Kuwait J. Sci. Eng. **36** (1B) pp. 211-230, 2009

Measuring the bullwhip effect in supply chains with a vector autoregressive demand process

S.K. CHAHARSOOGHI AND A. SADEGHI

Tarbiat Modares University, Department of Industrial Engineering, College of Engineering, P.O. Box 14115-143, Tehran, Iran.
E-mail: a.sadeghi@gazviniau.ac.ir

ABSTRACT

In this paper, we study the bullwhip effect problem in a two-echelon supply chain, which includes two products according to our assumptions. The main aim of the paper is to provide a measure for bullwhip effect that enables the analysis and reduction of this phenomenon in supply chains with two products. VAR (1) time series, a suitable pattern for demand modeling in a two product supply chain is utilized. The use of a simple moving average method for lead-time demand forecasting, and OUT ordering policy are due to their wide range application in the real world. A general form of bullwhip measure has been derived and demonstrated to prove that an explicit expression for bullwhip effect measure does not exist based on the present approach on the bullwhip measure. However, the bullwhip effect measure may still exist in limited cases. A/the Condition in which a bullwhip effect can be removed is described, and finally, the bullwhip effect in a two-product supply chain is demonstrated via a numerical example.

Keywords: Bullwhip effect; forecasting; supply chain; time series

INTRODUCTION

Nowadays, outsourcing has an important role in industrial environments. Consequently, manufacturers are settling in supply chain management systems (SCMs) that are growing at a very high rate (exponentially). Raw material suppliers, part manufacturers, final product assemblers, distributors, retailers, and final customers are various sectors of supply chains. Demand amplification is a major obstacle to achieve coordination and maintain harmony within different stages of supply chains. Many companies have observed increasing fluctuation in orders while moving up from downstream sites to upstream sites. The result is a loss of supply chain profitability. The first recorded documentation of this status was by Forrester (1958). He used the industrial dynamics approach to show amplification of demand variability among supply chains. After that time, many researchers such as Goodman (1974), Blinder (1982 & 1986), Blanchard (1983), Kahn (1987), Baganha & Cohen (1995), Metters (1997) continued investigation about ordering variation. Sterman (1989) developed the Beer Game at MIT. He proposed it as an evidence for existence demand amplification in supply chains. Lee *et al.* (1997) introduced five main causes of this phenomenon: demand forecast updating, order batching, price fluctuation, rationing, and non-zero lead-time (1997). Understanding these causes of the bullwhip effect can be useful for managers to find suitable solutions for haltering and controlling it.

Chen *et al.* (2000) quantified and derived a lower bound for the bullwhip effect in a simple supply chain. Dejonckheere *et al.* (2003) proposed a control theory approach for measuring bullwhip effect and suggested a new general replenishment rule that can reduce variance amplification significantly. Disney and Towill (2003) introduced an ordering policy that results in taming the bullwhip effect. Zhang (2004) considered three forecasting methods for a simple inventory control system and presented three measures for the bullwhip effect based on three forecasting methods. Kim *et al.* (2006) investigated stochastic lead-time and the role of information sharing on the bullwhip effect. Chandra & Grabis (2005) measured the bullwhip effect when order size is calculated according to multiple step forecasts using autoregressive models. Luong (2007) investigated the effects of autoregressive coefficient and lead-time on the bullwhip effect when MMSE forecasting method is used. Luong and Phien's (2007) research was based on the order of an autoregressive demand pattern. They showed that in a high order of demand pattern, the bullwhip effect could be reduced when lead-time increases. Makui & Madadi (2007) utilized the Lyapunov exponent and provided a measure for the bullwhip effect. They presented useful results on the behavior of the bullwhip effect by investigating the mathematical relationships. Gaalman & Disney (2007) investigated the behavior of the proportional order up to policy for ARMA (2, 2) demand with arbitrary lead-times. Jaksic & Rusjan (2008) demonstrated that certain replenishment policies could be inducers of the bullwhip effect.

Although many researches have investigated the bullwhip effect, more investigations are still needed to study it, minimize its effect, and quantify it in order to propose solutions for complex supply chains. This research considers a two-echelon supply chain consisting of one retailer and one supplier. This supply chain produces two products in which the demand pattern is based on a first order vector autoregressive (VAR (1)) demand process. Ordering policy for each product is: "order up to policy" and the forecasting method is: "moving average." According to these assumptions, the bullwhip effect is quantified in a two-product supply chain then it was demonstrated via a numerical example.

SUPPLY CHAIN ASSUMPTIONS

In this paper, a two-echelon supply chain consists of one retailer and one supplier. Retailer encounters market demand and orders it from the supplier according to his/her ordering policy. Supplier complies with received orders. Hence, demand information flow is from retailer to supplier, whereas product flow is from supplier to retailer. There are two products in the supply chain, in essence, retailer has to meet demand for two products. Each product demand has an effect on demand of the other product. Therefore, we must consider a suitable pattern for demand modeling that includes relationship between products. In the next part, we explain a proposed demand model.

Demand Pattern

Considering relationship between products, a first order vector autoregressive process, VAR (1) is taken into consideration for demand modeling. Because of VAR (1) properties, it can be used not only for demand modeling in a two product supply chain, but also in multi-product supply chains. In our VAR (1) model, demand of each product affects demand of the other one as follows: demand of each product in every period depends on the demand of the same product and the demand of the other product in the last period. For example, consider a supply chain of dairy products in which two products, cheese and butter, are produced. In this supply chain, demand of cheese in period (t) is dependent on the demand of cheese in period (t-1), as is the demand of butter in period ($t-1$). This situation also exists for the demand of butter in each period. Suppose that (D_t^i) is the demand of (i^{th}) product in period (t). The, VAR (1) process for the demand of the two products can hence be determined by:

$$\begin{cases} D_t^1 = \phi_{11} D_{t-1}^1 + \phi_{12} D_{t-1}^2 + a_t^1 \\ D_t^2 = \phi_{21} D_{t-1}^1 + \phi_{22} D_{t-1}^2 + a_t^2 \end{cases}, \tag{1}$$

where, $a_t^i (i=1,2$ and $t=1,2,\ldots)$ is the forecast error of (i^{th}) product for the period (t) and is i.i.d. normally distributed with mean zero and variance σ_{ii}. The relationship between demands of two products is clarified in equation (1). In order for the demand process to be stationary, the following relationship must hold:

$$\left| \frac{(\phi_{11} + \phi_{22}) \pm \sqrt{(\phi_{11} - \phi_{22})^2 + 4\phi_{12}\phi_{21}}}{2} \right| \prec 1.$$

It can be shown that variance of each product (i.e.γ_{11}, γ_{22}) can be derived by:

$$\gamma_{11} = \frac{\sigma_{11}[(1-\phi_{11}\phi_{22})(1-\phi_{22}^2) - \phi_{12}\phi_{21}(1+\phi_{22}^2)] + 2\phi_{12}\sigma_{12}[\phi_{11}(1-\phi_{22}^2) + \phi_{12}\phi_{21}\phi_{22}] + \phi_{12}^2\sigma_{22}[1-\phi_{12}\phi_{21}+\phi_{11}\phi_{22}]}{1+(\phi_{11}\phi_{22}-\phi_{12}\phi_{21}-1)(\phi_{11}\phi_{22}+\phi_{12}\phi_{21}+\phi_{11}^2+\phi_{22}^2)+(\phi_{12}\phi_{21}-\phi_{11}\phi_{22})^3} \quad (2)$$

$$\gamma_{22} = \frac{\phi_{21}^2\sigma_{11}[1+\phi_{11}\phi_{22}-\phi_{12}\phi_{21}] + 2\phi_{21}\sigma_{12}[\phi_{22}(1-\phi_{11}^2) + \phi_{11}\phi_{12}\phi_{21}] + \sigma_{22}[(1-\phi_{11}\phi_{22})(1-\phi_{11}^2) - \phi_{12}\phi_{21}(1+\phi_{11}^2)]}{1+(\phi_{11}\phi_{22}-\phi_{12}\phi_{21}-1)(\phi_{11}\phi_{22}+\phi_{12}\phi_{21}+\phi_{11}^2+\phi_{22}^2)+(\phi_{12}\phi_{21}-\phi_{11}\phi_{22})^3} \quad (3)$$

Moreover, covariance between two products (i.e.γ_{12}) can be determined by equation (4):

$$\gamma_{12} = \frac{\phi_{21}\sigma_{11}[\phi_{11}(1-\phi_{22}^2) + \phi_{12}\phi_{21}\phi_{22}] + \sigma_{12}[(1-\phi_{11}^2)(1-\phi_{22}^2) - \phi_{12}^2\phi_{21}^2] + \phi_{12}\sigma_{22}[\phi_{22}(1-\phi_{11}^2) + \phi_{11}\phi_{12}\phi_{21}]}{1+(\phi_{11}\phi_{22}-\phi_{12}\phi_{21}-1)(\phi_{11}\phi_{22}+\phi_{12}\phi_{21}+\phi_{11}^2+\phi_{22}^2)+(\phi_{12}\phi_{21}-\phi_{11}\phi_{22})^3} \quad (4)$$

Ordering Policy

In this research, we consider an order up to (OUT) policy for a retailer inventory control system. OUT policy is a standard ordering algorithm in many MRP systems (Gilbert 2005). OUT policy is easy to understand and is often utilized by companies to coordinate orders from suppliers where setup costs may be reasonably ignored (Gaalman & Disney 2007). In an OUT system, the level of inventory is reviewed periodically, and an order is placed to bring the inventory position to a predefined level. In a considered inventory control system, at the beginning of each period, the inventory position is observed and in order to raise the inventory level to (S_t), an order (Q_t)/ is placed. After the order is placed, customer demand (D_t) occurs. This sequence is consistent with equation (5):

$$Q_t = S_t - S_{t-1} + D_{t-1} \quad (5)$$

Using a base stock policy, order up to level (S_t) at the beginning of period (t) can be determined by equation (6):

$$S_t = \hat{D}_t^L + z.\hat{\sigma}_t^L \quad (6)$$

where $\hat{D}_t^L$ is the lead-time demand forecast and $\hat{\sigma}_t^L$ is the standard deviation of the lead time demand forecast error. Moreover, z is normal z score and can be determined by a normal table based on the favorable service level of the inventory system. Substituting from equation (6), for stock values (S_t) and (S_{t-1}), into equation (5), the result would be:

$$Q_t = \hat{D}_t^L - \hat{D}_{t-1}^L + z(\hat{\sigma}_t^L - \hat{\sigma}_{t-1}^L) + D_{t-1} \quad (7)$$

Since i.i.d., then each of the products is ordered independently. Thus, equation (7) can be used for both of them separately according to their parameters.

Forecasting Method

Because of the lead-time between placing an order and receiving the products into stock, we need to forecast demand (Gaalman & Disney 2007). Among the forecasting methods, moving average and exponential smoothing models have been used widely in the real world in different industrial factories because of their ease of use, flexibility, and robustness in dealing with non-linear demand processes (Silver *et al.*, 2000). In this research, we suggest that a retailer uses the moving average forecasting method to forecast lead-time demand. By definition, we have:

$$\bar{D}_t = \frac{\sum_{j=1}^{p} D_{t-j}}{p} \quad . \tag{8}$$

Therefore, lead-time demand estimation can be expressed as follows:

$$\hat{D}_t^L = L.\bar{D}_t \quad . \tag{9}$$

Substituting for moving average demand (D_t) from equation (8) into equation (9) would result in:

$$\hat{D}_t^L = L\left(\frac{\sum_{j=1}^{p} D_{t-j}}{p}\right) \quad . \tag{10}$$

Equation (10) can be used for lead-time demand forecasting for both products.

QUANTIFYING THE BULLWHIP EFFECT

Many investigations on the bullwhip effect are developed by a ratio of order quantity that is ordered to a supplier and a variance of market demand that is seen by a retailer. This definition for the bullwhip effect measurement is due to its nature: the amplification of demand while moving from upstream to downstream in supply chains. Therefore, the ratio expressed in equation (11) is a direct translation for measuring the bullwhip effect phenomenon:

$$BE = \frac{Var(Q_t)}{Var(D)} \quad . \tag{11}$$

Thus, the above relationship can provide a measurement of the bullwhip effect. In addition, according to equation (11), it is sufficient to determine the variance of order and the variance of demand for quantifying the bullwhip effect. However, let us introduce the following proposition that would help in determining variance of orders.

Proposition 1:

A Standard deviation of lead-time demand forecast error for each product is constant during periods and does not depend on (t). Equation (12) can represent it as:

$$(\hat{\sigma}_t^L)^2 = (L + \frac{L^2}{p})\gamma + 2\sum_{i=1}^{L-1}(L-i)\gamma(i) + 2(\frac{L}{p})[(\frac{L}{p})(\sum_{i=1}^{p-1}(p-i)\gamma(i)) - \sum_{j=0}^{p-1}\sum_{i=j+1}^{j+L}\gamma(i)] \quad (12)$$

in which γ is a covariance of each product and $\gamma(i) = Cov(D_t, D_{t+i})$. In fact, the above proposition implies that $\hat{\sigma}_t^L = \hat{\sigma}_{t-1}^L$.

Proof:

To simplify calculations, we did not consider each product separately in equation (12) as the result can be used for both of them. By variance definition, we have:

$$(\hat{\sigma}_t^L)^2 = Var(D_t^L - \hat{D}_t^L) = Var(D_t^L) + Var(\hat{D}_t^L) - 2Cov(D_t^L, \hat{D}_t^L) \ ,$$

in which:

$$D_t^L = D_t + D_{t+1} + ... + D_{t+L-1} = \sum_{i=0}^{L-1} D_{t+i} \quad , \quad (13)$$

where, $\hat{D}_t^L$ has been derived before in equation (10). For appointment $(\hat{\sigma}_t^L)^2$, we must determine three relationships for $Var(D_t^L)$, $Var(\hat{D}_t^L)$ and $Cov(D_t^L, \hat{D}_t^L)$.

Derivation of: $Var(D_t^L)$

By definition, we have $Cov(D_t, D_{t+k}) = \gamma(k)$. Moreover, based on stationary conditions:

$$Var(D_t^L) = \gamma + \gamma + ... + \gamma + 2\gamma(1) + 2\gamma(2) + ... + 2\gamma(L-1) +$$

$$2\gamma(1) + 2\gamma(2) + ... + 2\gamma(L-2) + 2\gamma(1) + 2\gamma(2) + ... + 2\gamma(L-3) + ... + 2\gamma(1)$$

$$= L\gamma + 2[\gamma(1) + \gamma(1) + ... + \gamma(1) + \gamma(2) + ... + \gamma(2) + ... + \gamma(L-1)]$$

$$= L\gamma + 2[(L-1)\gamma(1) + (L-2)\gamma(2) + ... + \gamma(L-1)]$$

$$= L\gamma + 2[\sum_{i=1}^{L-1}(L-i)\gamma(i)]$$

Derivation of: $Var(\hat{D}_t^L)$

Consider equation (10) which shows the lead-time demand forecast. According to variance calculation rules, we can provide the variance of the lead-time demand forecast as follows:

$$Var(\hat{D}_t^L) = (\frac{L}{p})^2 Var(\sum_{i=1}^{p} D_{t-i}) = (\frac{L}{p})^2 Var(D_{t-1} + D_{t-2} + ... + D_{t-p})$$

$$= (\frac{L}{p})^2 [\gamma + \gamma + ... + \gamma + 2\gamma(1) + 2\gamma(2) + ... + 2\gamma(p-1) +$$

$$2\gamma(1) + 2\gamma(2) + ... + 2\gamma(p-2) +$$

$$2\gamma(1) + 2\gamma(2) + ... + 2\gamma(p-3) + ... + 2\gamma(1)]$$

$$= (\frac{L}{p})^2 [p\gamma + 2(\gamma(1) + \gamma(1) + ... + \gamma(1) + \gamma(2) + ... + \gamma(2) + ... + \gamma(p-1))]$$

$$= (\frac{L}{p})^2 [p\gamma + 2((p-1)\gamma(1) + (p-2)\gamma(2) + ... + \gamma(p-1))]$$

$$= (\frac{L}{p})^2 [p\gamma + 2(\sum_{i=1}^{p-1} (p-i)\gamma(i))] \quad .$$

Derivation of: $Cov(D_t^L, \hat{D}_t^L)$

Using the definition of D_t^L and $\hat{D}_t^L$ that are mentioned before, we have:

$$Cov(D_t^L, \hat{D}_t^L) = Cov[(D_t + D_{t+1} + ... + D_{t+L-1}), (\frac{L}{p})(D_{t-1} + D_{t-2} + ... + D_{t-p})]$$

$$= (\frac{L}{p})[Cov(D_t, D_{t-1}) + Cov(D_t, D_{t-2}) + ... + Cov(D_t, D_{t-p}) +$$

$$Cov(D_{t+1}, D_{t-1}) + Cov(D_{t+1}, D_{t-2}) + ... + Cov(D_{t+1}, D_{t-p}) + ... +$$

$$Cov(D_{t+L-1}, D_{t-1}) + Cov(D_{t+L-1}, D_{t-2}) + ... + Cov(D_{t+L-1}, D_{t-p})]$$

$$= (\frac{L}{p})[\gamma(1) + \gamma(2) + ... + \gamma(p) + \gamma(2) + \gamma(3) + ... + \gamma(p+1) + ...$$

$$+ \gamma(L) + \gamma(L+1) + ... + \gamma(L+p-1)]$$

$$= (\frac{L}{p})[\sum_{i=1}^{L} \gamma(i) + \sum_{i=2}^{L+2} \gamma(i) + ... + \sum_{i=p}^{L+p-1} \gamma(i)]$$

$$= (\frac{L}{p})[\sum_{j=0}^{p-1} \sum_{i=j+1}^{j+L} \gamma(i)] \quad .$$

Now consider again the previous variance relationship:

$$(\hat{\sigma}_t^L)^2 = Var(D_t^L) + Var(\hat{D}_t^L) - 2Cov(D_t^L, \hat{D}_t^L) \quad .$$

Substituting three derived variances in the above equation, we have:

$$= L\gamma + 2[\sum_{i=1}^{L-1}(L-i)\gamma(i)] + (\frac{L}{p})^2[p\gamma + 2(\sum_{i=1}^{p-1}(p-i)\gamma(i))] - 2(\frac{L}{p})[\sum_{j=0}^{p-1}\sum_{i=j+1}^{j+L}\gamma(i)]$$
$$= (L + \frac{L^2}{p})\gamma + 2\sum_{i=1}^{L-1}(L-i)\gamma(i) + 2(\frac{L}{p})[(\frac{L}{p})(\sum_{i=1}^{p-1}(p-i)\gamma(i)) - \sum_{j=0}^{p-1}\sum_{i=j+1}^{j+L}\gamma(i)] \quad .$$

Thus, it is clear that the above relationship does not depend on (t) and merely depends on problem parameters(L, p, γ).

Proposition (1) implies that standard the deviation of lead-time demand for each product does not impress the bullwhip effect and is only needed for the determination of (S_t) in OUT) for each product in every period.

According to the result of proposition (1), equation (7) can be reduced to equation (13) that is used to derive the variance of orders. After summarizing the order quantity relationship, we have:

$$Q_t = \hat{D}_t^L - \hat{D}_{t-1}^L + D_{t-1} \quad . \tag{14}$$

Variance of Order Quantity

To provide variance of order we should have an explicit expression for order measure. Thus, substituting equation (10) in equation (13) would result in:

$$Q_t = (1 + \frac{L}{p})D_{t-1} - (\frac{L}{p})D_{t-p-1} \quad . \tag{15}$$

Equation (14) provides the order quantity of a product based on OUT policy when a retailer uses the moving average forecasting method for lead-time demand estimation. The above relationship can be used for both products if we replace relevant parameters in equation (14) as follows:

$Q_t^i = (1 + \frac{L_i}{p_i})D_{t-1}^i - (\frac{L_i}{p_i})D_{t-p-1}^i$ and $i = 1,2$.

Specifying variance of order quantity, we have:

$$Var(Q_t) = (1 + \frac{L}{p})^2\gamma + (\frac{L}{p})^2\gamma - 2(1 + \frac{L}{p})(\frac{L}{p})\gamma(p) \quad . \tag{16}$$

Thus, to determine $Var(Q_t)$ it is necessary to calculate $\gamma(p)$. Substituting from equation (15) in equation (11) results in a relationship for the bullwhip effect measurement that can be used for both products. This general form of the bullwhip effect measurement is defined by:

$$BE = \frac{Var(Q_t)}{Var(D)} = \frac{(1+\frac{L}{p})^2\gamma + (\frac{L}{p})^2\gamma - 2(1+\frac{L}{p})(\frac{L}{p})\gamma(p)}{\gamma}$$

$$BE = [(1+\frac{L}{p})^2 + (\frac{L}{p})^2] - 2(1+\frac{L}{p})(\frac{L}{p})[\frac{\gamma(p)}{\gamma}] \qquad (17)$$

The following demonstrates conditions in which the bullwhip effect exists in our two-product supply chain.

Problem: When does the bullwhip effect not exist in a two-product supply chain, according to the proposed model assumptions?

Equation (17) may be viewed as a special case of equation (16), similar to Chen *et al.* (2000) in a single product supply chain:

$$BE = 1 + (\frac{2L}{p} + \frac{2L^2}{p^2})(1 - \frac{\gamma(p)}{\gamma}) \quad . \qquad (18)$$

We know if $BE \leq 1$, then a steady state exists between demand and orders in the supply chain. Hence, we do not have the bullwhip effect in the supply chain. Therefore, we must have $1 + (\frac{2L}{p} + \frac{2L^2}{p^2})(1 - \frac{\gamma(p)}{\gamma}) \leq 1$ or $(\frac{2L}{p} + \frac{2L^2}{p^2})(1 - \frac{\gamma(p)}{\gamma}) \leq 0$. In our supply chain, we can prove that $\gamma \succ \gamma(p)$ (see Appendix A) results in $(1 - \frac{\gamma(p)}{\gamma}) \succ 0$. Because $L \succ 0$ and $p \succ 0$, therefore, $(\frac{2L}{p} + \frac{2L^2}{p^2})(1 - \frac{\gamma(p)}{\gamma}) \succ 0$, or $BE \succ 1$. In addition, if $p \to \infty$ then:

$(\frac{2L}{p} + \frac{2L^2}{p^2})(1 - \frac{\gamma(p)}{\gamma}) \to 0$, or $BE \to 1$. Accordingly, we can conclude:

In a two-product supply chain base on our assumptions, the bullwhip effect always exists except status in which the number of observations in moving average calculations increases as much as possible. Consequently, the bullwhip effect can be removed from the supply chain by increasing the number of observation, *p*.

Can consider equation (16) to quantify the bullwhip effect in the proposed supply chain. Obviously, equation (16) does not submit an explicit expression of the bullwhip effect because there is no specific relationship for $\gamma(p)$. However, knowing p, we can compute $\gamma(p)$; hence, the bullwhip effect can be calculated. In this research, we derived $\gamma(p)$ for some specific values of p as the bullwhip effect relationships are needed for analytical purposes in the fourth section of this paper. To provide a relationship for the bullwhip effect measurement for both products, let us consider $BE^i_{p_i}$ to be the bullwhip effect of (i^{th}) product ($i = 1, 2$).

In addition, we will use L_i and p_i to show lead-time and the number of observations in the forecasting method for each of the products (i.e. L and p), respectively. In addition, we consider $\gamma_{ii}(p_i)$ as a covariance between two measures of i^{th} product demand at lag p_i. Moreover, γ_{ii} represents demand variance of i^{th} product. Accordingly, substituting the aforementioned terms in equation (16) results in equation (19). Thus the bullwhip effect for each product will be:

$$BE^i_{p_i} = [(1+\frac{L_i}{p_i})^2 + (\frac{L_i}{p_i})^2] - 2(1+\frac{L_i}{p_i})(\frac{L_i}{p_i})[\frac{\gamma_{ii}(p_i)}{\gamma_{ii}}] \quad i = 1,2 \quad . \tag{19}$$

Derivation of: $\gamma_{ii}(p_i)$:

The covariance matrix function for VAR (1) process is as follows:

$$\Gamma(p_i) = \begin{cases} \Gamma(-1)\phi'_1 + \Sigma & p_i = 0 \\ \Gamma(p_i - 1)\phi'_1 = \Gamma(0)(\phi'_1)^{p_i} & p_i \geq 1 \end{cases} \tag{20}$$

It is clear that we need $\Gamma(0)$ and $\phi'^{p_i}_1$ to calculate $\gamma_{ii}(p_i)$ with $p_i \geq 1$. In the last section, we described that we do not have any explicit relationship for $\gamma_{ii}(p_i)$. Indeed, equation (20) shows that due to the non-existence of any determined form of $\phi'^{p_i}_1$ when ϕ'_1 is a [2x2] matrix and p_i is greater than one, we cannot provide a general formula for $\gamma_{ii}(p_i)$.

Now consider $\Gamma(0) = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{12} & \gamma_{22} \end{bmatrix}$ in which γ_{11} and γ_{22} are demand variance of the first and second product, respectively, and γ_{12} is the covariance between demands of two products. In addition, ϕ_1 is a coefficient matrix in a VAR (1) process for a two-product supply chain based on equation (1) and can be specified by $\phi_1 = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$. So if we know p_i, then we can obtain $\gamma_{ii}(p_i)$ by equation (18). In the next section, $\gamma_{ii}(p_i)$ is determined for $p_i = 1,2,3$ and $i = 1$, 2. Then the bullwhip effect measurements for both products are calculated.

Derivation of: BE^1_1 *and* BE^2_1

When we use only the last observation in the moving average forecasting method (i.e.$p_i = 1$), matrices $\Gamma(1)$ and ϕ'_1 are needed for specifying the bullwhip effect. Consequently, $\Gamma(1) = \Gamma(0)\phi'_1$ and $\Gamma(0) = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{12} & \gamma_{22} \end{bmatrix}$. Hence,

$$\Gamma(1) = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{12} & \gamma_{22} \end{bmatrix} \begin{bmatrix} \phi_{11} & \phi_{21} \\ \phi_{12} & \phi_{22} \end{bmatrix} \text{ or } \Gamma(1) = \begin{bmatrix} \gamma_{11}\phi_{11} + \gamma_{12}\phi_{12} & \gamma_{11}\phi_{21} + \gamma_{12}\phi_{22} \\ \gamma_{12}\phi_{11} + \gamma_{22}\phi_{12} & \gamma_{12}\phi_{21} + \gamma_{22}\phi_{22} \end{bmatrix}.$$

Thus $\gamma_{11}(1) = \gamma_{11}\phi_{11} + \gamma_{12}\phi_{12}$ and $\gamma_{22}(1) = \gamma_{12}\phi_{21} + \gamma_{22}\phi_{22}$. Therefore, using equation (19) the bullwhip effect for the first product can be provided by:

$$BE_1^1 = [(1 + L_1)^2 + (L_1)^2] - 2(1 + L_1)(L_1)[\phi_{11} + (\frac{\gamma_{12}}{\gamma_{11}})\phi_{12}] \quad . \tag{21}$$

Moreover, for the second product we have:

$$BE_1^2 = [(1 + L_2)^2 + (L_2)^2] - 2(1 + L_2)(L_2)[\phi_{22} + (\frac{\gamma_{12}}{\gamma_{22}})\phi_{21}] \quad . \tag{22}$$

Derivation of: BE_2^1 and BE_2^2

To determine $\gamma_{11}(2)$ and $\gamma_{22}(2)$, we need the square of the transposed coefficient matrix, ϕ_1'. Based on matrices rules, we have $\phi_1'^2 = \begin{bmatrix} \phi_{11}^2 + \phi_{21}\phi_{12} & \phi_{21}(\phi_{11} + \phi_{22}) \\ \phi_{12}(\phi_{11} + \phi_{22}) & \phi_{22}^2 + \phi_{21}\phi_{12} \end{bmatrix}$. Hence, by equation (18) we have: $\Gamma(2) = \Gamma(0)(\phi_1')^2$. Consequently, $\gamma_{11}(2) = \gamma_{11}(\phi_{11}^2 + \phi_{21}\phi_{12}) + \gamma_{12}\phi_{12}(\phi_{11} + \phi_{22})$ and $\gamma_{22}(2) = \gamma_{12}\phi_{21}(\phi_{11} + \phi_{22})$ $+\gamma_{22}(\phi_{22}^2 + \phi_{21}\phi_{12})$. Substituting for $\gamma_{11}(2)$ and $\gamma_{22}(2)$ in equation (19) results in a bullwhip effect relationship for the first product:

$$BE_2^1 = [(1 + \frac{L_1}{2})^2 + (\frac{L_1}{2})^2] - 2(1 + \frac{L_1}{2})(\frac{L_1}{2})[(\phi_{11}^2 + \phi_{21}\phi_{12}) + \phi_{12}(\phi_{11} + \phi_{22})(\frac{\gamma_{12}}{\gamma_{11}})]. \tag{23}$$

The bullwhip effect measure for the second product can be provided too:

$$BE_2^2 = [(1 + \frac{L_2}{2})^2 + (\frac{L_2}{2})^2] - 2(1 + \frac{L_2}{2})(\frac{L_2}{2})[(\phi_{22}^2 + \phi_{21}\phi_{12}) + \phi_{21}(\phi_{11} + \phi_{22})(\frac{\gamma_{12}}{\gamma_{22}})]. \tag{24}$$

In fact, equations (21) and (22) show relationships for quantifying the bullwhip effect, when a retailer uses only the last two observations for lead-time demand forecasting for both products.

Derivation of: BE_3^1 and BE_3^2

To provide $\gamma_{11}(3)$ and $\gamma_{22}(3)$, we need to calculate $\phi_1'^3$. The steps are similar to the previous sections and result in:

$$\phi_1'^{\,3} = \begin{bmatrix} \phi_{11}^3 + \phi_{21}\phi_{12}(2\phi_{11} + \phi_{22}) & \phi_{21}(\phi_{11}^2 + \phi_{21}\phi_{12} + \phi_{11}\phi_{22} + \phi_{22}^2) \\ \phi_{12}(\phi_{11}^2 + \phi_{11}\phi_{22} + \phi_{21}\phi_{12} + \phi_{22}^2) & \phi_{22}^3 + \phi_{21}\phi_{12}(\phi_{11} + 2\phi_{22}) \end{bmatrix}$$

$$BE_3^1 = [(1+\frac{L_1}{3})^2 + (\frac{L_1}{3})^2] - 2(1+\frac{L_1}{3})(\frac{L_1}{3})[\phi_{11}^3 + \phi_{21}\phi_{12}(2\phi_{11}+\phi_{22}) + \phi_{12}(\phi_{11}^2 + \phi_{11}\phi_{22} + \phi_{21}\phi_{12} + \phi_{22}^2)(\frac{\gamma_{12}}{\gamma_{11}})] \quad (25)$$

$$BE_3^2 = [(1+\frac{L_2}{3})^2 + (\frac{L_2}{3})^2] - 2(1+\frac{L_2}{3})(\frac{L_2}{3})[\phi_{22}^3 + \phi_{21}\phi_{12}(\phi_{11}+2\phi_{22}) + \phi_{21}(\phi_{11}^2 + \phi_{21}\phi_{12} + \phi_{11}\phi_{22} + \phi_{22}^2)(\frac{\gamma_{12}}{\gamma_{22}})]. \quad (26)$$

Because it is necessary to determine BE_4^1, BE_4^2, BE_5^1 and BE_5^2 for our analytical approach in the next section, we have provided them in Appendix B.

Note: If we represent ϕ_1' to the power of p by $\phi_1'^{\,p} = \begin{bmatrix} \phi_1'^{\,p}(1,1) & \phi_1'^{\,p}(1,2) \\ \phi_1'^{\,p}(2,1) & \phi_1'^{\,p}(2,2) \end{bmatrix}$, considering past relationships about the bullwhip effect, we can provide general forms of the bullwhip effect for two products as follow:

$$BE_{p_1}^1 = (1+\frac{L_1}{p_1})^2 + (\frac{L_1}{p_1})^2 - 2(1+\frac{L_1}{p_1})(\frac{L_1}{p_1})[\phi_1'^{\,p_1}(1,1) + (\frac{\gamma_{12}}{\gamma_{11}})\phi_1'^{\,p_1}(2,1)]$$

$$BE_{p_2}^2 = (1+\frac{L_2}{p_2})^2 + (\frac{L_2}{p_2})^2 - 2(1+\frac{L_2}{p_2})(\frac{L_2}{p_2})[\phi_1'^{\,p_2}(2,2) + (\frac{\gamma_{12}}{\gamma_{11}})\phi_1'^{\,p_2}(1,2)] \; .$$

These relationships can be used for quantifying the bullwhip effect when p is large and we intend to obtain the matrix $\phi_1'^{\,p}$ by mathematical software.

NUMERICAL EXAMPLE

In this section, an analytical discussion about the bullwhip effect behavior in a two-product supply chain is represented using a numerical example. Consider the demand process of two products in a supply chain defined by:

$\begin{cases} D_t^1 = 0.7D_{t-1}^1 + 0.6D_{t-1}^2 + a_t^1 \\ D_t^2 = 0.2D_{t-1}^1 + 0.5D_{t-1}^2 + a_t^2 \end{cases}$. In this demand process, we have: $\phi_1 = \begin{bmatrix} 0.7 & 0.6 \\ 0.2 & 0.5 \end{bmatrix}$ and according to the stationary condition, we must have: $\left| \dfrac{(\phi_{11}+\phi_{22}) \pm \sqrt{(\phi_{11}-\phi_{22})^2 + 4\phi_{12}\phi_{21}}}{2} \right| \prec 1.$

Substituting the coefficients will result in:

$$\left|\frac{(0.7+0.5)+\sqrt{(0.7-0.5)^2+4(0.6)(0.2)}}{2}\right| = 0.96 \quad ;\text{and}\quad \left|\frac{(0.7+0.5)-\sqrt{(0.7-0.5)^2+4(0.6)(0.2)}}{2}\right| = 0.23.$$

Therefore, the stationary condition is satisfied. Now, we can obtain the bullwhip effect measure for different values of L and limited measures of p ($p = 1,2,3,4,5$) for two products by the previous determined relationships. For simplicity in mathematical calculations, we assume that error terms have standard normal distribution and are uncorrelated. This hypothesis does not affect the generality of the problem. Table 1(a) and Table 1(b) contain various values of the bullwhip effect for the first and second product, respectively.

Table 1(a). Bullwhip effect values for p1

		p1				
		1	**2**	**3**	**4**	**5**
L1	1	1.215	1.142	1.116	1.103	1.095
	2	1.644	1.377	1.291	1.248	1.222
	3	2.287	1.708	1.524	1.434	1.381
	4	3.145	2.132	1.814	1.661	1.571
	5	4.218	2.651	2.164	1.93	1.793
	6	5.505	3.265	2.571	2.24	2.047

Table 1(b). Bullwhip effect values for p2

		p2				
		1	**2**	**3**	**4**	**5**
L2	1	1.73	1.374	1.255	1.198	1.165
	2	3.191	1.997	1.638	1.476	1.386
	3	5.383	2.869	2.148	1.832	1.661
	4	8.305	3.99	2.786	2.268	1.992
	5	11.96	5.36	3.551	2.783	2.378
	6	16.34	6.979	4.444	3.378	2.819

The following figures depict the bullwhip effect behavior, while L and p are varying for each of the products separately. It is clear that the bullwhip effect is related to lead-time directly and is relevant to number of observations in moving average calculations (p), reversely. Fig. 1(a) shows that the bullwhip effect of the first product increases when its lead-time increases. A steady rise can be seen for $p = 2$, 3, 4, 5; when the $p = 1$ the bullwhip effect curve dramatically increases. The difference between the bullwhip effects when p changes from 1 to 2 indicates that only the last observation is not sufficient for lead-time demand forecasting and causes a huge demand fluctuation.

Fig. 1(b) shows the bullwhip effect of the second product with respect to its lead-time. All above notes about the bullwhip effect of the first product is valid for the second product, but the difference between bullwhip effects for $p = 1$ and $p = 2$ is more than the first product. Nevertheless, a considerable rise in the bullwhip effect curve is evident, while we are using the last observation alone for lead-time demand forecasting.

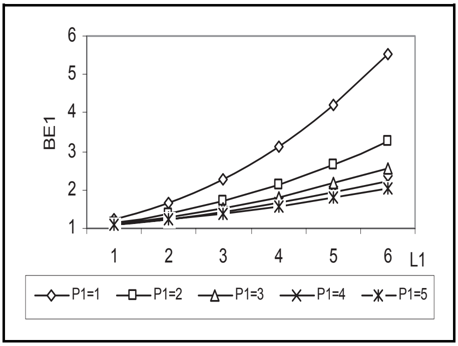

Fig. 1(a). Bullwhip effect variation with respect to L1.

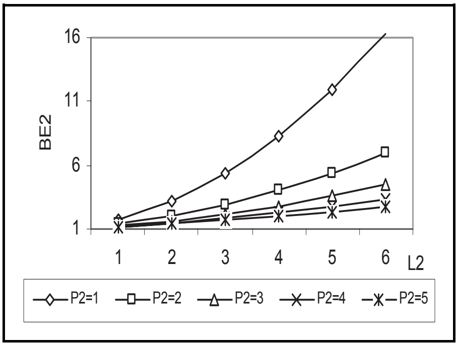

Fig. 1(b). Bullwhip effect variation with respect to L2.

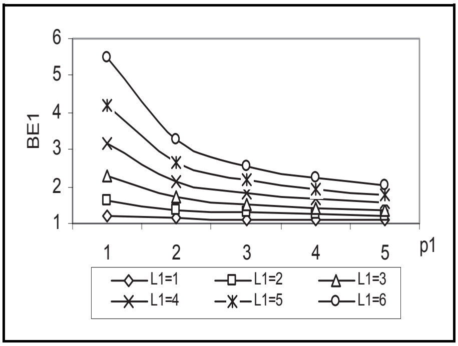

Fig. 2(a). Bullwhip effect variation with respect to p1.

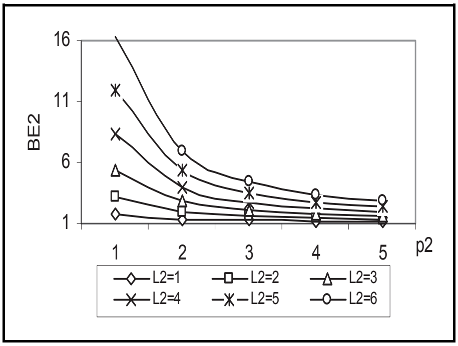

Fig. 2(b). Bullwhip effect variation with respect to p2.

Fig. 2(a) depicts bullwhip effect variations of the first product with respect to number of observations in lead-time demand forecasting. It is clear that more observations result in less bullwhip effect. In addition, a sharp fall can be seen when the number of observations changes from 1 to 2. This is the same result in Fig. (1), and it means that using only the last observation in lead-time demand forecasting is not satisfactory. Moreover, Fig. 2(a) shows that the slope of the bullwhip effect curve decreases dramatically when lead-time increases from 1 to 6. In the other words, if lead-time is large, (i.e. 6), then increasing *p* from 1 to 2 can reduce the bullwhip effect considerably.

Finally, Fig. 2(b) shows the bullwhip effect curve for the second product with respect to the number of observations, *p*. It is clear that the bullwhip effect decreases when the number of observations increases. A dramatic fall can be seen when the lead-time of the second product increases from 1 to 2. The slope of the curves between the bullwhip effects of two products differs in Fig. 2(a) and Fig. 2(b). A comparison between these two figures shows that the slant of the bullwhip effect graph for the first product is less than the second one.

As a result, we can conclude by the above graphs as well as by Table 1(a) and 1(b) that decreasing lead-time along with increasing the number of observations in lead-time demand forecasting based on the moving average forecasting method can reduce the bullwhip effect of each product. Accordingly, in our example, the minimum measure of the bullwhip effect occurs when $L = 1$ and $p = 5$, and its measure is 1.0952 for the first product and is equal to 1.1653 for the second one. In addition, when lead-time is long and the number of observations is small, we encounter the maximum value of the bullwhip effect. In fact, a longer L and a smaller p result in larger the bullwhip effects for both products. Therefore, the maximum value of the bullwhip effect for two products is 5.5051 and 16.339; when $L = 6$ and $p = 1$, respectively.

Another interpretation of the bullwhip effect variation is when there are two products in a supply chain. It is clear that ϕ_{12} and ϕ_{21} represent effects of demand for each product on demand of another one as demonstrated in equation (1). Thus, we need to determine the effects of variation of ϕ_{12} and ϕ_{21} on the bullwhip effect of each product. In our example, when ϕ_{11}, ϕ_{21}, ϕ_{22} are constant then ϕ_{12} must be $-.05 \prec \phi_{12} \prec .75$, based on stationary conditions. In addition, for the second product we must have $-.016 \prec \phi_{21} \prec .25$ when ϕ_{11}, ϕ_{12}, ϕ_{22} are constant. Fig. 3(a,b) show the bullwhip effect of two products according to their coefficient variations.

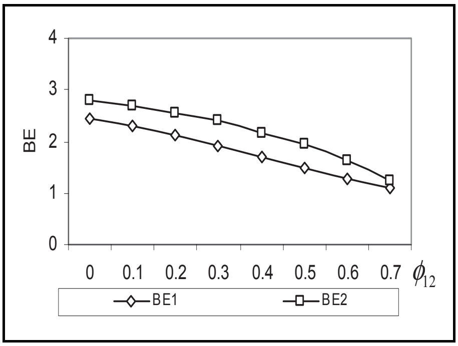

Fig. 3(a). Bullwhip effect variation forϕ_{12}, when $\phi_{11} = 0.7$, $\phi_{21} = 0.2$, $\phi_{22} = 0.5$, $L1 = 2$, and $p1 = 3$.

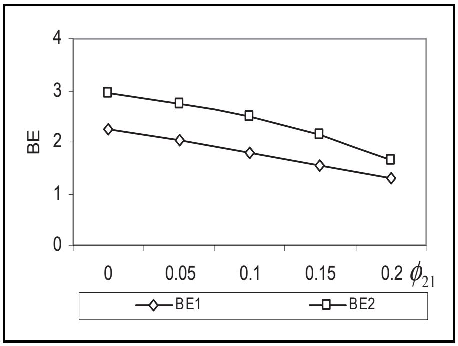

Fig. 3(b). Bullwhip effect variation forϕ_{21}, when $\phi_{11} = 0.7$, $\phi_{12} = 0.6$, $\phi_{22} = 0.5$, $L2 = 2$, and $p2 = 3$.

It is clear that when ϕ_{12} and ϕ_{21} increases the bullwhip effect of the two products decreases. In essence, dependence between products has an important role on bullwhip reduction. In other words, when we do not have any relation between products (i.e. $\phi_{12} = 0$ or $\phi_{21} = 0$), the bullwhip effect has a maximum value, whereas when the dependence rate is high (i.e. ϕ_{12} and ϕ_{21} are as much as possible), the bullwhip effect has a minimum value. Moreover, we can continue our investigations on the bullwhip effect measure with respect to ϕ_{11} and ϕ_{22} variation. According to stationary conditions, we can find limitations to these coefficients that imply: $\phi_{11} \prec 0.76$ when other coefficients are constant, and $-0.93 \prec \phi_{22} \prec 0.6$ when others are constant. Fig. 4(a) depicts the bullwhip effect variation when ϕ_{12} varies in its range.

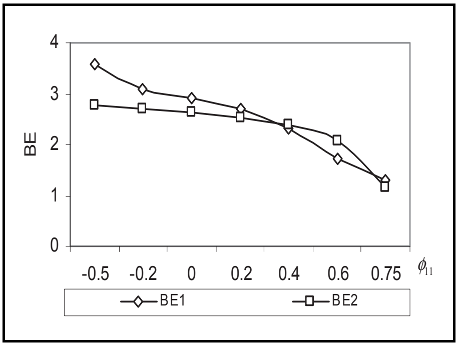

Fig. 4(a). Bullwhip effect variation forϕ_{11}, when $\phi_{12} = 0.6$, $\phi_{21} = 0.2$, $\phi_{22} = 0.5$, L1 = 2, and p1 = 3

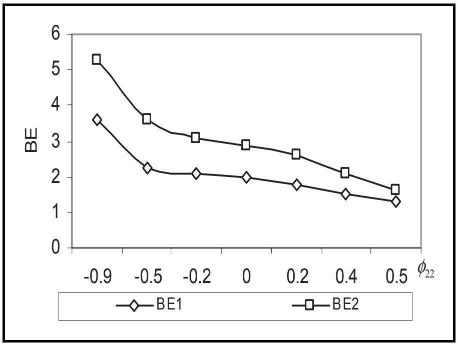

Fig. 4(b). Bullwhip effect variation forϕ_{22}, when $\phi_{11} = 0.7$, $\phi_{12} = 0.6$, $\phi_{21} = 0.2$, L2 = 2, and p2 = 3

Obviously, we can see that the bullwhip effect of the two products can be reduced by increasing ϕ_{11}. It means that if we increase the dependence of each product on the demand of the last period of the same product, not only with the bullwhip effect of the same product be reduced, but the bullwhip effect of another one can be decreased. This condition is valid for ϕ_{22} and is clear in Fig. 4(b), as ϕ_{22} increases the bullwhip effect as the two products decrease.

CONCLUSION

In this research, we have investigated the bullwhip effect in a two echelon supply chain consisting of two products. Demand of each product was correlated to the demand of another one. This relationship is described by the VAR (1) model. Retailers used OUT policy for ordering for both of the products. The order of the first product does not depend on the order of the second product. We assumed the retailer simple moving average method to forecast the lead-time demand of each product independently. After a description of the model, we derived a general expression for the bullwhip effect and mentioned that it is not possible to provide an explicit equation for the bullwhip effect of two products when we use a covariance function of the VAR (1) process in order to quantify the bullwhip effect. Then we provided the bullwhip effect measure of each product in limited cases for better analysis. In addition, we determined conditions in which the bullwhip effect could be removed from the supply chain without affecting the other calculations. In the last section, we analyzed the behavior of the bullwhip effect by a numerical example. We showed that lead-time reduction, as well as, increasing the number of observations in lead-time demand forecasting can reduce the bullwhip effect for both products, simultaneously. In other words, if a retailer uses more historical information in forecasting lead-time demand, the bullwhip effect can be decreased. In addition, if a manufacturer decreases lead-time, then the phenomenon can be removed from the supply chain. After that, we provided an analytical approach on the bullwhip effect based on the dependence of the demand of each product on the demand of the same product on the last period and the demand of the other product in the last period. It was clear that increasing the dependence between products has an important role in the bullwhip reduction of both products.

This research would be incomplete if we did not mention its drawbacks. Our aim was to provide mathematical relationships for quantifying the bullwhip effect. Yet, a more analytical approach is still needed. In this paper, our assumptions are based on Chen *et al.* (2000a), therefore, disadvantages of that paper (such as cost considerations, ordering policy, and forecasting method) exist in this current article. In addition, similar to Zhang (2004), a study on different forecasting methods, as well as, more analytical approaches on conditions that the bullwhip effect exists in our supply chain can be accomplished. It is more interesting when a retailer uses different forecasting methods for two products. This study is the first basic investigation on the bullwhip effect in more than one product supply chain and continuing the research in more complex supply chains is strongly recommended.

Appendix A

Due to the fact that the bullwhip effect is an increasing function in lead-time, we must have: $\frac{\partial BE}{\partial L} \succ 0$. Therefore, taking the first derivative of the bullwhip effect with respect to L we have: $\frac{\partial BE}{\partial L} = (1 + \frac{2L}{p})(\frac{2}{p})(1 - \frac{\gamma(p)}{\gamma})$ which should be positive. Because L and p are positive, we can conclude that $(1 - \frac{\gamma(p)}{\gamma}) \succ 0$ or $\gamma \succ \gamma(p)$.

Appendix B

$$BE_4^1 = (1+\frac{L_1}{4})^2 + (\frac{L_1}{4})^2 - 2(1+\frac{L_1}{4})(\frac{L_1}{4})[\phi_{11}^4 + \phi_{11}\phi_{21}\phi_{12}(3\phi_{11}+2\phi_{22}) + \phi_{21}\phi_{12}(\phi_{21}\phi_{12}+\phi_{22}^2)+ \phi_{12}(\phi_{11}+\phi_{22})(\phi_{11}^2+2\phi_{21}\phi_{12}+\phi_{22}^2)(\frac{\gamma_{12}}{\gamma_{11}})]$$

$$BE_4^2 = (1+\frac{L_2}{4})^2 + (\frac{L_2}{4})^2 - 2(1+\frac{L_2}{4})(\frac{L_2}{4})[\phi_{22}^4 + \phi_{21}\phi_{12}\phi_{22}(3\phi_{22}+2\phi_{11}) + \phi_{21}\phi_{12}(\phi_{21}\phi_{12}+\phi_{11}^2)+ \phi_{21}(\phi_{11}+\phi_{22})(\phi_{11}^2+2\phi_{21}\phi_{12}+\phi_{22}^2)(\frac{\gamma_{12}}{\gamma_{22}})]$$

$$BE_5^1 = (1+\frac{L_1}{5})^2 + (\frac{L_1}{5})^2 - 2(1+\frac{L_1}{5})(\frac{L_1}{5})[\phi_{11}^5 + \phi_{21}\phi_{12}(4\phi_{11}^3 + (\phi_{11}\phi_{22}+\phi_{21}\phi_{12})(3\phi_{11}+2\phi_{22}) + \phi_{22}^3)+ \phi_{12}(\phi_{22}(\phi_{11}+\phi_{22})(\phi_{11}^2+2\phi_{21}\phi_{12}+\phi_{22}^2) + \phi_{21}\phi_{12}(\phi_{11}(3\phi_{11}+2\phi_{22}) + (\phi_{21}\phi_{12}+\phi_{22}^2)) + \phi_{11}^4)(\frac{\gamma_{12}}{\gamma_{11}})]$$

$$BE_5^2 = (1+\frac{L_2}{5})^2 + (\frac{L_2}{5})^2 - 2(1+\frac{L_2}{5})(\frac{L_2}{5})[\phi_{22}^5 + \phi_{21}\phi_{12}(4\phi_{22}^3 + (\phi_{11}\phi_{22}+\phi_{21}\phi_{12})(3\phi_{22}+2\phi_{11}) + \phi_{11}^3)+ \phi_{21}(\phi_{11}(\phi_{11}+\phi_{22})(\phi_{11}^2+2\phi_{21}\phi_{12}+\phi_{22}^2) + \phi_{21}\phi_{12}(\phi_{22}(3\phi_{22}+2\phi_{11}) + (\phi_{21}\phi_{12}+\phi_{11}^2)) + \phi_{22}^4)(\frac{\gamma_{12}}{\gamma_{22}})]$$

REFERENCES

Bagahnha, M.P. & Cohen M.A. 1995. The stabilizing effect of inventory in supply chain. Management Science **46:** 436-443.

Blanchard, O.J. 1983. The production and inventory behavior of the American automobile industry. Journal of Political Economy **91:** 365-400.

Blinder, A.S. 1982. Inventories and sticky process. American Economics Review **72:** 334-349.

Blinder, A.S. 1986. Can the production smoothing model of inventory behavior be saved? Quarterly Journal of Economics **101:** 431-454.

Chandra, C. & Grabis, J. 2005. Application of multi-steps forecasting for restraining the bullwhip effect and improving inventory performance under autoregressive demand. European Journal of Operational Research **166:** 337-350.

Chen, F., Drezner, Z., Ryan, J.K. & Smith-Levi, D. 2000a. Quantifying the bullwhip effect in a simple supply chain: The impact of forecasting, lead times and information. Management Science **46:** 436-443.

Chen, F., Drezner, Z., Ryan, J.K. & Smith-Levi, D. 2000b. The impact of exponential smoothing forecasts on the bullwhip effect. Naval Research Logistics **47:** 269-286.

Dejonckheere, J., Disney, S.M., Lambrecht, M.R. & Towill, D.R. 2003. Measuring and avoiding the bullwhip effect: A control theoretic approach. European Journal of Operational Research **147:** 567-590.

Disney, S.M. & Towill D.R. 2003. On the bullwhip and inventory variance produced by an ordering policy. Omega **31:** 157-167.

Forrester, J.W. 1958. Industrial dynamics - A major break through for decision-making. Harvard Business Review.

Gilbert, K. 2005. An ARIMA supply chain model. Management Science **51:** 305-310.

Goodman, M. 1974. A new look at higher-order exponential smoothing for forecasting. Operational Research **22:** 880-888.

Hosoda, T. & Disney, S.M. 2006. On variance amplification in a three-echelon supply chain with minimum mean square error forecasting. Omega **34:** 344-358.

Jaksic, M. & Rusjan, B. 2008. The effect of replenishment policies on the bullwhip effect: A transfer function approach. European Journal of Operational Research **184:** 946-961.

Kahn, J. 1987. Inventories and volatility of production. American Economics Review **77:** 667-679.

Kim, J.G., Chatfield, D., Harrison, T.P. & Hayya, J.C. 2006. Quantifying the bullwhip effect in supply chain with stochastic lead time. European Journal of the Operational Research **173:** 617-636.

Lee, H.L., Padmanabhan, V. & Whang, S. 1997. The bullwhip effect in supply chains. Sloan Management Review **38:** 93-102.

Luong, H.T. 2007. Measure of the bullwhip effect in supply chains with autoregressive demand process. European Journal of Operational Research **180:** 1086-1097.

Luong, H.T. & Phien, N.H. 2007. Measure of the bullwhip effect in supply chains: The case of high order autoregressive demand process. European Journal of Operational Research **183:** 197-209.

Makui, A. & Madadi, A. 2007. The bullwhip effect and Lyapunov exponent. Applied Mathematics and Computation **189:** 35-40.

Metters, R. 1997. Quantifying the bullwhip effect in supply chains. Journal of Operations Management **15:** 89-100.

Silver, E., Peterson, R. & Pyke, D.F. 2000. Decision Systems for Inventory Management and Production. Willey, New York.

Sterman, J.D. 1989. Optimal policy for a multi-product, dynamic, nonstationary inventory problem. Management Science **12:** 206-222.

Wei W.W.S. 1990. Time Series Analysis: Univariate and Multivariate Methods. Addison-Wesley, New York.

Zhang, X. 2004. The impact of forecasting methods on the bullwhip effect. International Journal of Production Economics **88:** 15-27.

Submitted : *7/10/2007*
Revised : *11/3/2008*
Accepted : *25/3/2008*

قياس أثر البولويب في سلاسل التمويل مع عملية طلب الارتداد الأوتوماتيكي الكمي

سيد كمال جهار سوقى وأحمد صادقي

جامعة تربية المدرسين – كلية الهندسة – قسم الهندسة الصناعية
ص. ب. 143 – 14115 – طهران – الجمهورية الإسلامية الإيرانية

خلاصة

في هذا البحث تمت دراسة مشكلة أثر البولويب في سلسلة التمويل الثنائية والتي تشتمل على منتجين بناء على فرضياتنا.

إن الهدف الرئيسي من هذا البحث هو تزويد قياس لتأثير البولويب الذي له القدرة على تحليل التسلسل الزمني VAR (1) وهو الشكل المناسب لنموذج الطلب في سلاسل التمويل للمنتجين.

لقد تم استخدام طريقة المتوسط المتحرك البسيطة للتنبؤ بحاجة الوقت المتقدم، وسياسة الطلب OUT نظراً لأنهما الأكثر تطبيقاً وشيوعاً.

لقد تم اشتقاق وعرض الشكل العام لقياس البولويب لإثبات أنه لا يوجد تعبير واضح لقياس أثر البولويب بناء على الأسلوب الحالي المتبع لقياس البولويب. ومع هذا، قد يوجد قياس أثر البولويب في سلاسل التمويل ذات المنتجين من خلال مثال عددي.