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Exact solutions for the wick-type stochastic time-fractional KdV equations

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ABSTRACT

Our aim in this paper is to explore white noise functional solutions for the variable coefficients Wick-type stochastic time-fractional KdV equations. Using the modified fractional sub-equation method, we can find out new exact solutions for the time-fractional KdV equations. Subsequently, the Hermite transform and the inverse Hermite transform are employed to find white noise functional solutions for the variable coefficients Wick-type stochastic time-fractional KdV equations.

Keywords: Time-fractional KdV equations; wick product; white noise; hermite transform.

INTRODUCTION

In this paper, with the help of Hermite transform, white noise theory and modified fractional sub-equation method, we will deduce white noise functional solutions for the variable coefficients Wick-type stochastic time-fractional KdV equations as the following form:

$$D_t^{\alpha} U + P(t) \diamond U \diamond U_x + Q(t) \diamond U_{xxx} = 0, \quad (x,t) \in \mathbb{R} \times \mathbb{R}_+, \quad 0 < \alpha \leq 1, \quad (1.1)$$

where D_t^{α} is the modified Riemann-Liouville derivative defined by Jumarie, (2006), $P(t)$ and $Q(t)$ are non-zero white noise functions, and $'' \diamond ''$ is the Wick product on the Kondratiev distribution space $(S)_{-1}$ which was defined by Holden *et al.* (2010). Moreover, when Wick product is replaced by ordinary product in Eq. (1.1), we obtain the variable coefficients time-fractional KdV equations:

$$D_t^{\alpha} u + p(t) u u_x + q(t) u_{xxx} = 0, \qquad (x,t) \in \mathbb{R} \times \mathbb{R}+, 0 < \alpha \leq 1, \qquad (1.2)$$

where $P(t)$ and $Q(t)$ are non-zero, bounded measurable or integrable functions on . The KdV equation has arisen in a number of physical contexts as collisionfree hydromagnetic waves, stratified internal waves, ion-acoustic waves, plasma physics, lattice dynamics, etc (Fung, 1997). Certain theoretical physics phenomena in the quantum mechanics domain are explained by means of a KdV model. It is used in fluid dynamics, aerodynamics, and continuum mechanics as a model for shock wave formation, solitons, turbulence, boundary layer behavior, and mass transport. All of the physical phenomena may be considered as non-conservative, so they can be described using fractional differential equations. Therefore, in this work, our motive mainly devoted to formulate a time-fractional KdV equation version. In El-Wakil *et al.* (2011), the authors assert that Eq. (1.2) is the mathematical model for small but finite amplitude electron-acoustic solitary waves in plasma of cold electron fluid with two different temperature isothermal ions. Therefore, if this model is perturbed by Gaussian white noise, we can regard Eq. (1.1) as the mathematical model for the resultant phenomenon. Since Wadati first introduced and studied stochastic KdV equation (Wadati, 1983), many authors, e.g., (Xie, 2003; Xie, 2004; Chen & Xie, 2006; Chen & Xie, 2007; Ghany, 2011) and so on, have investigated more intensively the stochastic partial differential equations (SPDE).

On the basis of the homotopy analysis method, Dehghan *et al.* developed a scheme to obtain the approximate solution of the deterministic time-fractional KdV equation with constant coefficients and Caputo's derivative (Dehghan *et al.,* 2010). The present letter is motivated by the desire to propose the modified fractional sub-equation method to construct exact analytical solutions for the variable coefficients Wick-type stochastic time-fractional KdV equations with the modified Riemann-Liouville derivative defined by:

$$D_x^{\alpha} f(x) = \begin{cases} \dfrac{1}{\Gamma(1-\alpha)} \displaystyle\int_0^x (x-\theta)^{-\alpha-1}[f(\theta)-f(0)]d\theta, & \alpha < 0, \\ \dfrac{1}{\Gamma(1-\alpha)} \dfrac{d}{dx} \displaystyle\int_0^x (x-\theta)^{-\alpha}[f(\theta)-f(0)]d\theta, & 0 < \alpha < 1, \\ [f^{(\alpha-n)}(x)]^{(n)} & n \leq \alpha < n+1, \end{cases} \tag{1.3}$$

Where n = 1,2,3,... which has merits over the original one, for example, the $\alpha-$order derivative of a constant is zero. Some useful formulas and results of Jumarie's modified Riemann-Liouville derivative can be found in (Jumaire, 2006; Jumaire, 2009). Our first interest in this work is implementing new strategies that give white noise functional solutions of the variable coefficients Wick-type stochastic time-fractional KdV equations. The strategies that will be pursued in this work rest mainly on Hermite transform, white noise theory and modified fractional sub-equation method, all of which are employed to find white noise functional solutions of Eq. (1.1). The proposed schemes, as we believe, are entirely new and introduce new solutions in addition to the well-known traditional solutions. The ease of using these methods, to determine shock or solitary type of solutions, shows its power.

White Noise Functional Solutions of Eq. (1.1)

In this section, we will give new strategies that give white noise functional solutions of Eq. (1.1). Taking the Hermite transform of Eq. (1.1), we get the deterministic equation:

$$D_t^\alpha \tilde{U}(x,t,z) + \tilde{P}(t,z)\tilde{U}(x,t,z)\tilde{U}_x(x,t,z) + \tilde{Q}(t,z)\tilde{U}_{xxx}(x,t,z) = 0, \quad (2.1)$$

where $z = (z_1, z_2, ...) \in (\mathbb{C}^{\mathbb{N}})_c$ is a vector parameter. For the sake of simplicity we denot $P(t,z) = \tilde{P}(t,z)$, $Q(t,z) = \tilde{Q}(t,z)$and $u(x,t,z) = \tilde{U}(x,t,z)$. To convert the fractional partial differential equation in two independent variables (1.2) into an fractional ordinary differential equation, we take the traveling wave transformation:

$$u = u(\xi), \qquad \xi = kx + ct, \quad (2.2)$$

where k, c are arbitrary constants which satisfy $kc \neq 0$, then Eq.(2.1) is reduced into a nonlinear fractional ordinary differential equation:

$$c^\alpha D_\xi^\alpha u + kPu\frac{du}{d\xi} + k^3 Q\frac{d^3u}{d\xi^3} = 0. \quad (2.3)$$

We next suppose Eq. (2.3) has a series expansion solution in the form:

$$u = \sum_{i=0}^{n} l_i(t,z)\phi^i(\xi) + \sum_{i=1}^{n} m_i(t,z)\phi^{-i}(\xi), \quad (2.4)$$

where l_i $(i = 0, 1, ..., n)$, $m_i(i = 1, ..., n)$ are functions to be determined later, is a positive integer and satisfies the fractional Riccati equation:

$$D_\xi^\alpha \phi = \sigma + \phi^2, \qquad 0 < \alpha \leq 1, \quad (2.5)$$

where σ is an arbitrary constant. Very recently, (Zhang *et al.*, 2010) first generalized the Exp-function method (Zhang, 2010) to fractional differential equations and obtained the following five solutions of Eq. (2.5).

$$\begin{cases} \phi_1(\xi) = -\sqrt{-\sigma}\tanh_\alpha(\sqrt{-\sigma}\xi), & \sigma < 0, \\ \phi_2(\xi) = -\sqrt{-\sigma}\coth_\alpha(\sqrt{-\sigma}\xi), & \sigma < 0, \\ \phi_3(\xi) = \sqrt{\sigma}\tan_\alpha(\sqrt{\sigma}\xi) & \sigma > 0, \\ \phi_4(\xi) = \sqrt{\sigma}\cot_\alpha(\sqrt{\sigma}\xi) & \sigma > 0, \\ \phi_5(\xi) = -\dfrac{\Gamma(1+\alpha)}{\xi^\alpha + \nu} & \sigma = 0, \quad \nu = const., \end{cases} \tag{2.6}$$

where $\tanh_\alpha(x)$, $\coth_\alpha(x)$, $\tan_\alpha(x)$ and $\cot_\alpha(x)$ are the generalized hyperbolic and trigonometric functions which were defined in (Podlubny, 1999) by:

$$\tanh_\alpha(x) = \frac{E_\alpha(x^\alpha) - E_\alpha(-x^\alpha)}{E_\alpha(x^\alpha) + E_\alpha(-x^\alpha)},$$

$$\coth_\alpha(x) = \frac{E_\alpha(x^\alpha) + E_\alpha(-x^\alpha)}{E_\alpha(x^\alpha) - E_\alpha(-x^\alpha)},$$

$$\tan_\alpha(x) = \frac{E_\alpha(ix^\alpha) - E_\alpha(-ix^\alpha)}{i(E_\alpha(ix^\alpha) + E_\alpha(-ix^\alpha))},$$

$$\cot_\alpha(x) = \frac{i(E_\alpha(ix^\alpha) + E_\alpha(-ix^\alpha))}{E_\alpha(ix^\alpha) - E_\alpha(-ix^\alpha)},$$

$$E_\alpha(x) = \sum_{j=0}^{\infty} \frac{x^j}{\Gamma(1+j\alpha)}$$

is the Mittag-Leffler function. Balancing $u\frac{du}{d\xi}$ and $\frac{d^3u}{d\xi^3}$ in Eq. (2.3), we have . By substituting Eq. (2.4) along with Eq. (2.5) into Eq. (2.3) and collect the coefficients of $\phi^i(i = -5, -4, ..., 5)$ and set them to be zero, we will obtain the following set of algebraic equations in the unknowns $l_i(i = 0, 1, 2)$ and $m_i(i = 1, 2)$

where

$$a_0 = l_1\sigma - m_1, \quad a_1 = 2l_2\sigma, \quad a_2 = l_1,$$

$$a_3 = 2l_2, \quad a_4 = -2m_2, \quad a_5 = -m_1\sigma,$$

$$a_6 = -2m_2\sigma,$$

$$b_0 = a_0l_0 + a_4l_1 + a_5l_2 + a_1m_1 + a_2m_2,$$

$$b_1 = a_1l_0 + a_0l_1 + a_4l_2 + a_2m_1 + a_3m_2,$$

$$b_2 = a_2l_0 + a_1l_1 + a_0l_2 + a_3m_1,$$

$$b_3 = a_3l_0 + a_2l_1 + a_1l_2,$$

$$b_4 = a_3l_1 + a_2l_2, b_5 = a_3l_2,$$

$$b_6 = a_4l_0 + a_5l_1 + a_6l_2 + a_0m_1 + a_1m_2,$$

$$b_7 = a_5l_0 + a_6l_1 + a_4m_1 + a_0m_2,$$

$$b_8 = a_6l_0 + a_5m_1 + a_4m_2,$$

$$b_9 = a_6m_1 + a_5m_2, b_{10} = a_6m_2,$$

$$c_0 = 2\big(a_2\sigma^2 + a_5\big), c_1 = 2\sigma(3a_3\sigma + a_1), c_2 = 8a_2\sigma,$$

$$c_3 = 2(9a_3\sigma + a_1), c_4 = 6a_2, c_5 = 12a_3, c_6 = 2(a_4\sigma + 3a_6), c_7 = 8a_5, c_8 = 2(a_4\sigma + 9a_6),$$

With the aid of MATHEMATICA, we can find the following sets of solutions of the system (2.7)

$$l_0 = -\frac{8k^3 + c^\alpha}{kP}, \quad l_1 = m_1 = 0, \quad l_2 = m_2 = -12k^2\frac{Q}{P}, \tag{2.8}$$

$$l_0 = -8k^2\frac{Q}{P}, \ l_1 = m_2 = 0, \ l_2 = \frac{1}{k\sigma P}\big[2k^3(\sigma - 4)Q + c^\alpha\big],$$

$$m_1 = \frac{2}{c^\alpha}\sigma\big[4k^3\sigma\big(7k^3\sigma + 16k^3 - 32\big)Q^2 + 32c^\alpha k^3(\sigma - 1)Q + 2c^{2\alpha}\big], \tag{2.9}$$

$$l_0 = -\frac{8k^3\sigma Q + c^\alpha}{kP}, \ m_1 = l_2 = 0, \ l_1 = -2c^\alpha kP, \ m_2 = -12k^2\frac{Q}{P}. \tag{2.10}$$

Using Eqs.(2.2), (2.4), (2.8), (2.9), (2.10) and (2.5), we obtain fifteen solutions of Eq. (2.1) as follows:

$$u_i(x,t,z) = -\frac{8k^3 + c^\alpha}{kP(t,z)} - 12k^2\frac{Q(t,z)}{P(t,z)\phi_i^2(kx + ct)}\big[\phi_i^4(kx + ct) + 1\big], i = 1, ..., 5, \tag{2.11}$$

$$u_{i+5}(x,t,z) = -8k^2\frac{Q(t,z)}{P(t,z)} + \frac{2k^3(\sigma-4)Q(t,z)+c^\alpha}{kP(t,z)}\phi_i^2(kx+ct)$$

$$+\frac{2\sigma\left[4k^3\sigma\left(7k^3\sigma+16k^3-32\right)Q^2(t,z)+32c^\alpha k^3(\sigma-1)Q(t,z)+2c^{2\alpha}\right]}{c^\alpha\phi_i(kx+ct)}, \; i=1,...,5, \quad (2.12)$$

$$u_{i+10}(x,t,z) = -\frac{8k^3\sigma Q(t,z)+c^\alpha}{kP(t,z)} - \frac{2c^\alpha}{P(t,z)}\phi_i(kx+ct) - \frac{12k^2Q(t,z)}{P(t,z)\phi_i^2(kx+ct)}, i=1,...,5. \quad (2.13)$$

Lemma 1. (Xie 2003) Suppose $u(x,t,z)$ is a solution (in the usual strong pointwise sense) of Eq. (2.1) for (x,t) in some bounded open set $G\subset\mathbb{R}\times\mathbb{R}_+$, and for all $z\in K_m(n)$ for some $m<\infty,\; n>0$. Moreover, suppose that $u(x,t,z)$ and all its derivatives, which are involved in Eq. (2.1) are (uniformly) bounded for $(x,t,z)\in G\times K_m(n)$, continuous with respect to $(x,t)\in G$ for all $z\in K_m(n)$ and analytic with respect to $z\in K_m(n)$ for all $(x,t)\in G$. Then there exists $U(x,t)\in(S)_{-1}$ such that $u(x,t,z)=\tilde{U}(x,t)(z)$ for all $(x,t,z)\in G\times K_m(n)$ and $U(x,t)$ solves (in the strong sense) Eq. (1.1) in $(S)_{-1}$.

From Lemma 1, we know that there exists $U(x,t)\in(S)_{-1}$ such that $u(x,t,z)=\tilde{U}(x,t)(z)$ for all $(x,t,z)\in G\times K_m(n)$ where $U(x,t)$ is the inverse Hermite transformation of $u(x,t,z)$. Consequently, $U(x,t)$ solves Eq. (1.1). Hence, the white noise functional solutions of Eq. (1.1) are as follows:

$$U_i(x,t) = -\frac{8k^3+c^\alpha}{kP(t)} - 12k^2\frac{Q(t)}{P(t)\Diamond\phi_i^{\Diamond 2}(kx+ct)}\Diamond\left[\phi_i^{\Diamond 4}(kx+ct)+1\right], \; i=1,...,5, \quad (2.14)$$

$$U_{i+5}(x,t) = -8k^2\frac{Q(t)}{P(t)} + \frac{2k^3(\sigma-4)Q(t)+c^\alpha}{kP(t)}\Diamond\phi_i^{\Diamond 2}(kx+ct)$$

$$+\frac{2\sigma\left[4k^3\sigma\left(7k^3\sigma+16k^3-32\right)Q^{\Diamond 2}(t)+32c^\alpha k^3(\sigma-1)Q(t)+2c^{2\alpha}\right]}{c^\alpha\phi_i^{\Diamond}(kx+ct)}, i=1,...,5, \quad (2.15)$$

$$U_{i+10}(x,t) = -\frac{8k^3\sigma Q(t)+c^\alpha}{kP(t)} - \frac{2c^\alpha}{P(t)}\Diamond\phi_i^{\Diamond}(kx+ct) - \frac{12k^2Q(t)}{P(t)\Diamond\phi_i^{\Diamond 2}(kx+ct)}, \; i=1,...,5. \quad (2.16)$$

where $\phi_i^{\Diamond}$ is the Wick version of the solution ϕ_i.

Remark

Suppose that $f(t)$ is a deterministic integrable function on $R_{/+}$ and

$$Q(t)=\lambda P(t), \qquad P(t)=f(t)+\mu W(t), \quad (2.17)$$

where $\lambda,\ \mu$ are constants and $W(t)$ is Gaussian white noise, that is $W(t) = \dot{B}(t)$, $B(t)$ is Brownian motion. Considering the Hermite transform of Eq. (2.17), we have $\tilde{Q}(t,z) = \lambda\tilde{P}(t,z)$, $\tilde{P}(t,z) = f(t) + \mu\tilde{W}(t,z)$, where $\tilde{W}(t,z) = \sum_{j=1}^{\infty} z_j \int_0^t \eta_j(s)ds$ and the and the function $\eta_j(s)$ can be found in Holden *et al.,* (2010). In this case, the solutions of Eq. (1.1) are given in non-Wick versions as follows:

$$U_{i+15} = -\frac{8k^3 + c^{\alpha}}{kP(t)} - \frac{12k^2\lambda\left[\phi_i^4(\pi(x,t)) + 1\right]}{\phi_i^2(\pi(x,t))}, \quad i = 1, ..., 5, \tag{2.18}$$

$$\begin{aligned} U_{i+20}(x,t) &= -8k^2\lambda + \left(2k^3\lambda(\sigma-4) + \frac{c^{\alpha}}{kP(t)}\right)\phi_i^2(\pi(x,t)) \\ &+ \frac{2\sigma\left[4k^3\sigma\lambda^2\left(7k^3\sigma + 16k^3 - 32\right)P^2(t) + 32c^{\alpha}k^3\lambda(\sigma-1)P(t) + 2c^{2\alpha}\right]}{c^{\alpha}\phi_i(\pi(x,t))}, i = 1, ..., 5, \end{aligned} \tag{2.19}$$

$$U_{i+25}(x,t) = -8k^2\sigma\lambda - \frac{c^{\alpha}}{kP(t)} - \frac{2c^{\alpha}}{P(t)}\phi_i(\pi(x,t)) - \frac{12k^2\lambda}{\phi_i^2(\pi(x,t))}, i = 1, ..., 5. \tag{2.20}$$

$$\pi(x,t) = kx + cf(t) + c\mu\left(B(t) - \frac{t^2}{2}\right).$$

In Eqs. (2.18) - (2.20), we have already used the following relation (Holden *et al.*, 2010)

$$E_{\alpha}^{\diamondsuit}(B(t)) = E_{\alpha}\left(B(t) - \frac{t^2}{2}\right).. \tag{2.21}$$

Special Cases. If we put $\alpha = 1$, this implies that $\tan_{\alpha}(x) = \tan(x)$, $\cot_{\alpha}(x) = \cot(x), \tanh_{\alpha}(x) = \tanh(x)$, $\coth_{\alpha}(x) = \coth(x)$ and $E_{\alpha}(x) = \exp(x)$. So, Eq. (2.8) gives:

$$U_{i+15} = \frac{\mathrm{K}_1}{P(t)} + \mathrm{K}_2[\phi_i^2(\pi(x,t)) + \phi_i^{-2}(\pi(x,t))], \quad i = 1, ..., 5 \tag{2.19}$$

This result, compatible with the results given by Wazwaz (2007).

Summary and Discussion

In general, the solution of SPDE will be a stochastic distribution, and we have to interpret possible products that occur in the equation, as one cannot in general take the product of two distributions. In our paper, products are considered to be Wick products, which overcome this difficulty through white noise functional approach. Subsequently, we take the Hermite transform of the resulting equation and obtain an equation that we try to solve, where the random variables have been replaced by complex-valued functions of infinitely many complex variables. Finally, we use the inverse Hermite transform to obtain a solution of the regularized, original equation (Ghany, 2011). Since $\Psi^{\diamond}(x) = \Psi(x)$ for any non-random function $\Psi(x)$, hence (2.14)-(2.16) are solutions of the variable coefficients time-fractional KdV equations (1.2). our method in this paper is a standard, direct and computerized which allows us to do complicated and tedious algebraic calculations. This method is valid for any type of time-fractional KdV equations, such as KdV-Burger equation, modified KdV equations, Benjamin- Bona-Mahony, KdV-Burger -Kuramoto, ... etc. On the other hand, for any other type (not belongs to KdV) we cannot use our method directly, but we need some necessary modifications, see (Zhang, 2012; Kim & Sakthivel, 2011; Holden *et al.,* 2010). In Benth & Gjerde, (1998), we can find a unitary mapping between the Gaussian white noise space and the Poisson white noise space. Hence we can obtain the solutions of the Poisson SPDE simply by applying this mapping to the solutions of the corresponding Gaussian SPDE. We note that as $\alpha \to 1$, all the obtained exact solutions give a new set of exact solutions of the well known Wick-type stochastic KdV equations (Xie, 2003). Moreover, we observe that we can get different white noise functional solutions of Eq. (1.1) from (2.14)-(2.16) for different forms of the coefficients $P(t)$and $Q(t)$.

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الحلول المضبوطة للمعادلات التصادفية كسرية الزمن من النوع ويك

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خلاصة

هدفنا في هذا البحث هو دراسة حلول داليات الضجيج البيضاء للمعادلات التصادفية الكسرية الزمن من النوع ويك. نستخدم طريقة المعادلة الجزئية الكسرية المعّدلة وذلك لإيجاد حلول مضبوطة جديدة للمعادلات كسرية الزمن من طراز ك. دي. في. ونستخدم فيما محول هرمايت ومحول هرمايت المعاكس وذلك لإيجاد حلول داليات الضجيج البيضاء للمعادلات التصادفية الكسرية الزمن من النوع ويك وذات المعاملات المتغيرة.