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Ricci curvature of slant submanifolds in (k, μ)-contact space forms

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ABSTRACT

B.Y. Chen (1999) established a clear relationship involving intrinsic invariants, namely the sectional curvature and the scalar curvature and the main extrinsic invariants, namely the squared mean curvature for a submanifold in real space form with arbitrary co-dimension.

In this article, we establish inequalities between the Ricci curvature and the squared mean curvature, and also between the s-Ricci curvature and the scalar curvature for a slant, semi-slant and bi-slant submanifolds in (k,μ)-contact space forms.

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INTRODUCTION

A $(2m+1)$-dimensional Riemannian manifold $(\bar{M}, g)$ is said to be an almost contact-metric manifold if there exist on $\bar{M}$ a $(1,1)$tensor field ϕ, a vector field ξ and a 1-form η such that

$$\eta(\xi) = 1, \phi^2 X = -X + \eta(X)\xi \qquad and \qquad g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \quad (1)$$

for any vector fields X, Y on $\bar{M}$. Then, $\phi(\xi) = 0$ and $\eta o \phi = 0$. Such a manifold is said to be a contact-metric manifold if $d\eta = \Phi$, where $\Phi(X, Y) = g(X, \phi Y)$ is called the fundamental 2-form of $\bar{M}$. The almost-contact structure is said to be normal if the induced almost-complex structure J on the product manifold $\bar{M} \times IR$ defined by

$$J\left(X, \lambda \frac{d}{dt}\right) = \left(\phi X - \lambda\xi,\ \eta(X)\frac{d}{dt}\right) \qquad (2)$$

is integrable, where X is tangent to $\bar{M}$, t is the coordinate on IR and λ is a smooth function on $\bar{M}\times IR$. The condition for (ϕ, ξ, η, g) being normal is equivalent to the vanishing of the torsion tensor

$$[\phi, \phi] + 2d\eta\otimes\xi, \tag{3}$$

where $[\phi, \phi]$ is the Nijenhuis tensor of ϕ.

An almost-contact metric manifold is called a Sasakian manifold if

$$(\overline{\nabla}_X\phi)Y = g(X, Y)\xi - \eta(Y)X, \overline{\nabla}_X\xi = -\phi X, \tag{4}$$

for any vector fields X, Y on $\bar{M}$, where$\overline{\nabla}$denotes the Riemannian connection of g.

T. Koufogiorgos (1997) introduced the notion of (k,μ)-contact space forms, which contains the well-known class of Sasakian space forms for $k = 1$.

Let $\bar{M}$ be a contact metric manifold. The (k,μ)-nullity distribution of $\bar{M}$, for the pair (k,μ) is a distribution

$$\begin{aligned} N(k,\mu) : p \to N_p(k,\mu) \quad &= \{Z \in T_pM/\overline{R}(X,Y)Z \quad = k[g(Y,Z)X - g(X,Z)Y] \\ &+\mu[g(Y,Z)hX - -g(X,Z)hY]\}, \end{aligned} \tag{5}$$

where $k,\ \mu \in IR$ and the (1,1)-tensor field h defined by $2h = L_\xi\phi$ is symmetric and satisfies

$$h\xi = 0, h\phi + \phi h = 0, \overline{\nabla}_X\xi = - \quad \phi X - \phi hX, trace(h) = trace(\phi h) = 0. \tag{6}$$

If $\xi \in N(k,\mu)$, $\bar{M}$ is called a (k,μ)-contact metric manifold. Since in a (k,μ)-contact metric manifold one has $h^2 = (k\text{-}1)\phi^2$, therefore $k \leq 1$, and if $k = 1$, then the structure is Sasakian (D.E. Blair 1995). Characteristic examples of non-Sasakian (k,μ)-contact metric manifolds are the tangent sphere bundles of Riemannian manifolds of constant sectional curvature not equal to one.

The sectional curvature $\overline{K}(X, \phi X)$ of a plane section spanned by a unit vector X orthogonal to ξ is called a ϕ-sectional curvature. If the (k,μ)-contact metric manifold $\bar{M}$ has constant ϕ-sectional curvature c, then it is called (k,μ)-contact space form and is denoted by $\bar{M}(c)$. The curvature tensor of $\bar{M}(c)$ is given by (T: Koufogiorgos, 1997).

$$\begin{aligned}\overline{R}(X,Y)Z &= \frac{c+3}{4}\{g(Y,Z)X-g(X,Z)Y\}\\ &+\frac{c-1}{4}\{2g(X,\phi Y)\phi Z+g(X,\phi Z)\phi Y-g(Y,\phi Z)\phi X\}\\ &+\frac{c+3-4k}{4}\{\eta(X)\eta(Z)Y-\eta(Y)\eta(Z)X+g(X,Z)\eta(Y)\xi-g(Y,Z)\eta(X)\xi\}\\ &+\frac{1}{2}\{g(hY,Z)hX--g(hX,Z)hY+g(\phi hX,Z)\phi hY-g(\phi hY,Z)\phi hX\}\\ &+g(\phi Y,\phi Z)hX-g(\phi X,\phi Z)hY+g(hX,Z)\phi^2Y-g(hY,Z)\phi^2X\\ &+\mu\{\eta(Y)\eta(Z)hX-\eta(X)\eta(Z)hY+g(hY,Z)\eta(X)\xi\quad -g(hX,Z)\eta(Y)\xi\}\end{aligned} \tag{7}$$

for all X, Y, Z∈T$\bar{M}$, where $c\ +\ 2k\ =\ -1\ =\ k\text{-}\mu$ if $k < 1$.

Let $\bar{M}$(c) be a $(2m+1)$-dimensional (k,μ)-contact space form. For each plane section $\pi \subset T_p\bar{M},\ p \in \bar{M}$, we denote $\overline{K}(\pi)$ the sectional curvature of the plane section π.

Let $\{e_1, e_2 \ldots, e_n, \ldots, e_{2m+1}\}$ be an orthonormal basis of the tangent space $T_p\overline{M}(c)$.Then the scalar curvature τ of $\overline{M}$(c) is defined by

$$\tau = \sum_{i<j} \overline{K}(e_i\,,\,e_j), i,j\ = 1, \ldots., n, \ldots..2m+1, \tag{8}$$

where $\overline{K}\,(e_i, e_j)$ is the sectional curvature of the section spanned by e_i and e_j.

Let M be an $n-$dimensional submanifold of $\overline{M}(c)$ and denote by σ, ∇ and $\nabla^{\perp}$ the second fundamental form of M, the induced connection on M and the normal bundle $T^{\perp}M$, respectively. Then the Gauss and Weingarten formulae are given by

$$\begin{aligned}\overline{\nabla}_X Y &= \nabla_X Y + \sigma(X,\ Y),\\ \bar{\nabla}_X N &= -A_N X + \nabla^{\perp}_X N,\end{aligned} \tag{9}$$

for any vector fields X, Y tangent to M and a vector field N normal to M, whereA_N is the shape operator in the direction of N. The second fundamental form and the shape operator are related by

$$g(\sigma(X,\ Y), N) = g(A_N X,\ Y). \tag{10}$$

Let R be Riemannian curvature tensor of M, then the Gauss equation is given by

$$R(X,Y,Z,W) = \overline{R}(X,Y,Z,W) + g(\sigma(X,W),\sigma(Y,Z)) - g(\sigma(X,Z),\sigma(Y,W)), \tag{11}$$

for all $X, Y, Z, W \in TM$.

Let $\{e_1, e_2, \ldots, e_n\}$ be an orthonormal basis of T_pM, $p \in M$, then the mean curvature vector $H(p)$ is given by

$$H(p) = \frac{1}{n} \sum_{i=1}^{n} \sigma(e_i, e_i). \tag{12}$$

For any vector X tangent to M, we put $\phi X = PX + FX$, where PX and FX are the tangential and the normal component of ϕX, respectively. We define the squared norm of P by

$$\|P\|^2 = \sum_{i,j=1}^{n} g^2(\phi e_i, e_j). \tag{13}$$

Also, we set $\sigma_{ij}^r = g(\sigma(e_i, e_j), e_r)$, $i,j \in \{1, \ldots, n\}$, $r \in \{n+1, \ldots, 2m+1\}$, and

$$\|\sigma\|^2 = \sum_{i,j=1}^{n} g(\sigma(e_i, e_j), \sigma(e_i, e_j)), \tag{14}$$

where $\{e_{n+1}, \ldots, e_{2m+1}\}$ is an orthonormal basis of $T_p^{\perp}M$.

A submanifold M in $\bar{M}(c)$ is called totally geodesic if the second fundamental form vanishes identically, and totally umbilical if there is a real number λ such that

$$\sigma(X, Y) = \lambda g(X, Y) H,$$

for any tangent vectors X,Y on M .

Let $K(\pi)$ denote the sectional curvature of M associated with a plane section $\pi \subset T_pM, p \in M$. For an orthonormal basis $\{e_1, e_2, \ldots, e_n\}$ of the tangent space T_pM, the scalar curvature τ is defined by

$$\tau = \sum_{i<j} K_{ij}, \tag{15}$$

where K_{ij} denotes the sectional curvature of the 2-plane section spanned by e_i and e_j. Suppose L is a s-plane section of T_pM and X a unit vector in L. We choose an orthonormal basis $\{e_1, e_2, \ldots, e_s\}$ of L such that $e_1 = X$. Define the Ricci curvature Ric_L of L at X by respectively

$$Ric_L(X) = K_{12} + \ldots.. + K_{1s}. \quad (16)$$

We call such a curvature $s-$Ricci curvature. The scalar curvature τ of the $s-$plane section L is given by

$$\tau(L) = \sum_{1\le i<j\le s} K_{ij}. \quad (17)$$

For each integer $s, 2 \le s \le n$, the Riemannian invariant Θ_s of an n-dimensional Riemannian manifold M is defined by

$$\Theta_s(p) = \frac{1}{s-1} \inf_{L,X} Ric_L(X), \qquad p \in M, \quad (18)$$

where L runs over all s-plane sections in T_pM and X runs over all unit vectors in L.

The relative null space of a submanifold M in a Riemannian manifold is defined by

$$N_p = \{X \in T_pM : \sigma(X, Y) = 0, \ \ for\ all\ \ Y \in T_pM\}.$$

Now, we recall

Definition 1 (J.L. Cabrerizo *et al.,* 2000): A submanifold M tangent to ξ is said to be slant if for any $p\in M$ *and* X$\in T_pM$, linearly independent on ξ, the angle between ϕX and $T_pM$is a constant $\theta \in [0, \pi/2]$, called the slant angle of M in $\bar{M}(c)$.Invariant and anti-invariant submanifolds of $\bar{M}(c)$ are slant submanifolds with slant angle $\theta = 0$ and $\theta = \frac{\pi}{2}$ respectively.

Definition 2 (J.L. Cabrerizo *et al.,* 1999): A submanifold M tangent to ξ is said to be a semi-slant submanifold of $\bar{M}(c)$ if there exist two orthogonal distributions D_1 andD_2 on Msuch that:

(1) TM admits the orthogonal direct decomposition

$$TM = D_1 \oplus D_2 \oplus \{\xi\},$$

(2) The distribution D_1 is an invariant distribution i.e. $\phi(D_1) = D_1$, and

(3) The distribution D_2 is slant with slant angle $\theta \neq 0$.

Definition 3 (J.L. Cabrerizo *et al.,* 1999): A submanifold M tangent to ξ is said to be a bi-slant submanifold of $\bar{M}(c)$ if there exist two orthogonal distributions D_1 and D_2 on M such that

(1) TM admits the orthogonal direct decomposition

$$TM = D_1 \oplus D_2 \oplus \{\xi\},$$

(2) For any $i=1, 2$, D_i is slant distribution with slant angle θ_i.

On the other hand, CR-submanifolds of $\bar{M}(c)$ are bi-slant submanifolds with $\theta_1 = 0, \theta_2 = \frac{\pi}{2}$.

Let $2d_1 = dim\ D_1$ and $2d_2 = dim\ D_2$.

The invariant distribution of a semi-slant submanifold is a slant distribution with zero slant angle. Thus, it is obvious that semi-slant submanifolds are particular cases of bi-slant submanifolds.

RICCI CURVATURE AND SQUARED MEAN CURVATURE

B.Y. Chen established a sharp relationship between the Ricci curvature and the squared mean curvature for submanifolds in real space form. K. Arslan *et al.* (2001) established inequalities between the Ricci curvature and the squared mean curvature for submanifolds normal to structure vector field ξ in a (k,μ)-contact space form. Such a submanifold is called C-totally real submanifold (F. Defever *et al.*, 1997). In this section, we estimate Ricci curvature for slant, semi-slant and bi-slant submanifolds tangent to ξ in a (k,μ)-contact space form.

Theorem 1. Let M be an n-dimensional submanifold tangent to ξ in a $(2m+1)$-dimensional (k,μ)-contact space form $\bar{M}(c)$. Then, we have

(I) For each unit vector $X \in T_pM$ orthogonal to ξ

$$\begin{aligned} Ric(X) \leq\ & \frac{1}{4}\left\{n^2\|H\|^2+(n-1)(c+3)+3(c-1)\|PX\|^2-(c+3-4k)+\left\|(\phi h)^T X\right\|^2-\left\|h^T X\right\|^2\right\} \\ & +(\mu+n-2)g(h^T X, X). \end{aligned} \tag{19}$$

(II) If H (p) = 0, then a unit tangent vector X orthogonal to ξ at p satisfies the equality case of (19) if and only if X belongs to relative null space.

(III) The equality case of (19) holds identically for all unit tangent vectors orthogonal to ξ at p if and only if p is a totally geodesic point.

Proof. Let $X \in T_pM$ be a unit tangent vector at p orthogonal to ξ .We choose an orthonormal basis $\{e_1, e_2, \ldots, e_n = \xi, e_{n+1}, \ldots, e_{2m+1}\}$ of $T_pM$such that $e_1, e_2, \ldots, e_n$ are tangent to M at p with $e_1 = X$. Then, from the equation of Gauss, We have

$$\begin{aligned} n^2\|H\|^2 =\ & 2\tau + \|\sigma\|^2 - \frac{1}{4}\{n(n-1)(c+3)+3(c-1)\|P\|^2-2(n-1)(c+3-4k)\} \\ & -\frac{1}{2}\left\{\|(\phi h)^T\|^2-\|h^T\|^2-\left(trace(\phi h)^T\right)^2+\left(trace(h)^T\right)^2\right\} - 2(\mu+n-2)trace(h)^T, \end{aligned} \tag{20}$$

where

$$\|\sigma\|^2 = \sum_{i,j=1}^{n} g(\sigma(e_i,e_j),\sigma(e_i,e_j)) \quad , \quad \|Q\|^2 = \sum_{i,j=1}^{n} g(e_i, Qe_j) \quad , \quad Q \in \left\{P, (\phi h)^T, h^T\right\}$$

and $(\phi h)^T X$ and $h^T X$ are the tangential parts of ϕhX and hX respectively for $X \in TM$.

From (20) we get

$$\begin{aligned} n^2\|H\|^2 &= 2\tau + \sum_{r=n+1}^{2m+1} [(\sigma_{11}^r)^2 + (\sigma_{22}^r + ... + \sigma_{nn}^r)^2 + 2\sum_{2\le i<j\le n} \left(\sigma_{ij}^r\right)^2] \\ &-2\sum_{r=n+1}^{2m+1}\sum_{2\le i<j\le n} \sigma_{ii}^r\sigma_{jj}^r - \frac{1}{4}\{n(n-1)(c+3) + 3\|P\|^2(c-1) - 2(n-1)(c+3-4k)\} \\ &-\frac{1}{2}\left\{\|(\phi h)^T\|^2 - \|h^T\|^2 - \left(trace(\phi h)^T\right)^2 + \left(trace(h)^T\right)^2\right\} - 2(\mu+n-2)trace(h)^T, \end{aligned}$$

which implies

$$\begin{aligned} n^2\|H\|^2 &= 2\tau + \frac{1}{2}\sum_{r=n+1}^{2m+1} [(\sigma_{11}^r + \sigma_{22}^r + ... + \sigma_{nn}^r)^2 + (\sigma_{11}^r - \sigma_{22}^r - ... - \sigma_{nn}^r)^2 \\ &+2\sum_{r=n+1}^{2m+1}\sum_{1\le i<j\le n} (\sigma_{ij}^r)^2 - 2\sum_{r=n+1}^{2m+1}\sum_{2\le i<j\le n} \sigma_{ii}^r\sigma_{jj}^r \\ &- \frac{1}{4}\{n(n-1)(c+3) + 3(c-1)\|P\|^2 - 2(n-1)(c+3-4k)\} \\ &- \frac{1}{2}\left\{\|(\phi h)^T\|^2 - \|h^T\|^2 - \left(trace(\phi h)^T\right)^2 + \left(trace(h)^T\right)^2\right\} - 2(\mu+n-2)trace(h)^T. \end{aligned} \tag{21}$$

By using Gauss' equation, we have

$$\begin{aligned} \sum_{2\le i<j\le n} K_{ij} &= \sum_{r=n+1}^{2m+1}\sum_{2\le i<j\le n} [\sigma_{ii}^r\sigma_{jj}^r - (\sigma_{ij}^r)^2] \\ &+\frac{1}{8}(n-1)(n-2)(c+3) + \frac{3}{8}(c-1)\sum_{i=2}^{n}\|Pe_i\|^2 - \frac{2}{8}(n-2)(c+3-4k) \\ &+\frac{1}{4}\{\|(\phi h)^T\|^2 - \|h^T\|^2 + 2\|h^T e_1\|^2 - 2\|(\phi h)^T e_1\|^2 - \sum_{i=2}^{n} g^2((\phi h)^T e_i, e_i) + \sum_{i=2}^{n} g^2(h^T e_i, e_i)\} \\ &+(\mu+n--2)\sum_{i=2}^{n} g(h^T e_i, e_i). \end{aligned} \tag{22}$$

By substituting (22) in (21), we get

$$\frac{1}{2}n^2\|H\|^2 \geq 2Ric(X) - \frac{1}{2}(n-1)(c+3) - \frac{3}{2}(c-1)\|PX\|^2 - \frac{1}{2}\left\|(\phi h)^T X\right\|^2 + \frac{1}{2}\left\|h^T X\right\|^2 + \frac{1}{2}(c+3-4k)$$
$$- \quad 2(\mu + n - 2)g(h^T X, X),$$

which is equivalent to (19).

(II) Now, we assume that $H\ (p) = 0$. The equality case holds in (19) if and only if

$$\begin{cases} \sigma^r_{12} = \ldots\ldots = \sigma^r_{1n} = 0\,, \\ \sigma^r_{11} = \sigma^r_{22} + \ldots + \sigma^r_{nn}\,, r \in \{n+1, \ldots\ldots, 2m+1\}. \end{cases}$$

Then $\sigma^r_{1j} = 0$ for all $j \in \{1 \ldots n\}, r \in \{n+1, \ldots\ldots, 2m+1\}$, that is $X \in N_p$.

(III) The equality case of (19) holds for all unit tangent vectors orthogonal to ξ at p if and only if

$$\begin{cases} \sigma^r_{ij} = 0, \quad i \neq j, \quad r \in \{n+1, \ldots\ldots, 2m+1\} \\ \sigma^r_{11} + \ldots\ldots + \sigma^r_{nn} - 2\sigma^r_{ii} = 0, \quad i \in \{1, \ldots, n\},\ r \in \{n+1, \ldots\ldots, 2m+1\}. \end{cases}$$

Thus, we have two cases, namely either $n = 2$ or $n \neq 2$. In the first case p is a totally umbilical point, while in the second case p is a totally geodesic point. Since $\xi \in TM$, therefore each totally umbilical point is totally geodesic. Thus in both cases p is a totally geodesic point.

From the above theorem, it follows:

Corollary2. (I. Mihai, 2002): Let M be an n-dimensional submanifold in a (2m + 1)-dimensional Sasakian space form $\bar{M}$(c) tangent to the structure vector field ξ . Then

(I) For each unit vector $X \in T_p$M orthogonal to ξ , we have

$$Ric(X) \leq \frac{1}{4}\left\{ n^2\|H\|^2 + (n-1)(c+3) + 3(c-1)\,\|PX\|^2 - (c-1) \right\}. \quad (23)$$

(II) If $H\ (p) = 0$, then a unit tangent vector X orthogonal to ξ at p satisfies the equality case of (23) if and only if X belongs to relative null space.

(III) The equality case of (23) holds identically for all unit tangent vectors orthogonal to ξ at p if and only if p is a totally geodesic point.

Corollary3. Let M be an n-dimensional θ-slant submanifold tangent to ξ in a (2m + 1)-dimensional (k,μ)-contact space form $\bar{M}$(c). Then, we have

(I) For each unit vector X$\in$ T_pM orthogonal to ξ

$$\begin{aligned} Ric(X) \leq\ & \frac{1}{4}\left\{ n^2\|H\|^2+(n-1)(c+3)+3(c-1)\cos^2\theta-(c+3-4k)+\left\|(\phi h)^T X\right\|^2-\left\|h^T X\right\|^2\right\} \\ & +(\mu+n-2)g(h^T X, X). \end{aligned} \tag{24}$$

(II) If $H\ (p)\ =\ 0$, then a unit tangent vector X orthogonal to ξ at p satisfies the equality case of (24) if and only if X belongs to relative null space.

(III) The equality case of (24) holds identically for all unit tangent vectors orthogonal to ξ at p if and only if p is a totally geodesic point.

Proof. A θ-slant submanifold M of an almost contact metric manifold satisfies

$$g(PX,PY) = cos^2\theta g(\phi X,\phi Y), g(FX,FY) = sin^2\theta g(\phi X,\phi Y)\ for\ all\ X, Y \in TM. \tag{25}$$

In view of (25), for a unit tangent vector $X \in T_pM$, we have

$$\|PX\|^2 = g(PX,PX) = \cos^2\theta.$$

Using this in (19), we have (24).

Theorem 4. Let M be an $n-$dimensional bi-slant submanifold satisfying $g(X,\phi Y)=0$ for any $X\in D_1$ and $Y\in D_2$, tangent to ξ in a $(2m+1)$-dimensional (k,μ)-contact space form $\bar{M}(c)$. Then,

(I) For each unit vector $X \in T_pM$ orthogonal to ξ, if

(i) X is tangent to D_1, we have

$$\begin{aligned} Ric(X) \leq\ & \frac{1}{4}\left\{ n^2\|H\|^2+(n-1)(c+3)+\frac{3}{2}(c-1)\cos^2\theta_1-(c+3-4k)+\left\|(\phi h)^T X\right\|^2-\left\|h^T X\right\|^2\right\} \\ & +(\mu+n-2)g(h^T X, X) \end{aligned} \tag{26}$$

and if

(ii) X is tangent to D_2, we have

$$\begin{aligned} Ric(X) \leq\ & \frac{1}{4}\left\{ n^2\|H\|^2+(n-1)(c+3)+\frac{3}{2}(c-1)\cos^2\theta_2-(c+3-4k)+\left\|(\phi h)^T X\right\|^2-\left\|h^T X\right\|^2\right\} \\ & +(\mu+n-2)g(h^T X, X). \end{aligned} \tag{27}$$

(II) If $H(p)\ =\ 0$, then a unit tangent vector X orthogonal to ξ at p satisfies the equality case of (26) and (27) if and only if X belongs to relative null space.

(III) The equality case of (26) and (27) holds identically for all unit tangent vectors orthogonal to ξ at p if and only if p is a totally geodesic point.

Proof. Let $X\in T_pM$ be a unit tangent vector at p orthogonal to ξ .We choose an orthonormal basis $\{e_{1,}, e_2, \ldots, e_n\} = \xi, e_{n+1}, \ldots, e_{2m+1}\}$ of $T_pM$such that $e_{1,}e_{2,\ldots,}e_n$ are tangent to M at p with $e_1 = X$.

Then, from Gauss' equation, we have

$$\begin{aligned} n^2\|H\|^2 &= 2\tau + \|\sigma\|^2 \\ &- \frac{1}{4}\{n(n-1)(c+3) + 6(d_1\cos^2\theta_1 + d_2\cos^2\theta_2)(c-1) - 2(n-1)(c+3-4k)\} \\ &-\frac{1}{2}\left\{\|(\phi h)^T\|^2 - \|h^T\|^2 - \left(trace(\phi h)^T\right)^2 + \left(trace(h)^T\right)^2\right\} - 2(\mu+n-2)trace(h)^T, \end{aligned} \tag{28}$$

where $2d_1 = \dim D_1$ and $2d_2 = \dim D_2$.

From (28) we have,

$$\begin{aligned} n^2\|H\|^2 &= 2\tau + \sum_{r=n+1}^{2m+1}[(\sigma_{11}^r)^2 + (\sigma_{22}^r + \ldots. + \sigma_{nn}^r)^2 + 2\sum_{2\le i<j\le n}\left(\sigma_{ij}^r\right)^2] - 2\sum_{r=n+1}^{2m+1}\sum_{2\le i<j\le n}\sigma_{ii}^r\sigma_{jj}^r \\ &-\frac{1}{4}\{n(n-1)(c+3) + 6(d_1\cos^2\theta_1 + d_2\cos^2\theta_2)(c-1) - 2(n-1)(c+3-4k)\} \\ &-\frac{1}{2}\left\{\|(\phi h)^T\|^2 - \|h^T\|^2 - \left(trace(\phi h)^T\right)^2 + \left(trace(h)^T\right)^2\right\} - 2(\mu+n-2)trace(h)^T \end{aligned}$$

which implies

$$\begin{aligned} n^2\|H\|^2 = {} & 2\tau + \frac{1}{2}\sum_{i=n+1}^{2m+1}[(\sigma_{11}^r + \sigma_{22}^r + \ldots. + \sigma_{nn}^r)^2 + (\sigma_{11}^r - \sigma_{22}^r - \ldots - \sigma_{nn}^r)^2 \\ & +2\sum_{r=n+1}^{2m+1}\sum_{1\le i<j\le n}(\sigma_{ij}^r)^2 - 2\sum_{r=n+1}^{2m+1}\sum_{2\le i<j\le n}\sigma_{ii}^r\sigma_{jj}^r \\ & - \frac{1}{4}\{n(n-1)(c+3) + 6(d_1\cos^2\theta_1 + d_2\cos^2\theta_2)(c-1) - 2(n-1)(c+3-4k)\} \\ & -\frac{1}{2}\left\{\|(\phi h)^T\|^2 - \|h^T\|^2 - \left(trace(\phi h)^T\right)^2 + \left(trace(h)^T\right)^2\right\} - 2(\mu+n-2)trace(h)^T. \end{aligned} \tag{29}$$

Now, we consider two cases:

(i) If X is tangent to D_1, then we have

$$\begin{aligned}\sum_{2\le i<j\le n} K_{ij} &= \sum_{r=n+1}^{2m+1}\sum_{2\le i<j\le n}[\sigma^r_{ii}\sigma^r_{jj}-(\sigma^r_{ij})^2]\\ &+\frac{1}{8}[(n-1)(n-2)(c+3)+\{6(d_1\cos^2\theta_1+d_2\cos^2\theta_2)-3\cos^2\theta_1\}(c-1)-(n-2)(c+3-4k)]\\ &+\frac{1}{4}\{\left\|(\phi h)^T\right\|^2-\left\|h^T\right\|^2+2\left\|h^Te_1\right\|^2-2\left\|(\phi h)^Te_1\right\|^2-\sum_{i=2}^n g^2((\phi h)^Te_i,e_i)+\sum_{i=2}^n g^2(h^Te_i,e_i)\}\\ &+(\mu+n--2)\sum_{i=2}^n g(h^Te_i,e_i).\end{aligned} \tag{30}$$

By substituting (30) in (29), we get

$$\begin{aligned}\frac{1}{2}n^2\|H\|^2 \ &\geq\ 2Ric(X)-\frac{1}{2}(n-1)(c+3)-\frac{3}{4}(c-1)\cos^2\theta_1-\frac{1}{2}\left\|(\phi h)^TX\right\|^2+\frac{1}{2}\left\|h^TX\right\|^2+\frac{1}{2}(c+3-4k)\\ &-2(\mu+n--2)g(h^TX,X)\end{aligned}$$

which is equivalent to (26).

(ii) If X is tangent to D_2, then we have

$$\begin{aligned}\sum_{2\le i<j\le n} K_{ij} &= \sum_{r=n+1}^{2m+1}\sum_{2\le i<j\le n}[\sigma^r_{ii}\sigma^r_{jj}-(\sigma^r_{ij})^2]\\ &+\frac{1}{8}[(n-1)(n-2)(c+3)+\{6(d_1\cos^2\theta_1+d_2\cos^2\theta_2)-3\cos^2\theta_2\}(c-1)-(n-2)(c+3-4k)]\\ &+\frac{1}{4}\{\left\|(\phi h)^T\right\|^2-\left\|h^T\right\|^2+2\left\|h^Te_1\right\|^2-2\left\|(\phi h)^Te_1\right\|^2-\sum_{i=2}^n g^2((\phi h)^Te_i,e_i)+\sum_{i=2}^n g^2(h^Te_i,e_i)\}\\ &+(\mu+n-2)\sum_{i=2}^n g(h^Te_i,e_i).\end{aligned} \tag{31}$$

By substituting (31) in (29), we get

$$\begin{aligned}\frac{1}{2}n^2\|H\|^2\geq\ &2Ric(X)-\frac{1}{2}(n-1)(c+3)-\frac{3}{4}(c-1)\cos^2\theta_2-\frac{1}{2}\left\|(\phi h)^TX\right\|^2+\frac{1}{2}\left\|h^TX\right\|^2+\frac{1}{2}(c+3-4k)\\ &-2(\mu+n-2)g(h^TX,X),\end{aligned}$$

which is equivalent to (27).

(II) Now, we assume that $H(p)=0$. The equality case holds in (26) and (27) if and only if

$$\begin{cases}\sigma^r_{12}=.......=\sigma^r_{1n}=0\,,\\ \sigma^r_{11}=\sigma^r_{22}+.....+\sigma^r_{nn}\,,r\in\{n+1,.......,2m+1\}.\end{cases}$$

Then $\sigma^r_{1j}=0$ for all $j\in\{1,\ldots.,n\}$ $r\in\{n+1,\ldots.,2m+1\}$, that is $X\in N_p$.

(III) The equality case of (26) and (27) holds for all unit tangent vectors orthogonal to ξ at p if and only if

$$\begin{cases} \sigma^r_{ij} = 0, \quad i \neq j, \quad r \in \{n+1, ..., 2m+1\} \\ \sigma^r_{11} + ... + \sigma^r_{nn} - 2\sigma^r_{ii} = 0, \quad i \in \{1, ..., n\}, \ r \in \{n+1, ..., 2m+1\}. \end{cases}$$

In this case, it follows that p is a totally geodesic point. The converse is trivial.

Now we can state the following:

Corollary5. Let M be an n-dimensional semi-slant submanifold in a (2m + 1)-dimensional (k,μ)-contact space form $\bar{M}$(c). Then,

(I) For each unit vector X$\in T_pM$ orthogonal to ξ, if

(i) X is tangent to D_1, we have

$$Ric(X) \leq \frac{1}{4}\left\{n^2\|H\|^2 + (n-1)(c+3) - (c+3-4k) + \left\|(\phi h)^T X\right\|^2 - \left\|h^T X\right\|^2\right\} + (\mu + n - 2)g(h^T X, X) \tag{32}$$

and if

(ii) X is tangent to D_2, we have

$$Ric(X) \leq \frac{1}{4}\left\{n^2\|H\|^2 + (n-1)(c+3) + \frac{3}{2}(c-1)\cos^2\theta - (c+3-4k) + \left\|(\phi h)^T X\right\|^2 - \left\|h^T X\right\|^2\right\} + (\mu + n - 2)g(h^T X, X). \tag{33}$$

(II) If $H(p) = 0$, then a unit tangent vector X orthogonal to ξ at p satisfies the equality case of (32) and (33) if and only if X belongs to relative null space.

(III) The equality case of (32) and (33) holds identically for all unit tangent vectors orthogonal to ξ at p if and only if p is a totally geodesic point.

Corollary6. Let M be an n-dimensional invariant submanifold in a (2m + 1)-dimensional (k,μ)-contact space form $\bar{M}$(c). Then,

(I) For each unit vector $X\in T_pM$ orthogonal to ξ , if

$$Ric(X) \leq \frac{1}{4}\left\{n^2\|H\|^2 + (n-1)(c+3) + \frac{3}{2}(c-1) - (c+3-4k) + \left\|(\phi h)^T X\right\|^2 - \left\|h^T X\right\|^2\right\} + (\mu + n - 2)g(h^T X, X). \tag{34}$$

(II) If $H\ (p) = 0$, then a unit tangent vector X orthogonal to ξ at p satisfies the equality case of (34) if and only if X belongs to relative null space.

(III) The equality case of (34) holds identically for all unit tangent vectors orthogonal to ξ at p if and only if p is a totally geodesic point.

Corollary7. Let M be an n-dimensional anti-invariant submanifold in a (2m + 1)-dimensional (k,μ) - contact space form $\bar{M}$(c). Then,

(I) For each unit vector $X \in T_pM$ orthogonal to ξ, if

$$Ric(X) \leq \frac{1}{4}\left\{ n^2\|H\|^2 + (n-1)(c+3) - (c+3-4k) + \left\|(\phi h)^T X\right\|^2 - \left\|h^T X\right\|^2 \right\} + (\mu + n - 2)g(h^T X, X). \quad (35)$$

(II) If $H(p) = 0$, then a unit tangent vector X orthogonal to ξ at p satisfies the equality case of (35) if and only if X belongs to relative null space.

(III) The equality case of (35) holds identically for all unit tangent vectors orthogonal to ξ at p if and only if p is a totally geodesic point.

s-RICCI CURVATURE AND SQUARED MEAN CURVATURE

In this section, we establish the relationship between the s-Ricci curvature and the squared mean curvature for slant, semi-slant and bi-slant submanifolds in a (k,μ)-contact space form $\bar{M}(c)$.

First, we state an inequality involving the scalar curvature and squared mean curvature for a slant submanifold M tangent to the structure vector field ξ .

Theorem8. Let M be an n-dimensional θ-slant submanifold tangent to ξ in a (2m + 1)- dimensional (k,μ)-contact space form $\bar{M}(c)$. Then, we have

$$\|H\|^2 \geq \frac{2\tau}{n(n-1)} - \frac{1}{4n(n-1)}\left[n(n-1)(c+3) + 3(n-1)(c-1)\cos^2\theta - 2(n-1)(c+3-4k)\right] - \frac{1}{2n(n-1)}\left\{\|(\phi h)^T\|^2 - \|h^T\|^2 - \left(trace(\phi h)^T\right)^2 + \left(trace(h)^T\right)^2\right\} - \frac{2}{n(n-1)}(\mu + n - 2)trace(h)^T. \quad (36)$$

The equality case of (36) holds at a point $p \in M$ if and only if p is a totally umbilical point.

Proof. Let p be a point of M. We choose an orthonormal basis $\{e_1, e_2, \ldots, e_n = \xi\}$ for the tangent space T_pM and $\{e_{n+1}, \ldots, e_{2m+1}\}$ for the normal space $T_p^{\perp}M$ at p such that the normal vector e_{n+1} is parallel to the mean curvature vector $H(p)$ and $e_1, e_2, \ldots, e_n$ diagonalizes the shape operator A_{n+1}. Then the shape operators take the forms

$$A_{n+1} = \begin{pmatrix} a & 0 & 0 & . & . & 0 \\ 0 & a & 0 & . & . & 0 \\ 0 & 0 & \mu & . & . & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & \mu \end{pmatrix}, \qquad \text{where a} + \text{b} = \mu \tag{37}$$

$$A_r = \begin{pmatrix} \sigma^r_{11} & \sigma^r_{22} & 0 & . & . & 0 \\ \sigma^r_{12} & -\sigma^r_{11} & o & . & . & 0 \\ 0 & 0 & 0 & . & . & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & 0 \end{pmatrix} \tag{38}$$

where we denote by $A_r = A_{e_r}, r = n + 2, \ldots., 2m + 1$
and trace $\mathrm{A}_r = \sum_{i=1}^{n} \sigma^r_{ii} = 0.$

From Gauss' equation, we have

$$\begin{aligned} n^2\|H\|^2 &= 2\tau + \sum_{i=1}^{n} a_i^2 + \sum_{r=n+2}^{2m+1} \sum_{i,j=1}^{n} \left(\sigma^r_{ij}\right)^2 \\ &-\frac{1}{4}\left[n(n-1)(c+3) + 3(n-1)(c-1)\cos^2\theta - 2(n-1)(c+3-4k)\right] \\ &- \frac{1}{2}\left\{\|(\phi h)^T\|^2 - \|h^T\|^2 - \left(trace(\phi h)^T\right)^2 + \left(trace(h)^T\right)^2\right\} - 2(\mu + n - 2)trace(h)^T. \end{aligned} \tag{39}$$

On the other hand,

$$0 \le \sum_{i<j} \left(a_i - a_j\right)^2 = (n-1)\sum_{i=1}^{n} a_i^2 - 2\sum_{i<j} a_i a_j. \tag{40}$$

So, from (39) and (40), we obtain

$$n^2\|H\|^2 = \left(\sum_{i=1}^{n} a_i\right)^2 = \sum_{i=1}^{n} a_i^2 + 2\sum_{i<j} a_i a_j \le n\sum_{i=1}^{n} a_i^2,$$

which implies

$$\sum_{i=1}^{n} a_i^2 \ge \; n\,\|H\|^2. \tag{41}$$

Using (40) and (41), we have

$$n^2\|H\|^2 \geq 2\tau + n\|H\|^2 + \sum_{r=n+2}^{2m+1}\sum_{i,j=1}^{n}\left(\sigma_{ij}^r\right)^2$$
$$-\frac{1}{4}\left[n(n-1)(c+3)+3(n-1)(c-1)\cos^2\theta-2(n-1)(c+3-4k)\right] \tag{42}$$
$$-\frac{1}{2}\left\{\|(\phi h)^T\|^2-\|h^T\|^2-\left(trace(\phi h)^T\right)^2+\left(trace(h)^T\right)^2\right\}-2(\mu+n-2)trace(h)^T,$$

which is equivalent to (36).

If the equality case of (36) holds at a point $p \in M$, then from (40) and (42), we get $A_r = 0, r = n+2, \ldots\ldots, 2m+1$ and $a_1 = \ldots\ldots = a_n$. Hence p is a totally umbilical point. The converse is trivial.

With similar computational steps, as in Theorem 8, we have

Theorem9. Let M be an n-dimensional bi-slant submanifold satisfying g(X,ϕY) = 0 for any $X \in D_1$ and $Y \in D_2$, tangent to ξ in a (2m + 1)-dimensional (k,μ)-contact space form $\bar{M}(c)$. Then, we have

$$\|H\|^2 \geq \frac{2\tau}{n(n-1)} - \frac{1}{4n(n-1)}\left[n(n-1)(c+3)+6(d_1\cos^2\theta_1+d_2\cos^2\theta_2)(c-1)-2(n-1)(c+3-4k)\right]$$
$$-\frac{1}{2n(n-1)}\left\{\|(\phi h)^T\|^2-\|h^T\|^2-\left(trace(\phi h)^T\right)^2+\left(trace(h)^T\right)^2\right\}-\frac{2}{n(n-1)}(\mu+n-2)trace(h)^T,$$

where $2d_1 = dimD_1$ *and* $2d_2 = dimD_2$.

Corollary10. Let M be an n-dimensional semi-slant submanifold in a $(2m+1)$-dimensional (k,μ)-contact space form $\bar{M}$(c). Then, we have

$$\|H\|^2 \geq \frac{2\tau}{n(n-1)} - \frac{1}{4n(n-1)}\left[n(n-1)(c+3)+6(d_1+d_2\cos^2\theta)(c-1)-2(n-1)(c+3-4k)\right]$$
$$-\frac{1}{2n(n-1)}\left\{\|(\phi h)^T\|^2-\|h^T\|^2-\left(trace(\phi h)^T\right)^2+\left(trace(h)^T\right)^2\right\}-\frac{2}{n(n-1)}(\mu+n-2)trace(h)^T,$$

where $2d_1 = dimD_1$ *and* $2d_2 = dimD_2$.

Theorem11. Let M be an n-dimensional θ-slant submanifold tangent to ξ in a $(2m+1)$- dimensional (k,μ)-contact space form $\bar{M}(c)$. Then, for any integer s$(2 \leq s \leq n)$, at any point $p \in M$, we have

$$\|H\|^2 \geq \Theta_s(p) - \frac{1}{4n(n-1)}\left[n(n-1)(c+3) + 3(n-1)(c-1)\cos^2\theta - 2(n-1)(c+3-4k)\right]$$
$$- \frac{1}{2n(n-1)}\left\{\|(\phi h)^T\|^2 - \|h^T\|^2 - \left(trace(\phi h)^T\right)^2 + \left(trace(h)^T\right)^2\right\} - \frac{2}{n(n-1)}(\mu+n-2)trace(h)^T. \tag{43}$$

Proof. Let $\{e_1, e_2, \ldots, e_n\}$ be an orthonormal basis of T_pM. Denote by $L_{i_1 \ldots\ldots i_s}$ the s-plane section spanned by $e_{i_1}, \ldots\ldots e_{i_s}$. It follows from (16) and (17) that

$$\tau\left(L_{i_1 \ldots\ldots i_s}\right) = \frac{1}{2} \sum_{i \in \{i_1, \ldots, i_s\}} Ric_{L_{i_1 \ldots\ldots i_s}}(e_i), \tag{44}$$

$$\tau(p) = \frac{1}{C_{n-2}^{s-2}} \sum_{1 \leq i_1 < \ldots\ldots i_s \leq n} \tau\left(L_{i_1 \ldots i_s}\right). \tag{45}$$

Combining (18), (44) and (45), we get

$$\tau(p) \geq \frac{n(n-1)}{2}\, \Theta_s(p). \tag{46}$$

From (36) and (46), we obtain (43).

Now, we can state the following

Theorem12. Let M be an n-dimensional bi-slant submanifold tangent to ξ in a (2m + 1)-dimensional (k,μ)-contact space form $\bar{M}(c)$. Then, for any integer $s(2 \leq s \leq n)$, at any point p$\in M$, we have

$$\|H\|^2 \geq \Theta_s(p) - \frac{1}{4n(n-1)}\left[n(n-1)(c+3) + 6(d_1\cos^2\theta_1 + d_2\cos^2\theta_2)(c-1) - 2(n-1)(c+3-4k)\right]$$
$$- \frac{1}{2n(n-1)}\left\{\|(\phi h)^T\|^2 - \|h^T\|^2 - \left(trace(\phi h)^T\right)^2 + \left(trace(h)^T\right)^2\right\} - \frac{2}{n(n-1)}(\mu+n-2)trace(h)^T,$$

where $2d_1 = dimD_1$ *and* $2d_2 = dimD_2$.

Corollary13. Let M be an n-dimensional semi-slant submanifold tangent to ξ in a (2m + 1)-dimensional (k,μ)-contact space form $\bar{M}(c)$. Then, for any integer s$(2 \leq s \leq n)$, at any point p$\in M$, we have

$$\|H\|^2 \geq \Theta_s(p) - \frac{1}{4n(n-1)}\left[n(n-1)(c+3) + 6(d_1 + d_2\cos^2\theta)(c-1) - 2(n-1)(c+3-4k)\right]$$
$$- \frac{1}{2n(n-1)}\left\{\|(\phi h)^T\|^2 - \|h^T\|^2 - \left(trace(\phi h)^T\right)^2 + \left(trace(h)^T\right)^2\right\} - \frac{2}{n(n-1)}(\mu+n-2)trace(h)^T,$$

where 2d$_1$ = $dimD_1$ *and* $2d_2 = dimD_2$.

Corollary14. Let M be an n-dimensional invariant submanifold tangent to ξ in a (2m + 1)-dimensional (k,μ)-contact space form $\bar{M}(c)$. Then, for any integer s($2 \le s \le n$), at any point $p \in M$, we have

$$\|H\|^2 \geq \Theta_s(p) - \frac{1}{4n(n-1)}[n(c+3) + 3(c-1) - 2(c+3-4k)]$$
$$- \frac{1}{2n(n-1)}\left\{\|(\phi h)^T\|^2 - \|h^T\|^2 - \left(trace(\phi h)^T\right)^2 + \left(trace(h)^T\right)^2\right\} - \frac{2}{n(n-1)}(\mu + n - 2)trace(h)^T,$$

where $2d_1 = dimD_1$ *and* $2d_2 = dimD_2$.

Corollary15. Let M be an n-dimensional anti-invariant submanifold tangent to ξ in a (2m + 1)-dimensional (k,μ)-contact space form $\bar{M}(c)$. Then, for any integer s($2 \le s \le n$), at any point $p \in M$, we have

$$\|H\|^2 \geq \Theta_s(p) - \frac{1}{4n(n-1)}[n(c+3) - 2(c+3-4k)]$$
$$- \frac{1}{2n(n-1)}\left\{\|(\phi h)^T\|^2 - \|h^T\|^2 - \left(trace(\phi h)^T\right)^2 + \left(trace(h)^T\right)^2\right\} - \frac{2}{n(n-1)}(\mu + n - 2)trace(h)^T,$$

where $2d_1 = dimD_1$ *and* $2d_2 = dimD_2$.

Corollary16. Let M be an n-dimensional contact CR- submanifold tangent to ξ in a ($2m + 1$)-dimensional (k,μ)-contact space form $\bar{M}(c)$. Then, for any integer s($2 \le s \le n$), at any point $p \in M$, we have

$$\|H\|^2 \geq \Theta_s(p) - \frac{1}{4n(n-1)}[n(n-1)(c+3) + 6d_1(c-1) - 2(n-1)(c+3-4k)]$$
$$- \frac{1}{2n(n-1)}\left\{\|(\phi h)^T\|^2 - \|h^T\|^2 - \left(trace(\phi h)^T\right)^2 + \left(trace(h)^T\right)^2\right\} - \frac{2}{n(n-1)}(\mu + n - 2)trace(h)^T,$$

where $2d_1 = dimD_1$ *and* $2d_2 = dimD_2$.

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REFERENCES

Arslan, K., Ezentas, R., Mihai, I., Murathan, C. & Ozgur, C. 2001. Certain inequalities for submanifolds in (k,)-contact space forms. Bulletin of Australian Mathematical Society, 64 (2): 201-212.

Arslan, K., Ezentas, R., Mihai, I., Murathan, C. & Ozgur, C. 2002. Ricci curvature of submanifolds in Kenmotsu space forms. International Journal of Mathematics and Mathematical Sciences, Vol. 29 (12): 719-726.

Blair, D.E. 2002. Riemannian geometry of contact and symplectic manifolds. Progress in Mathematics, 203, Birkhauser Boston.

Blair,D.E., Koufogiorgos, T. & Papantoniou, B.J. 1995. Contact metric manifolds satisfying a nullity condition. Israel Journal of Mathematics, 91: 189-214.

Cabrerizo, J.L., Carriazo, A., Fernandez L.M. & Fernandez, M. 1999. Semi-slant submanifolds of Sasakian manifolds. Geometria Dedicata, 78(2): 183 -199.

Cabrerizo, J.L., Carriazo, A., Fernandez, L.M. & Fernandez, M. 2000. Slant submanifolds in Sasakian manifolds. Glasgow Mathematical Journal, 42(1): 125-138.

Chen, B.Y. 1999. Relations between Ricci curvature and shape operator for submanifolds with arbitrary co-dimensions. Glasgow Mathematical Journal, 41: 33-41.

Defever, F., Mihai, I. & Verstraelen, L. 1997. B.Y. Chen s inequality for C-totally real submanifolds in Sasakian space forms. Bollettino dell'Unione Matematica Italiana. 11: 365-374.

Hong, S. & Tripathi, M. M. 2005. On Ricci curvature of submanifolds of generalized Sasakian space forms. International Journal Pure and applied Mathematical Sciences, 2(2): 173 - 201.

Koufogiorgos, T. 1997. Contact Riemannian manifolds with constant -sectional curvature. Tokyo Journal of Mathematics, 20(1): 13-22.

Mihai, I. 2002. Ricci curvature of submanifolds in Sasakian space forms, Journal of the Australian Mathematical Society, 72(2): 247-256.

Tripathi, M. M. & Jeong-Sik Kim. 2004. On the concircular curvature tensor of a (k,)-manifold. Balkan Journal of Geometry and its Applications, 9(1): 104-114.

Yoon, D.W. 2006. Inequality for Ricci curvature of slant submanifolds in cosymplectic space forms, Turkish Journal of Mathematics, 30: 43 -56.

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منحنى ريكي المائل للفتحات الفرعية المائلة في (K, μ) أشكال الاتصال الفضائي

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خلاصة

لقد أسس الباحث B. Y Chen (1999) علاقة فاصلة تشتمل على الثوابت الحقيقية، تسمى انحناءات قطعية وانحناءات قياسية والثوابت غير الحقيقية الأساسية، ويسمى مربع متوسط الانحناء للفتحات الفرعية في شكل الفضاء الحقيقية مع بعد مشارك عشوائي في هذا البحث، تم تأسيس التفاوت ما بين منحنى ريكي ومربع متوسط الانحناء، وأيضاً بين منحني ريكي – s والمنحنى القياسي للميل، نصف الميل ومزدوج الميل للفتحات الفرعية في (k, μ) – أشكال الاتصال الفضائي.