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A simple digital deadbeat controller for a servo system

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ABSTRACT

A digital controller which gives a deadbeat type of response was designed for a laboratory servo system. The controller was then realised with R-C elements. A study of the system for different gains indicates that the controller is effective for small signals when a linear model of the servo system is valid.

INTRODUCTION

Compensation of sample-data control systems may be provided with the help of a continuous network of pulsed-data network. The latter, commonly known as a digital controller, is usually preferred for its versatility and effectiveness. Design principles of such controllers are introduced in standard texts (Jury 1958; Kuo 1963) while specific examples of design, their realisation and also difficulties encountered are available in the literature (Sekey 1975; Terrel 1977). It is often required that the output of a control system should track or follow the input in such a way that the final steady-state value is reached with minimum rise time and without any oscillation. The controller which performs this, referred to as a digital deadbeat controller, may be placed in cascade with the plant or in the feedback loop as usual in a conventional control system design.

This paper gives a design procedure of a deadbeat controller for a laboratory servo system. The concepts of z-transform, bilinear transform, etc. are discussed in brief for completeness only. The controller transfer function has been derived considering a second order dynamic model of the servo system. A modification has been suggested in order to make the controller R-C realisable. Various controller circuits constructed for small and medium gains gave satisfactory response.

THE SERVO SYSTEM

The basic components of the servo system are: a d.c. servo motor driving the load, a servo amplifier which controls the current through the motor field, an operational amplifier, a tachogenerator which provides a signal proportional to the angular position and a set of input–output potentiometers. The servo motor is a separately excited d.c. motor with a constant armature supply, the field coil being differentially excited from the servo amplifier. A block diagram of the overall servo system is given in Fig. 1. The forward and feedback loop gains can be adjusted with the respective selector switches shown. For a constant feedback gain (K_f) of 65%, the servo system gives rise to growing oscillations for K_e greater than 15% in the forward path. The open-loop transfer function of the servo system, henceforth referred to as 'plant' for $K_e = 20\%$ and $K_f = 65\%$, obtained from a number of tests, is

$$\frac{1910}{s(s+40{\cdot}6)}. \tag{1}$$

A continuous controller could be used to improve the response of this system, the design being carried out in the s-domain. However, since sampled-data systems require less energy and operate with a high degree of sensitivity, sampling was intentionally introduced. The continuous signal is sampled, fed to the controller and then reconstructed with a hold device before being fed to the plant. The simplest way to reconstruct the signal from the sampled value is to assume that the value remains constant during the sampling interval, or with 'zero order hold'. The block diagram of the sampled-data system is given in Fig. 2.

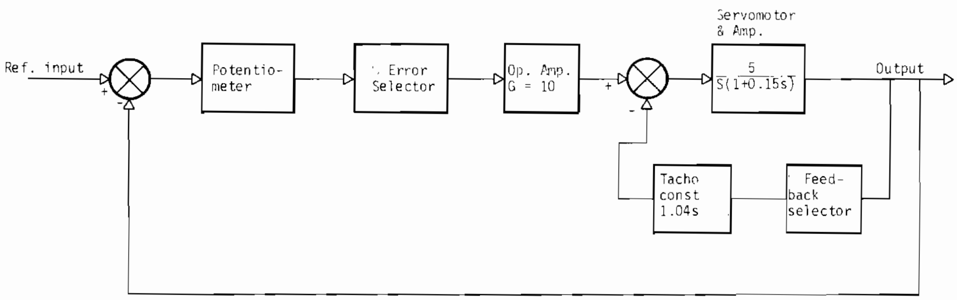

Fig. 1. Servo system block diagram.

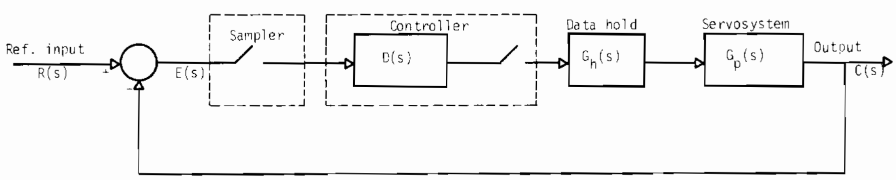

Fig. 2. Block diagram of a sampled-data system with controller.

Z AND r TRANSFORMS

If a continuous signal $e(t)$ is sampled at a sampling period of T sec, then the sampled signal can be represented mathematically as

$$e^*(t) = e(nT)\,\delta\,(t-nT);\; n = 0, 1, 2, \ldots \tag{2}$$

where $e(nT)$ denotes the values of the function $e(t)$ at the discrete sampling instants 0, $T, 2T, \ldots$, and $\delta(t)$ is an impulse function. Taking the Laplace transform on both sides,

$$\mathscr{L}[e^*(t)] = E^*(s) = \sum_0^n e(nT)\,e^{-nTs}. \tag{3}$$

The expression for $E^*(s)$ contains the factor e^{-nTs}, which makes it a nonalgebraic function of s and greatly magnifies the work of taking the inverse Laplace transform. Introducing a simple change of variable

$$z = e^{sT},$$

or

$$s = \frac{1}{T} \ln z, \tag{4}$$

reduces equation (3) to a power series in z^{-1} as follows:

$$E^*(s)|_{s = 1/T \ln z} = \sum_{0}^{n} e(nT)\, z^{-n}. \tag{5}$$

Expression (4) defines a correspondence or mapping from the s-plane to the z-plane and vice versa. As shown in Fig. 3, the entire left half of the s-plane is mapped into the unit circle in the z-plane, and the right half outside of it. The imaginary axis (jw) is

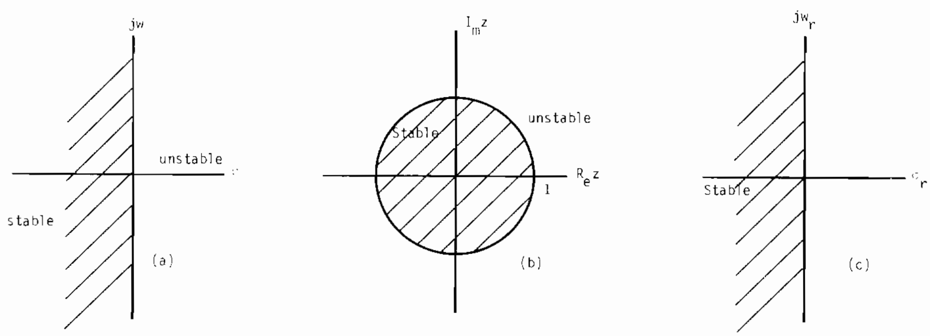

Fig. 3. Regions of stability for continuous and sampled-data systems; (a) s-plane, (b) z-plane, and (c) r-plane.

mapped on the unit circle; details of the mapping are given in Jury (1958) and Kuo (1963).

The stability of linear continuous systems can be determined directly from the location of the roots of the characteristic equation in the s-plane. A similar procedure can be applied to the sampled-data system by a transformation from the z to the r-plane as follows:

$$z = \frac{r+1}{r-1}. \tag{6}$$

The r-plane plays the same role as the s-plane for continuous systems and the methods of analysis of continuous systems can be used for s.d. systems with this transformation. Fig. 3 illustrates the mappings of the s, z and r-planes. The output response of a zero order hold can be expressed as

$$y(t) = u(t) - u(t-T) \tag{7}$$

where $u(t)$ is a step input and T is the sampling period. Taking Laplace transform on both sides, the transfer function of the hold device in Fig. 2 is

$$\frac{Y(s)}{E(s)} = \frac{1-e^{-Ts}}{s}. \tag{8}$$

The combined transfer function of the hold device and plant, in the z-domain for a sampling period of $T = 0{\cdot}01$ sec is then

$$G_s(z) = G_{ho}G_p(z) = \mathfrak{z}\left[\frac{1-e^{-Ts}}{s}\cdot\frac{1910}{s(s+40{\cdot}6)}\right]$$

$$= 1910\,(1-z^{-1})\,\mathfrak{z}\left[-\frac{1}{40{\cdot}6^2}\frac{1}{s}+\frac{1}{40{\cdot}6}\frac{1}{s^2}+\frac{1}{40{\cdot}6^2}\cdot\frac{1}{s+40{\cdot}6}\right]$$

$$= 1910\,(1-z^{-1})\left[-\frac{1}{40{\cdot}6^2}\frac{z}{z-1}+\frac{1}{40{\cdot}6}\frac{0{\cdot}01z}{(z-1)^2}+\frac{1}{40{\cdot}6^2}\frac{1}{z-e^{-40{\cdot}6}}\right]$$

$$= 0{\cdot}083426\,\frac{z+0{\cdot}8847}{(z-1)\,(z-0{\cdot}666)}. \qquad (9)$$

Substituting

$$z = \frac{r+1}{r-1},$$

the transfer function in the r-plane is

$$G_s(r) = 0{\cdot}2353\,\frac{(-1+r)\,(0{\cdot}06117+r)}{(4{\cdot}988+r)}. \qquad (10)$$

The frequency response of the sampled-data system can be obtained by substituting $r = j\omega_r$, when

$$G_s(j\omega_r) = 0{\cdot}2353\,\frac{(-1+j\omega_r)\,(0{\cdot}06117+j\omega_r)}{(4{\cdot}988+j\omega_r)}. \qquad (11)$$

A simple Bode plot of expression (11) shows that the gain margin is negative, indicating that the plant (including the hold device) is not stable, requiring a controller or compensator for satisfactory operation of the servo system. In the following sections, the design of a controller, which provides a deadbeat response and its realisation, is discussed.

Deadbeat response

Let the overall transfer function of the system given in Fig. 2, expressed in the z-domain, be

$$M(z) = \frac{C(z)}{R(z)} = \frac{A(z)}{z^p}. \qquad (12)$$

For a deadbeat response, the output of the system must equal the input for all time after a given finite time, say the kth sample, and the time contribution of $A(z)$ should be unity at the kth sample and thereafter. In addition to the requirement of physical realisability of the controller, the rise time should equal a minimum number of sampling periods, and the settling time measured at the sampling instants should be finite.

The design procedure starts with the selection of an overall system function $M(z)$ meeting the specifications of minimum rise time, finite settling time, zero steady-state error, and at the same time satisfying the condition of realisability. From Fig. 2

$$M(z) = \frac{C(z)}{R(z)} = \frac{D(z)\,G_s(z)}{1+D(z)\,G_s(z)}, \tag{13}$$

wherefrom the controller transfer function is

$$D(z) = \frac{M(z)}{1-M(z)} \cdot \frac{1}{G_s(z)}. \tag{14}$$

The error signal $E(z)$ is expressed as

$$E(z) = R(z)[1-M(z)]. \tag{15}$$

From the final value theorem

$$Lt_{n\to\infty} e(nT) = Lt_{z\to 1}(1-z^{-1})R(z)[1-M(z)]. \tag{16}$$

For a unit step input

$$R(z) = \frac{1}{1-z^{-1}}. \tag{17}$$

One of the requirements for a deadbeat response is that steady-state error (16) should be zero, which requires $M(z)$ to be of the form z^{-k}. Also, the fastest response is obtained when $M(z)$ equals z^{-1}. Substituting the expression for $G_s(z)$ from (9) into (14) and simplifying, one obtains

$$D(z) = \frac{12(z-0{\cdot}666)}{z+0{\cdot}8847}. \tag{18}$$

R-C realisation of the deadbeat controller

The expression for $D(z)$ given in (18) is not R-C realisable due to the term in its denominator, which when converted to the s-domain gives a function which is not p-r (positive real). This difficulty can be overcome by placing the controller in the feedback path such that the overall transfer functions in Figs 4(a) and (b) remain the same. Equating the overall transfer functions in the two figures, one gets

$$\frac{G_{ho}G_s(z)D(z)}{1+G_{ho}G_s(z)D(z)} = \frac{G_{ho}G_s(z)}{1+G_{ho}H(z)+G_{ho}G_s(z)} \tag{19}$$

or

$$G_{ho}H(z) = \frac{1-D(z)}{D(z)}. \tag{20}$$

This can be rewritten as

$$\mathfrak{Z}\left[\frac{H(s)}{s}\right] = \frac{1}{1-z^{-1}}\left[\frac{1}{D(z)}-1\right]. \tag{21}$$

Substituting the expression for $D(z)$ from (18) into (21), gives us

$$\begin{aligned}\mathfrak{Z}\left[\frac{H(s)}{s}\right] &= \frac{1}{1-z^{-1}}\left[\frac{z+0{\cdot}8847}{12(z-0{\cdot}666)}-1\right]\\ &= 0{\cdot}7397\,\frac{z^{-1}-1{\cdot}23919}{(1-0{\cdot}666z^{-1})(1-z^{-1})}\\ &= \frac{-0{\cdot}5297}{1-0{\cdot}666z^{-1}} - \frac{0{\cdot}3869}{1-z^{-1}}.\end{aligned} \tag{22}$$

Taking the inverse z-transform

$$\frac{H(s)}{s} = \frac{-0{\cdot}5297}{z+40{\cdot}6} - \frac{0{\cdot}3869}{s}$$

or

$$H(s) = 0{\cdot}9166\,\frac{s+17{\cdot}1367}{s+40{\cdot}6}. \tag{23}$$

The transfer function (23) is of the form $K(s+a)/(s+b)$, which can be realised with R-C elements. A choice of $C_1 = 30\,\mu F$ in Fig. 5 gives $R_1 = 1{\cdot}945$K ohms and $R_2 = 1{\cdot}42$K ohms for the desired pole zero locations.

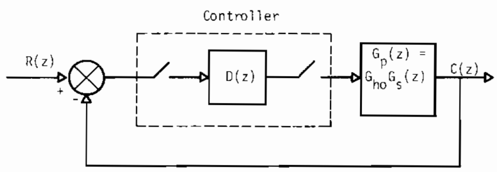

Fig. 4(a). System block diagram with $D(z)$ in the forward path.

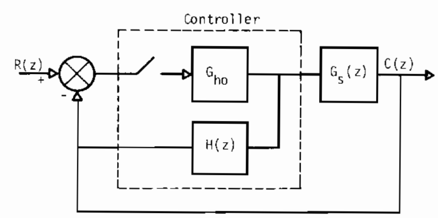

Fig. 4(b). Equivalent diagram of (a) with a controller in the feedback path.

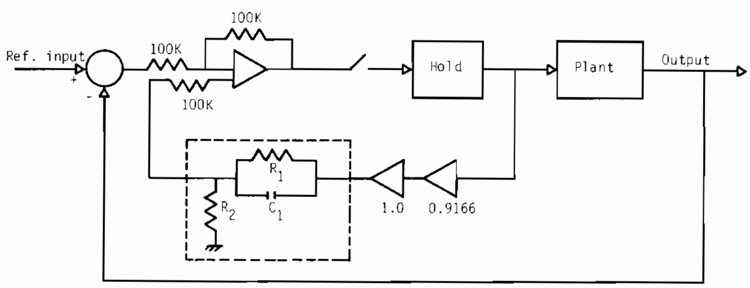

Fig. 5. Plant with a deadbeat controller.

RESULTS

The response of the servo system was recorded for different error settings (gains) in the forward path. The gain of the feedback path was kept constant at a 65% setting for all the studies. For a step input, the uncompensated system is unstable for an error setting of more than 15%. At lower settings, however, the system is stable, but response worsens as the gain increases.

The static loop sensitivity and, in turn, the location of the poles and zeroes of $H(s)$ depend on the error setting of the forward path. For each error setting, the control circuit had to be redesigned. The results of some of the cases studied are summarised in Table 1. For smaller gains, the response is almost deadbeat with no overshoot but the rise time and steady-state error are larger. For larger gains, the rise time decreases but there may be a slight overshoot. Fig. 6(a) shows the response of the uncompensated continuous system for a 20% error setting while Fig. 6(b) shows the response of the

Table 1. Summary of cases studied (C_1 is 30 μF for all the cases).

% Error setting	Static loop sensitivity of plant	Rise time (sec)	Peak overshoot (%)	Paramerers of $H(s)$		
				R_1 (Kohms)	R_2 (Kohms)	gain
0·1	9·6	5·4	—	821	39·60	1·00
5	477·5	1·7	—	0·83	0·51	0·95
10	955	1·5	—	4·19	0·91	0·93
20	1910	1·2	—	1·34	0·29	0·91
30	2685	0·9	12	2·26	0·40	0·87
40	3820	0·8	23	7·73	1·69	0·84
60	5730			—	—	—

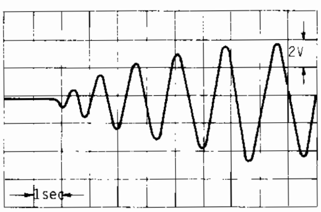

Fig. 6(a). Oscillogram of the uncompensated continuous system for 20% error setting.

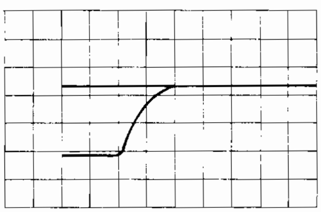

Fig. 6(b). Response corresponding to 6(a), with a deadbeat controller on the s.d. system.

same system sampled and with a deadbeat controller. For an error setting of 60% and above, it was not possible to realise a controller with the help of R-C elements.

CONCLUSIONS

The performance of a laboratory servo system was studied with continuous as well as sampled input. Sampling was intentionally introduced in order to improve the response of the system with additional compensators or controllers. The controller was so designed that it gave a deadbeat type of response for a step input to the servo system. The controller design was based on a linear second order dynamic model of the servo system. The servo-motor field was assumed to be unsaturated. Controllers were designed and constructed with R-C elements for small and moderate gains.

It was observed that sampling of the continuous signal made the system extremely sensitive and even with moderate gains it tended to be unstable in the absence of a compensator. This behaviour was also observed from analytical models. At lower gains, the controller gave rise to a response having virtually no overshoot but the rise time was relatively large. As the gain increased, the rise time decreased but the response was not exactly deadbeat. For very large gains, realisation of the controller with R-C elements was not possible.

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APPENDIX

Sampler and hold circuit: The function of the sampler circuit is merely that of a switch opening and closing at a rate determined by the multivibrator output frequency. The multivibrator is connected between the base and collector of the transistor shown in Fig. A1. When the base of the transistor is negative with respect to ground, the collector base diode of the transistor is forward biased and no output voltage is obtainable between emitter and collector. But when the base is positive, the diode is reverse biased and the output voltage is a sampled one, the rate of which depends on the multivibrator frequency. The sampler and hold circuit is given below (Fig. A1) with the values of different components labelled on them.

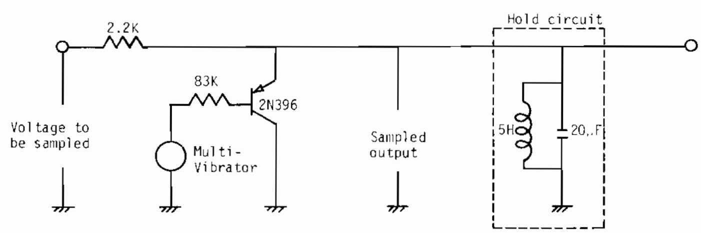

Fig. A1. Sampler and hold circuit.

جهاز تحكم رقمي بسيط يعطي تجاوبا مستمرا لنظام الاداء الآلي

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خلاصة

لقد تم تصميم جهاز رقمي يعطي تجاوبا مستمرا ومساويا للاشارة المطلوبة بعد فترة زمنية تعتمد على درجة نظام الاداء الآلي المتصل بالجهاز ، كما تم تحقيق هذا الجهاز بواسطة عناصر من المقاومات والمكثفات .

وعند دراسة تشغيل الجهاز مع نظام الاداء الآلي الموجود بالمختبر اتضحت فعالية هذا الجهاز عندما تكون الارشادات صغيرة ، وهي الحالة التي تناظر حالة تمثيل نظام الاداء الآلي بنموذج خطي ، مما يؤكد صلاحية الجهاز .