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Flow of oil and gas through chokes

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ABSTRACT

A theoretical technique for the two-phase flow of oil and gas through wellhead chokes has been developed. This technique assumes non-critical and critical flow conditions through a knife edge orifice where the mass flow rate through the choke reaches a definite maximum as the downstream pressure is further reduced. The paper also investigates the pressure differential ranges where critical velocity or the maximum flow rates is attained. The technique is verified by actual measured field data obtained in oilfield operation.

The purpose of this paper is to demonstrate the technique since it is believed that it will be of use to engineers in the field of oil and gas production in which measuring techniques are so difficult and expensive to carry out. It will be shown in the paper that the method is reliable and easy to apply.

INTRODUCTION

It is standard practice in oilfield operation to select a choke for flowing well oil in such a way that the fluctuations in downstream pressure do not affect the upstream pressure (wellhead), and thus the oil flow rate. To satisfy this condition, the flow of fluid through the choke must be critical flow conditions; this implies that the speed of the fluid through the choke should be that of sound. Various technical papers have been written on this subject with the same initial assumptions and some investigators have developed semi-rigorous equations whereas others have developed purely empirical formulae. Nind (1964) states that under the range of conditions found in oilfield operations an equation for choke performance under critical flow conditions has been developed by engineers in two major oil companies. He does not present the theory leading to the equation

$$Q = \frac{P_1 d^2\,*}{600\ R^{0\cdot5}} \tag{1}$$

Gilbert's correlation (Gilbert 1954) which led to an empirical formula was intended to aid in the prediction of oil and gas flow rates through wellhead chokes. He states that

$$Q = \frac{p_1 d^{1\cdot89}}{435\ R^{0\cdot546}} \tag{2}$$

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* The notations in this and other equations are given on pp. 161–162.

Poettmann & Beck (1963) derived similar correlations which were further developed by Ross (1961) using an average orifice discharge coefficient. Ashford (1974) presented a theoretical approach for critical multiphase flow performance through wellhead chokes. He used the orifice discharge coefficient as a fitted parameter to compensate for non-ideal factors excluded in his theoretical equation.

The objective of this study is to develop an equation for wellhead choke performance under critical and non-critical flow conditions, taking into account fluid properties as function of pressure and temperature. The method is based on the principle that each phase flows separately. No attempts were made to describe the orifice discharge coefficient as a function of fluid properties. An average discharge coefficient based on the orifice and pipe diameters was used (McCabe & Smith 1956).

OIL PHASE FLOW THROUGH CHOKES

In a nozzle, the cross-section of flow, density, pressure and velocity change smoothly but rapidly with distance along a streamline. Because of rapidity of the changes of pressure and temperature in a nozzle, the wall surface is insufficient to transfer any appreciable amount of heat to the fluid. The flow is therefore considered adiabatic. Because of the smooth variation of the dimensions and length of the nozzle, the effect of eddies and skin friction is negligible. The flow is thus considered as reversible, and since it is adiabatic, it must be also isentropic.

The derivation of the flow of oil and gas through a wellhead choke is based on the following assumptions which are usually made for ideal two-phase flow and are indicated by Ashford (1974): (1) each phase flows through the bean separately, (2) the fluid properties are considered to be function of pressure and temperature, (3) the gas behaves ideally, (4) during the flow of oil through the choke no gas is evolved, (5) the gas and oil are in heat equilibrium, (6) no slip occurs between the phases, and (7) the duct is short, so that wall friction need not be considered.

Bernoulli's energy balance equation can be applied to a flow in an isentropic nozzle (McCabe & Smith 1956)

$$\int_{u_1}^{u_2} u\,du = -g_c \int_{P_1}^{P_2} \frac{dp}{\rho} \tag{3}$$

in which u_1 and P_1 are the conditions at the entrance to the nozzle (choke). The flow is assumed to be one-dimensional, and the velocity and other variables are constant over any given cross-section. For an ideal gas

$$\rho_2 = \rho_1 \left(\frac{P_2}{P_1}\right)^{1/k} \tag{4}$$

Equation (4) relates the density to the pressure at any point under the conditions at the nozzle entrance. At the nozzle entrance u_1^2 is usually much less than u_2^2 and so it is neglected.

The substitution of eqn (4) into eqn (3), followed by integration gives the velocity of the gas flow through chokes

$$u_g = \left[\frac{2g_c\, P_1\, k}{\rho_g\,(k-1)}\left(1-\frac{P_2}{P_1}\right)\frac{k-1}{k}\right]^{\frac{1}{2}} \tag{5}$$

Similarly, eqn (3) can be integrated for incompressible fluid flow (oil phase)

$$u_o = \left[\frac{2g_c P_1}{\rho_o}\left(1-\frac{P_2}{P_1}\right)\right]^{\frac{1}{2}}. \tag{6}$$

But the volumetric flow rate per unit time $Q = uA$. Then it is necessary to introduce an orifice discharge coefficient C to allow for the fact that the minimum area of the flow stream will be somewhat less than the total area of the nozzle. The resulting volumetric flow equation for each phase is

$$Q_o = C_o A u_o \tag{7}$$

$$Q_g = C_g A u_g. \tag{8}$$

Since two-phase flow exists at the choke, then the effective choke cross-section through which only gas flows (A_g) is somewhat less than the choke area multiplied by the orifice discharge coefficient. It is assumed that oil and gas flow simultaneously through the choke; then it is logical to assume that the total area of the choke is equal to the effective area through which gas flows (A_g) plus the effective area through which oil flows (A_o). Then

$$A = A_g + A_o \tag{9}$$

Defining the following structural parameters as

$$\alpha_g = \frac{A_g}{A}$$

and

$$\alpha_o = \frac{A_o}{A}$$

then

$$\alpha_o + \alpha_g = 1$$

Equations (7) and (8) can be rewritten at standard conditions of pressure and temperature (1 atm and 15·54°C) as

$$Q_o = \frac{\pi}{4} - \frac{C_o u_o d^2 \alpha_o}{\beta_o} \tag{10}$$

and

$$Q_g = \frac{\pi}{4}\frac{C_g u_g d^2 \alpha_g}{\beta_g}. \tag{11}$$

The gas–oil ratio through the choke at standard conditions of pressure and temperature is

$$R = \frac{Q_g}{Q_o} = \frac{C_g u_g \alpha_g \beta_o}{C_o u_o \alpha_o \beta_g} \tag{12}$$

where β_o is the oil formation–volume factor, β_g is the gas compressibility factor and R is the gas–oil ratio. In the present study the gas–oil ratio refers to the liberated gas (R_s) since no free gas exists in the reservoir. All these parameters are obtained from laboratory pressure–volume–temperature data. The derivation here applies only for the case where the reservoir is above bubble-point condition and no free gas exists in the reservoir.

The gas compressibility factor is defined as

$$\beta_g = \frac{P_{sc}\,Z\,T_1}{T_{sc}P_1}. \tag{13}$$

The structural parameter α_g can be calculated from eqn (12)

$$\alpha_g = \alpha_o\,R_s\frac{u_o\,C_o\beta_g}{u_g\,C_g\,\beta_o} \tag{14}$$

but since $\alpha_o = 1-\alpha_g$, then eqn (14) can be re-written as

$$\alpha_g = \frac{(R_s u_o\beta_g o/u_g\beta_o\,C_g)}{1+(R_s u_o\beta_g C_o/u_g\beta_o C_g)}. \tag{15}$$

Substituting eqns (5), (6), (11) and (15) into eqns (8) and (9) yields the oil and gas flow rate equations at standard conditions of pressure and temperature as follows:

$$Q_o = \frac{\pi\,C_o\,d^2}{4\beta_o}\left[\frac{2g_c\,P_1}{\rho_o}\left(1-\frac{P_2}{P_1}\right)\right]^{\frac{1}{2}}(1-\alpha_g) \tag{16}$$

and

$$Q_g = \frac{\pi T_{sc}\,P_1 C_g d^2\alpha_g}{4\,P_{sc}\,Z\,T_1}\left[\frac{2g_c\,P_1 k}{\rho_{g_1}\,(k-1)}\left(1-\frac{P_2}{P_1}\right)^{k-1/k}\right]^{\frac{1}{2}} \tag{17}$$

The total fluid flow is $Q_T = Q_g + Q_o$ thus

$$Q_T = \frac{\pi C_o d^2(1-\alpha_g)}{4\beta_o}\left[\frac{2g_c P_1 k}{\rho_o}\left(1-\frac{P_2}{P_1}\right)\right]^{\frac{1}{2}} + $$

$$+\frac{\pi C_g d^2\,\alpha_g\,T_{sc}\,P_1}{4Z\,P_{sc}\,T_1}\left[\frac{2g_c P_1 k}{\rho_g(k-1)}\left(1-\frac{P_2}{P_1}\right)^{k-1/k}\right]^{\frac{1}{2}}. \tag{18}$$

The resulting gas–oil ratio at standard conditions of pressure and temperature is

$$R_p = \frac{Q_g}{Q_o} + (R_s - R). \tag{19}$$

CRITICAL FLOW

The flow through the choke is said to be critical flow when $\lambda = \lambda_c$ where λ_c is the critical pressure ratio $(P_2/P_1)_c$ and

$$\frac{d^{Q}T}{d\lambda} = 0. \tag{20}$$

This criterion can be obtained by differentiating eqn (18) and setting the resulting equation to zero. The condition following the solution of eqn (18) is

$$\lambda_c = 1-\left(\frac{C_g}{C_o}\cdot\frac{\alpha_g}{(1-\alpha_g)}\cdot\frac{\beta_o}{\beta_g}\right)^{2k}\left(\frac{\rho_o}{\rho_g}\frac{(k-1)}{k}\right)^{k}. \tag{21}$$

Equation (21) defines the critical pressure ratio (λ_c) at which the flow is described as critical flow.

More conveniently, the critical pressure ratio, λ_c, is obtained graphically by plotting the mixture flow rate, Q_T, versus P_2/P_1 at constant upstream pressure, P_1. The critical pressure ratio, λ_c, is found where the slope of the curve Q_T versus P_2/P_1 is zero.

RESULTS AND DISCUSSIONS

Equations (16) and (17) represent the oil and gas volumetric flow rates through chokes. A very useful family of curves could be obtained by calculating Q_o from eqn (16) and plotting it versus upstream wellhead pressure for various choke sizes. The results are shown in Fig. 1. Fluid properties used in the present predictions are given in the Appendix. It can be noted that fluid flow rates increased rapidly for initial pressure range and then levelled off asymptotically at higher pressures. A similar curve could be obtained for gas flow rates. Fig. 2 represents the effect of upstream pressure on the oil and gas structural parameters. It can be noted that a reduction of pressure below its saturation value has a considerable effect on the gas structural parameter (α_o) which is

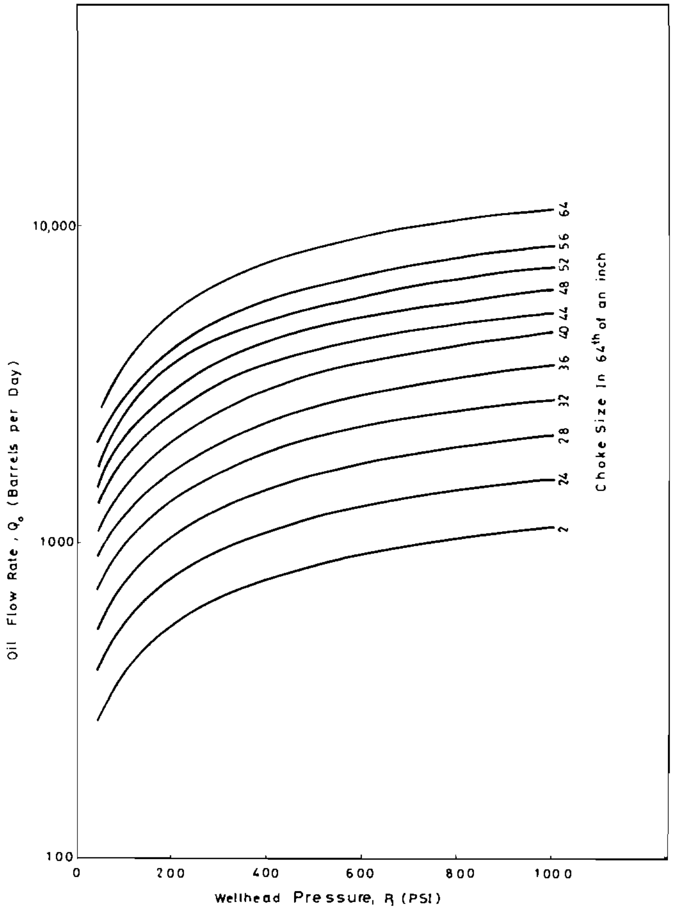

Fig. 1. Oil flow rate through chokes.

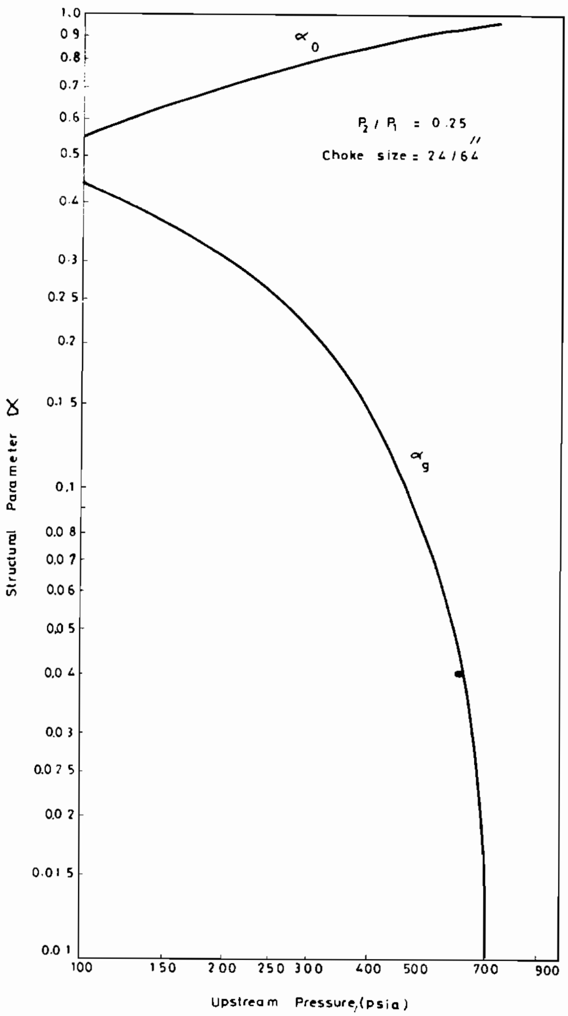

Fig. 2. Effect of upstream pressure on structural parameter.

less influenced by decrease in pressure. Since the only gas available in this system is the solution gas, the presented results are, therefore, expected. At pressures above the saturation pressure the only phase flowing is the oil phase in which α_g is equal to zero, and α_o is unity. If pressure is reduced to below the saturation value, gas will be liberated as free gas and α_g will increase exponentially, while α_o will decrease almost linearly on semi-logarithmic plot. In fact Fig. 2 shows, indirectly, the effect of gas–oil ratio on the oil and gas structural parameters.

The flow through the choke is described as critical flow when $dQ_T/d\lambda = Q$, where λ is (P_2/P_1) ratio. This criterion can be obtained by differentiating eqn (18). More conveniently the critical pressure ratios λ_c are obtained graphically by plotting the mixture flow rate, Q_T, versus P_2/P_1 at different upstream pressures P_1. The plot shown in Fig. 3 shows that λ_c decreases as P_1 increases or λ_c decreases with decrease in gas–oil ratio flowing through the choke but in all cases critical flow conditions were always established at P_2/P_1 below 0·5, as shown in Fig. 4.

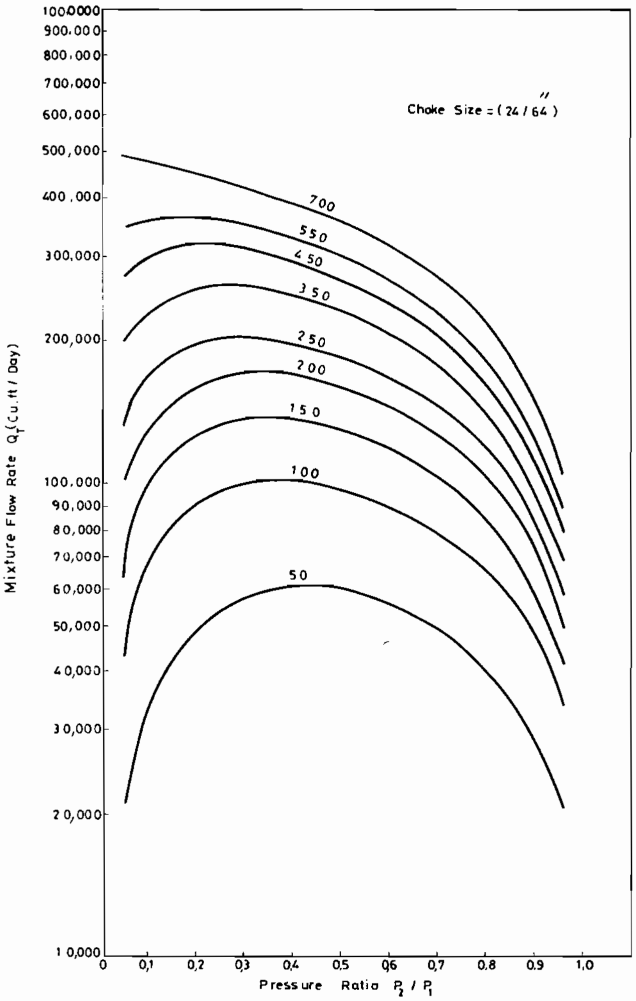

Fig. 3. Effect of pressure ratio on mixture fluid flow.

The specific heat capacity ratio, k, had little effect on the gas flow rate. For comparison purposes, the gas flow rates were predicted for k values of 1·04, 1·25 and 1·4 at different wellhead pressures. The results given in Table 1 show that Q_g decreased with increasing k.

The oil flow rates have been predicted using eqn (16). This prediction was compared with that from eqns (1) and (2), and the actual oil flow rate obtained from one of the oilfields in Kuwait.

The predicted oil flow rates, Q, in eqn (16) were calculated assuming an oil orifice discharge coefficient (C_o) of 0·61, which is based on the orifice and pipe diameters (McCabe & Smith 1956). A gas orifice discharge coefficient (C_g) of 0·98 was used, which is the same value reported by Nind (1964), if only gas phase was flowing. A comparison between these predictions and actual flow rates is made in Table 2 for different choke sizes and pressure ratios. The predicted values versus the actual data are also compared in Fig. 5. The fluid properties used in predicting Q are given in the Appendix.

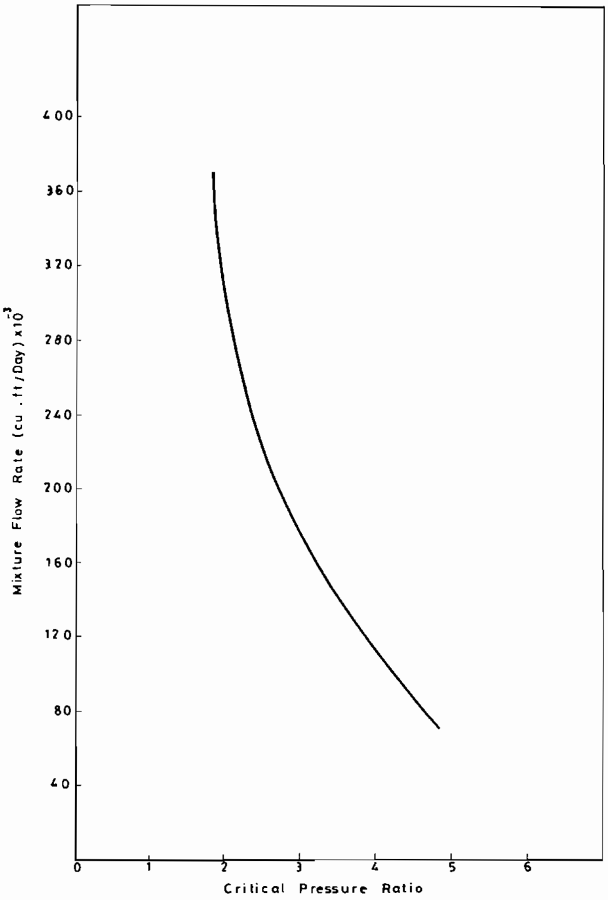

Fig. 4. Maximum mixture flow rate at critical pressure ratios.

Table 1. Effect of gas specific heat capacity ratio, k on Q_g (standard ft^3/day)

Upstream pressure P_1 Psia	$k = 1{\cdot}04$	$k = 1{\cdot}25$	$k = 1{\cdot}4$
700	481490	476361	473635
600	376590	365783	360197
500	299390	287691	281742
400	238754	228101	222850
300	187970	179470	175193
200	141787	135903	132929
100	92560	89783	88361
50	58123	57150	56643

The predicted values of Q using eqn (16) give better results than other empirical values. Still better results could have been obtained if C_o and C_g were used to fit the predicted results to field data and then using the resulting C_o and C_g for any further prediction. However, it is felt that the present method offers a simple theoretical prediction of oil and gas flow rates through using actual fluid properties under consideration.

Table 2. Comparison of actual field data with predicted oil flow rates

Choke size (1/64 in.)	P_1 (Psia)	P_2/P_1	Oil flow rates (barrels/day) Actual	Eqn 15	Nind	Gilbert
16	494	0·49	567	533	446	486
20	691	0·38	1134	1180	975	1036
24	681	0·38	1637	1697	1384	1442
24	590	0·59	1516	1247	1199	1249
28	455	0·56	1499	1499	1259	1289
28	433	0·53	1563	1499	1198	1127
28	590	0·58	2056	1704	1633	1672
28	580	0·57	2158	1712	1604	1643
28	567	0·55	2148	1725	1568	1607
28	569	0·61	1897	1609	1574	1612
32	506	0·49	2143	2237	1829	1846
32	440	0·43	2040	2170	1590	1605
32	458	0·50	2127	2095	1655	1671
32	464	0·48	2062	2138	1677	1693
32	460	0·53	2053	2056	1662	1678
32	265	0·81	1040	939	958	967
40	610	0·57	4808	3595	3444	3393

CONCLUSIONS AND RECOMMENDATIONS

(1) Oil and gas flow rates can be predicted accurately from the present method. The predictions are often better than the empirical equations available in the literature.

(2) A critical flow condition is established at a certain pressure ratio (λ_c) depending on the upstream wellhead pressure.

(3) The critical pressure ratio (λ_c) decreases with increase in gas–oil ratios flowing through the choke.

(4) The change of specific heat capacity ratios of gas had little effect on gas flow rates.

(5) The structural paramter (α) has a considerable effect on flow rates. However, the effect is much more pronounced in the case of gas.

(6) Further investigation should be carried out in order to study the effect of fluid properties on orifice discharge coefficient (C) and structural parameter (α).

(7) The present method should be extended for petroleum reservoirs where the pressure is below saturation pressure, in order to account for the free gas present.

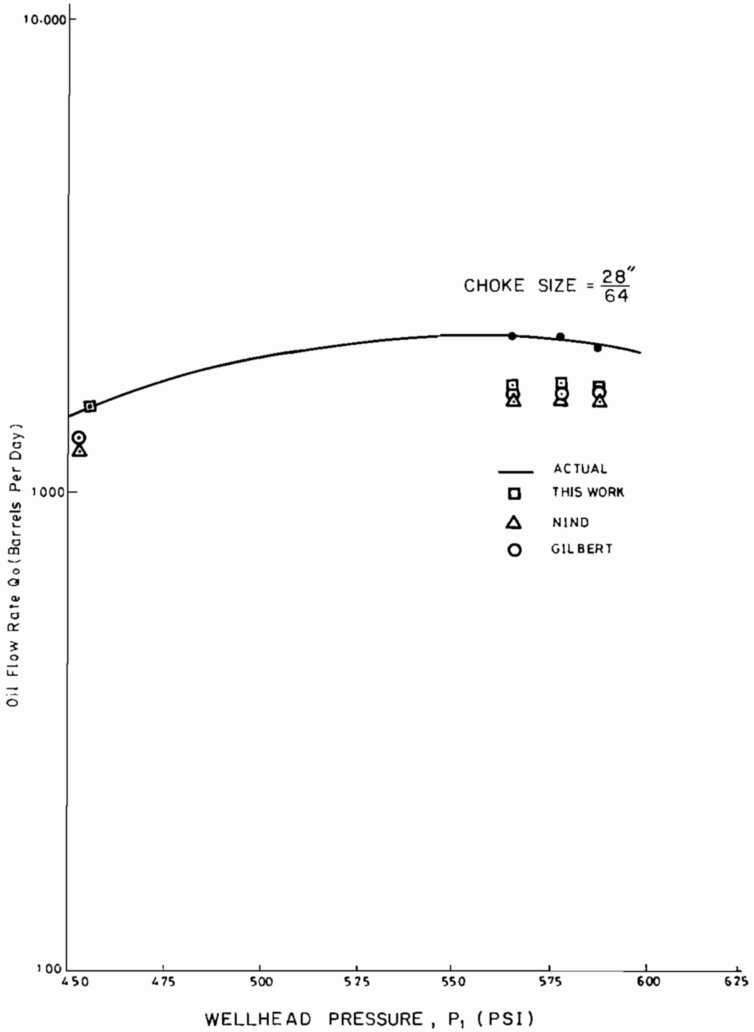

Fig. 5. Comparison of actual data with different empirical equations and the present work.

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NOTATIONS

A = Choke area, cm^2
A_g = Equivalent area to gas flow, cm^2
A_o = Equivalent area to oil flow, cm^2
C_g = Orifice discharge coefficient for gas flow
CR_o = Oil compressibility factor, atm^{-1}
C_o = Orifice discharge coefficient for oil flow
d = Choke diameter, cm
g_c = Conversion factor
k = Ratio of the specific heat of gas at constant pressure to its specific heat at constant volume
P = Pressure, atm
P_b = Bubble point pressure, atm
P_{sc} = Standard pressure (= 1 atm)
P_1 = Upstream choke pressure, atm
P_2 = Downstream choke pressure, atm
Q = Volume flow rate, cm^3/sec
Q_g = Gas volumetric rate, cm^3/sec
Q_o = Oil flow rate, cm^3/sec
Q_T = Total fluid volumetric flow rate, cm^3/sec
R = Gas–oil ratio, cm^3/cm^3
R_p = Production gas–oil ratio, cm^3/cm^3
R_s = Solution gas–oil ratio, cm^3/cm^3
SG = Gas specific gravity
T = Temperature, °C
T_{sc} = Standard temperature (15·54°C)
u = Velocity, cm/sec
u_1 = Upstream choke velocity, cm/sec
u_2 = Downstream choke velocity, cm/sec
Z = Gas compressibility factor

Greek symbols

α_g = Gas structural parameter
α_o = Oil structural parameter
β_g = Gas formation volume factor = $\dfrac{\text{cm}^3 \text{ of gas at } P}{\text{cm}^3 \text{ of gas at standard condition}}$
β_o = Oil formation volume factor = $\dfrac{\text{cm}^3 \text{ of oil at } P}{\text{cm}^3 \text{ of oil at standard condition}}$
λ = Pressure ratio, P_2/P_1
λ_c = Critical pressure ratio
ρ = Density, g/cm^3
ρ_g = Gas density, g/cm^3
ρ_o = Oil density, g/cm^3
ρ_{ob} = Oil density at bubble point pressure, g/cm^3

Subscripts

g = Gas

o = Oil

1 = Upstream choke condition

2 = Downstream choke condition

APPENDIX

The following data were used for the test system which yielded the results reported in this study. Conversion of units from field and laboratory tests to cgs units was carried out.

Oil discharge coefficient, C_o: 0·611.

Gas discharge coefficient, C_g: 0·98.

Ratio of the specific heat of the gas at constant pressure to its specific heat at constant volume, k: 1·04, 1·25, 1·4.

Reservoir bubble point pressure, P_b: 721 psi.

Oil formation volume factor at bubble point, β_{ob}: 1·23798 Reservoir volume/volume at standard condition.

Oil compressibility, $CR_o = 1{\cdot}49 \times 10^{-5}$ psi^{-1}.

Temperature at standard condition, T_{sc}: 60°F.

Pressure at standard condition, P_{sc}: 14·7 psi.

Solution gas–oil ratio at bubble point, R_{sb}: 223 standard ft^3 of gas/standard barrel of oil.

Oil density at bubble point, ρ_{ob}: 48·58 lb$_m$/ft^3.

Reservoir temperature, T: 120°F.

The fluid properties for the test system were suitably curve-fitted with empirical equations using the least squares technique.

Oil formation volume factor:

$\beta_o = 1{\cdot}00817\ P^{0{\cdot}0312}$, $P < P_b$ volume at P/volume at standard condition.

$\beta_o = 1{\cdot}23798 e^{-CR_o(P-P_b)}$, $P < P_b$, volume at P/volume at standard condition.

Solution gas–oil ratio:

$R_s = 5{\cdot}31726\ P^{0{\cdot}56546}$ ft^3 of gas/ft^3 of oil

Oil density:

$\rho_o = 53{\cdot}15041/P^{0{\cdot}01356}$, $P < P_b$ lb$_m$/ft^3.

$\rho_o = \rho_{ob} e^{-CR_o(P-P_b)}$, $P \leq P_b$ lb$_m$/ft^3.

Gas density:

$\rho_g = 0{\cdot}00534\ SG\ P$.

$SG = 1{\cdot}18836/P^{0{\cdot}4027}$.

سريان الزيت والغاز خلال الصمامات الخانقة

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خلاصة

تم في هذا البحث عمل نموذج نظري لتقدير معدل سريان الزيت والغاز خلال الصمامات الخانقة. ويفترض هذا النموذج وجود سرعة حرجة للسريان عند هذه الصمامات. وقد تم اختبار هذا النموذج بقياسات عملية من آبار النفط بالكويت.

والغرض من هذا البحث هو عمل هذا النموذج النظري الذي سوف يوفر على المهندسين في حقول البترول القيام بقياسات صعبة لتعيين معدل سريان الزيت والغاز.