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ON CONVERGENCE IN THE ULTRA-REAL SPACE

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Abstract. The purpose of this paper is to study convergence in the Ultra-real Space *R with respect to a suitable topology. Specifically we will investigate the relationship between the usual convergence in R of a real sequence and the convergence in *R of its extension (in some way) to *R .

1. PRELIMINARIES

The definitions and notations used in this work are in general those of Bourbaki (1955) and Robinson (1970).

Let u be a fixed ultra-filter on N containing the Fréchet filter. Let \mathcal{J} be the real algebra of all real sequences, addition, multiplication, and multiplication by scalar being defined by:

$$\begin{aligned} (f + g)_p &= f_p + g_p ; \\ (f \cdot g)_p &= f_p \cdot g_p ; \\ (\lambda f)_p &= \lambda \cdot f_p . \end{aligned}$$

The relation " \sim " on \mathcal{J} defined by $f \sim g$ if there exists $U \in u$ such that $f_U = g_U$, is an equivalence relation on \mathcal{J} . The equivalence class of $f \in \mathcal{J}$ will be denoted by *f . It is easy to verify that \sim is compatible with each operation on \mathcal{J} . Denote by θ the equivalence class of the 0-sequence. θ is two-sided ideal of \mathcal{J} and

$$f \sim g \iff f - g \in \theta$$

By extending, in the usual way, the operations on \mathcal{J} to the quotient \mathcal{J}/θ , we obtain an algebra (Bourbaki 1955, p. 104) which is in fact a field. This algebra will be denoted by *R . The order relation \leq on \mathcal{J} defined by

$$f \leq g \iff \forall n \in N : f_n \leq g_n$$

can be extended as usual to *R . *R , ordered by this extended order relation, is an ordered algebra (ordered field of course). It is called the ultra-real space. Denote by φ the mapping $a \rightarrow (a, a, \dots)$ from R to \mathcal{J} , and by π the canonical mapping from \mathcal{J} to *R . It is easy to see that $\pi \circ \varphi$ is an isomorphism between R , as an ordered

field, and $\pi \circ \varphi (R)$. So we can identify R with the subfield $\pi \circ \varphi (R)$ of *R .

Now let \mathcal{N} be the subset of \mathcal{J} defined by

$$n \in \mathcal{N} \iff \forall p \in N : n_p \in N$$

$\pi(\mathcal{N})$ will be denoted by *N and its elements will be called ultra-naturals.

2. *R AS A GENERALISED NORMED SPACE

Let ${}^*f \in {}^*R, f \in {}^*f$. It is easy to show that ${}^*|f|$ is independent of our choice of $f \in {}^*f$. So ${}^*|f| = {}^*|f|$ is well defined. The mapping ${}^*f \rightarrow {}^*|f|$ from *R into *R has the following properties:

$$(I) \quad |{}^*f| \geq \theta. \text{ and } |{}^*f| = \theta \iff {}^*f = \theta;$$

$$(II) \quad |{}^*f + {}^*g| \leq |{}^*f| + |{}^*g|;$$

$$(III) \quad \forall \lambda \in R : |\lambda {}^*f| = |\lambda| \cdot |{}^*f|.$$

These properties, being similar to those of a norm, it is natural to call $|{}^*f|$ an ultra-norm.

An element ${}^*f \in {}^*R$ will be called infinitesimal if $|{}^*f| \leq a$, for all $a > 0$ with $a \in R$. The set of all infinitesimals will be denoted by \mathcal{I} . *f is called finite if there exists $a \in R$ such that $|{}^*f| \leq a$. *f is called infinite if it is not finite. We note that $|{}^*f|$ defines a uniformity on *R . We shall consider always this uniformity and the topology induced by it.

3. THE EXTENDED SEQUENCES

If $f \in \mathcal{J}$, $*n \in *N$; $n, m \in *n$, we can verify that $f \circ n, f \circ m$ are defined in some set $U \in \mathcal{U}$. If we define arbitrarily $f \circ n$ and $f \circ m$ outside U , we have $f \circ n, f \circ m \in \mathcal{J}$ and $f \circ n \sim f \circ m$, so that $*(f \circ n) = *(f \circ m)$. If we denote $*(f \circ n)$ by f_{*n} then the mapping $F: *N \rightarrow f_{*n}$ from $*N$ into $*R$ is said to be the extended sequence of f . It is shown (Robinson 1970) by logical methods that f converges in the usual sense in R iff for all infinite ultra-naturals $*n, *m$ we have $f_{*n} - f_{*m} \in \mathcal{I}$. We prove directly the following:

Theorem

Let f be a real sequence. Then the following four statements are equivalent:

- (a) F is a Cauchy extended sequence;
- (b) $f_{*n} - f_{*m} \in \mathcal{I}$ for all infinite ultra-naturals $*n, *m$;
- (c) f is convergent in R (in the usual sense);
- (d) F is convergent in $*R$.

First we will prove the following lemma.

Lemma

If f is a real sequence and $*n, *m$ are infinite ultra-naturals such that $f_{*n} - f_{*m} \notin \mathcal{I}$ then for any $*n_0 \in *N$, there exists ultra-naturals $*n', *m' > *n_0$ such that $f_{*n'} - f_{*m'} \notin \mathcal{I}$.

Proof:

Let $*n, *m$ be infinite ultra-naturals such that $|f_{*n} - f_{*m}| > \varepsilon$ where ε is a positive standard real. Let $n \in *n, m \in *m$ and $n^0 \in *n_0$. Then there exists some $U \in \mathcal{U}$ such that:

$$i \in U \Rightarrow n_i, m_i, n_i^0 \in N \text{ and } |f_{n_i} - f_{m_i}| > \varepsilon.$$

Since $*n$ and $*m$ are infinite ultra-naturals, then we can choose $k' \in U$ for any $k \in U$ such that $n_{k'} > \max\{n_k, n_k^0\}$ and $m_{k'} > \max\{m_k, m_k^0\}$.

Then we have obviously $|f_{n_{k'}} - f_{m_{k'}}| > \varepsilon$.

Define n', m' by $n'_k = n_k, m'_k = m_k$ on U and arbitrarily on U^c . Now if $*n', *m'$ are defined by n', m' respectively, then we have:

$$*n', *m' > *n_0 \text{ and } |f_{*n'} - f_{*m'}| > \varepsilon$$

Now we present the proof of the theorem.

(a) \Rightarrow (b)

If F is a Cauchy extended sequence, then for the infinitesimal $*\mu > \theta, \exists *n_0$ such that:

$$*n, *m > *n_0 \Rightarrow |f_{*n} - f_{*m}| < *\mu.$$

Now suppose $*n, *m$ are infinite ultra-naturals such that $|f_{*n} - f_{*m}| > \varepsilon$ with ε a positive standard real. Hence by the lemma, there exists $*n', *m' > *n_0$ such that $f_{*n'} - f_{*m'} \notin \mathcal{I}$; a contradiction.

(b) \Rightarrow (c)

Suppose to the contrary that f is not convergent then f is not a Cauchy sequence (in the usual sense). Hence $\exists \varepsilon > 0$ (with $\varepsilon \in R$): $\forall p \in N, \exists n, m > p$ such that $|f_n - f_m| > \varepsilon$.

This means that there exist two infinite increasing sequences of real naturals, say (n_p) and (m_p) , such that $|f_{n_p} - f_{m_p}| > \varepsilon$ for all n_p and m_p .

Let $*n, *m$ be defined by (n_p) and (m_p) respectively. Then $*n, *m$ are obviously infinite ultra-naturals and $|f_{*n} - f_{*m}| > \varepsilon$; a contradiction.

(c) \Rightarrow (d)

Let f be convergent to $d \in R$ and $*\mu$ be an arbitrary positive infinitesimal. Let $\mu \in *U$ with $\mu_k > 0$ for all $k \in N$. Now for all $k, \exists n_k: p > n_k \Rightarrow |f_p - d| < \mu_k$. Let $*n_0$ be defined by (n_k) . Then obviously $|f_{*n} - d| < *\mu$ for all $*n > *n_0$.

(d) \Rightarrow (a)

Obvious.

REMARKS

- (1) This theorem shows that f and F converge or diverge together. Moreover, they have the same limit in the case of convergence.
- (2) Though $*R$ does not have the Dedekind property, it does have the Cauchy completeness property in the sense of this theorem.

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حول التقارب في الفضاء فوق الحقيقي

صلاح احمد وعادل ياسين

قسم الرياضيات بجامعة الكويت

خلاصة

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