\tau B.$$

Firstly, solving the system we obtain the two-component WKI equation,

$$\begin{aligned} u_t + \left[\frac{u_{xx}}{(1+u_x^2-v_x^2)^{3/2}}\right]_x &= 0 \\ v_t + \left[\frac{v_{xx}}{(1+u_x^2-v_x^2)^{3/2}}\right]_x &= 0 \end{aligned}, \qquad (44)$$

We call the system (44) to be the two-component WKI equation of hyperbolic type. Now, we consider

$$\mathbf{m}(t,u) = (t+1-u\frac{P_2}{\phi^*}\cosh\phi,\ 2t+u\frac{P_1}{\phi^*}\cosh\phi,\ 3t+u\sinh\phi)$$

or

$$\mathbf{m}(t,u) = (x(u,t),\ y(t,u),\ z(t,u))$$

the time-like curve, and we obtain $x^2+y^2-z^2=u^2$. A graph of curve on this surface is

$$\gamma = (x,\ \sinh(x-at),\ \cosh(x-at)).$$

This equation is the orbit curve equation in special relativity. The space of the theory of special relativity is the Minkowski space. Let

$$\gamma = (x,\ u(x,t),\ v(x,t)).$$

denote the curve equation in M^3. By using $a=2^{-3/2}$, this curve can be easily shown to satisfy the WKI equation. After necessary calculation, we obtain $g=2^{1/2}$, $h=2$, $\kappa=1/2$ and $\tau=-1/2$ and

$$\gamma_t = -\frac{1}{4}\vec{T} - \frac{1}{4}\vec{B}.$$

CONCLUSION

As it is well known, if the arc length of a curve is preserved, then the flow of curve is inextensible. Physically, inextensible curve and surface flows lead to a motion with no strain energy. Zhang & Hou (2007) discussed three-component WKI equation and curve motion flow in Euclidean & Minkowski spaces. Kwon *et al.* (2005) gave inextensible flows of curves and developable surfaces. Latifi and Razavi (2008) obtained inextensible flows of curves in Minkowskian space. Abdel-All and Abd-Ellah (2003) gave deformation of a kinematic surface in a hyperbolic space. As a final word we can say that, in this paper, we discuss a connection between two-component WKI equation and inextensible flows of a developable timelike ruled surface.

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Submitted : *28/3/2010*
Revised : *19/4/2011*
Accepted : *28/4/2011*

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خلاصة

لقد أعطى كوزي طريقة لتعيين سطح مسطر قابل للنشر. وفي هذا البحث نقوم، أولاً بإيجاد المعادلات الوسيطية لكرات الوحدة الثنوية الزائدية واللورنتزية ثم نعطي طريقة لتعيين سطح مسطر مثل-زمني وقابل للنشر. نقوم بعد ذلك بإعطاء تطبيق حول معادلة واداتي - إيشيكاوا الثنائية المركبة. ونعطي كذلك الانسيابات غير المدودة للأسطح القابلة للنشر. ونناقش في الجزء الأخير من البحث، الانسيابات غير المدودة لسطح مسطر مثل-زمني وقابل للنشر.

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- تنشر لأساتذة التربية والمختصين بها من مختلف الأقطار العربية والدول الأجنبية.

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في الدول العربيـة: أربعـة دنانير للأفراد، وخمسة عشر ديناراً للمؤسسات.

في الدول الأجنبية: خمسة عشـر دولاراً للأفراد، وستون دولاراً للمؤسسات.

توجه جميع المراسلات إلى:

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الكويت هاتف: ٢٤٨٤٦٨٤٣ (داخلي ٤٤٠٣ - ٤٤٠٩) - مباشر: ٢٤٨٤٧٩٦١ - فاكس: ٢٤٨٣٧٧٩٤

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