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A NOTE ON ENERGY BALANCE AND ELECTRON RUNAWAY IN A UNIFORM FULLY IONIZED PLASMA

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Abstract. The problem of temperature relaxation in a uniform fully ionized plasma is generalized to include the effects of a static electric field. An energy equation is derived from which we get an equation for the approach to a temperature-balance. It is deduced that the electrons runaway (energy increasing without limit) when the electron temperature is greater than three halves of the ion temperature. A general criterion is also deduced.

This paper concerns the effects of a static electric field E on temperature relaxation in a uniform fully ionized plasma. The special case when $\underset{\sim}{E} = \underset{\sim}{O}$ has been studied by several authors (e.g. Spitzer 1962 and Dreicer 1960) who assumed Maxwellian distributions for both electrons and ions. We assume here that electrons and ions in the plasma are supposed initially to possess different kinetic temperatures T_1 , T_2 and it is required to find the time in which equilibrium is attained. The energy equation of the electron gas is derived from the Boltzmann equation. Subsequent analysis of this equation leads to a criterion for electron runaway.

If the state of an electron-ion plasma is uniform, the velocity distribution function F_1 of the electrons satisfies the Boltzmann equation

$$\partial F_1/\partial t - (e_1\underset{\sim}{E}/m_1) . \partial F_1/\partial \underset{\sim}{C}_1 = (\partial F_1/\partial t)_{coll} \quad (1)$$

where $(\partial F_1/\partial t)_{coll}$ is the usual Boltzmann collision integral. Only elastic collisions are considered; likewise, ionization and recombination as a result of collisions are ignored. The symbols e_1, m_1, C_1 denote respectively the electronic charge, mass and peculiar velocity. The notation of Chapman and Cowling (1960) is used throughout.

The equation of energy of the electron gas is given by

$$(3Kn_1/2)\partial T_1/\partial t - e_1 n_1 \bar{\underset{\sim}{C}} . \underset{\sim}{E} = E_{12} \quad (2)$$

which can be obtained by multiplying (1) by $m_1 C_1^2$ /2 and integrating over all values of $\underset{\sim}{C}_1$ Here K is the Boltzmann constant, n_1 the electron number-density assumed to be constant and $\bar{\underset{\sim}{C}}_1$ - the electron-gas drift velocity. The first term on the left-hand side of (2) gives the change of the electrons thermal energy with time. The second term gives the power fed into the electron gas by the electric field. The term on the right-hand side is the rate at which the energy of the electron gas is being altered by collisions with the ions. Electron-electron collisions do not contribute to the right-hand side of (2) since these can not alter the total energy of the electron-gas.

To the first approximation we assume that the drift-velocity $\underset{\sim}{C}_1$ is linear in $\underset{\sim}{E}$ (Ohm's law). Thus the electric field term in (2) takes the form

$$e_1 n_1 \bar{\underset{\sim}{C}}_1 . \underset{\sim}{E} = \underset{\sim}{J}_1 . \underset{\sim}{E} = \sigma E^2, \quad (3)$$

where $\underset{\sim}{J}_1$ is the electric current-density produced by the diffused electrons and σ is the direct-current conductivity. After a few mean free collision times the electrical conductivity σ takes the form of the well-known (temperature)$^{3/2}$ law. This is true provided that the electric field $\underset{\sim}{E}$ is weak. The expression adopted for σ is that derived by Spitzer (1960) which is

$$\sigma = (3.5 m_1/3\pi^{3/2} e_1 e_2 \log u) \; (2KT_1/m_1)^{3/2} \quad (4)$$

where e_2 is the ionic charge and log(u) is the Coulomb logarithm whose argument u is the ratio between the Debye length and the classical distance of closest approach. It is worth pointing out that both electron-electron and electron-ion collisions contribute to the conductivity given by (4).

The collision term in (2) has been considered by Spitzer (1940) and we quote his result :

$$E_{12} = - (16\sqrt{\pi} \; Kn_1 n_2 e_1^2 e_2^2 \log u/m_1 m_2) . (2KT_1/m_1 + 2KT_2/m_2)^{-3/2} (T_1 - T_2) \quad (5)$$

where n_2, m_2 denote respectively the ionic number-desity and mass. Since $T_2/m_2 \ll T_1/m_1$ we may put (5) in the form

$$E_{12} = -(16\sqrt{\pi}\ Kn_1n_2e_1^2e_2^2\ \log u/m_1m_2).(m/2KT_1)^{3/2}\ (T_1-T_2). \qquad (6)$$

Combining (2)-(6) and dividing by $3Kn_1/2$, the energy equation then takes the following form

$$dT_1/dt = -(T_1-T_2)/t_{eq} + (244\ e_1e_2\ t_{eq}/27\pi\ Km_2)E^2 \qquad (7)$$

where t_{eq} is the time of equipartition, i.e. the time in which equilibrium is attained

$$t_{eq} = (3\ m_1m_2/32\sqrt{\pi}\ n_2e_1^2e_2^2\ \log u).(2KT_1/m_1)^{3/2}.$$

It should be pointed out that the electric field does not increase the time to reach equilibrium as one might think; it alters only the equilibrium value of T_2. For $\underset{\sim}{E} = \underset{\sim}{O}$ equation (7) is similar to equations derived by Landau (1965), Dreicer (1960) and Spitzer (1962), but for $\underset{\sim}{E} \neq \underset{\sim}{O}$ one gets an approach to a temperature balance equation of the form

$$T_1 - T_2 = (244\ e_1e_2/27\ \pi\ Km_2)\ t_{eq}^2\ E^2 \qquad (8)$$

in a time comparable with t_{eq}. In view of the approximation made in deriving (8), this equation may be correct only if $(T_1-T_2)/T_2$ is not fairly small.

Equation (8) gives the study value of T_1 if the ion-temperature T_2 remains a constant. Since t_{eq} is proportional to $T_1^{3/2}$, the equation is of the form

$$DT_1^3 = T_1 - T_2, \qquad (10)$$

where

$$D = \frac{244e_1e_2}{27\pi Km_2}\left[\frac{3m_1m_2(2K/m_1)^{3/2}\ E}{32\sqrt{\pi}\ n_2e_1^2e_2^2\ \log u}\right]^2$$

Equation (10) possesses a positive root T_1 only if $27DT_2^2 \leq 4$. When $27DT_2^2 > 4$, there is no positive root, and temperature balance is impossible. In the latter case the value of dT_1/dt given by (7) is positive for all values of T_1; the electron temperature cannot be restrained from attaining large values, and the electrons may be said to attain runaway energies.

If $27DT_2^2 = 4$, equation (9) has the unique positive root $T_1 = 3T_2/2$, and the value of dT_1/dt given by (7) is positive save if $T_1 = 3T_2/2$.

If $27DT_2^2 < 4$, equation (7) has two roots T_1', T_1'' where $T' < T''$, and dT/dt is negative if $T_1' < T_1 < T_1''$. In this case the value $T_1 = T_1'$ gives a state which is not only steady but stable to small perturbations.

The assumption that T_2 remains a constant is, of course, artificial; electron-ion collisions must ultimately warm up the ion-gas also, and so increase the probability of runaway effects. The main deduction from the above analysis is that T_1/T_2 cannot attain large values before runaway occurs. Our analysis gives a criterion that Joule heating should lead to a runaway even if T_2 is kept constant; in fact it shows that electrons run away as a whole if heating increases T_1 above $3T_2/2$.

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حول اتزان الطاقة وافلات الالكترونات من البلازما المنتظمة تامة التأين

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قسم الرياضيـــات بجامعة الكويت

خلاصـــــة

يدور هذا البحث حول تأثيرات مجال استاتيكي خارجي في البلازما تامة التأين . وهذا تعميم لابحاث مؤلفين آخرين لم يأخذوا تأثيرات مجال خارجي فيها بعين الاعتبار.

يبدأ البحث باشتقاق معادلة للطاقة أمكن تحويلها ، تحت شروط معينة ، الـــى معادلةاتزان حراري. وبتحليل هذه المعادلة توصل المؤلف الى أن الالكترونات تبدأ بالافلات من البلازما (أي أن طاقتها تزداد بدون حد) عندما تصبح درجة حرارتهـا درجة حرارة الايونات . كذلك تم ايجاد الشرط العام للافلات .