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Singularly loaded thin circular quadrant with clamped radial edges and free circular edge

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ABSTRACT

This paper deals with the deflections and boundary values of the bending moments due to a concentrated force acting normally on a thin plate having the shape of a quadrant of a circle whose boundary is clamped along the radial edges and free along the circular edge.

1. INTRODUCTION

Problems concerning the deflections and stresses of normally loaded thin sectorial plates subject to various combinations of boundary conditions along the circular and straight edges have been considered by many writers. References are to be found in a recent paper by the authors (1976). Two methods were used to analyse a semi-circular plate which is clamped along the diameter, free along the circular edge and acted upon by a transverse isolated force. In this paper similar procedures are adopted to study the case of an isotropic thin quadrant of a circle when the plate is clamped along the radial edges, free around the circular edge and subject to a normal concentrated force at any point. It is found that this requires the solving of two dual integral equations in two unknown functions. When the point of application of the concentrated force lies on the bisector radius of the plate, one integral equation is derived and numerically solved corresponding to a certain position of the point load. Numerical results are exhibited in the forms of tables and graphs.

2. FIRST METHOD

A. GENERAL SOLUTION

We assume that C denotes the free boundary of an isotropic homogeneous thin circular plate of centre O, radius c, flexural rigidity D and Poisson's ratio ν. Let $z = x+iy = r\,e^{i\theta} = c\zeta$ be the complex variable of any point P in the horizontal mid-plane of the circular plate which is subject to four point forces each equal to F acting vertically downwards at the points $\zeta_1 = se^{i\alpha}$, $\zeta_2 = \bar{\zeta}_1$, $\zeta_3 = -\zeta_1$, $\zeta_4 = -\zeta_2$, the plate being supported at the $4n$ points $z = \pm ct_k$, $\pm ict_k$ $(k = 1, 2, \ldots, n)$ and the

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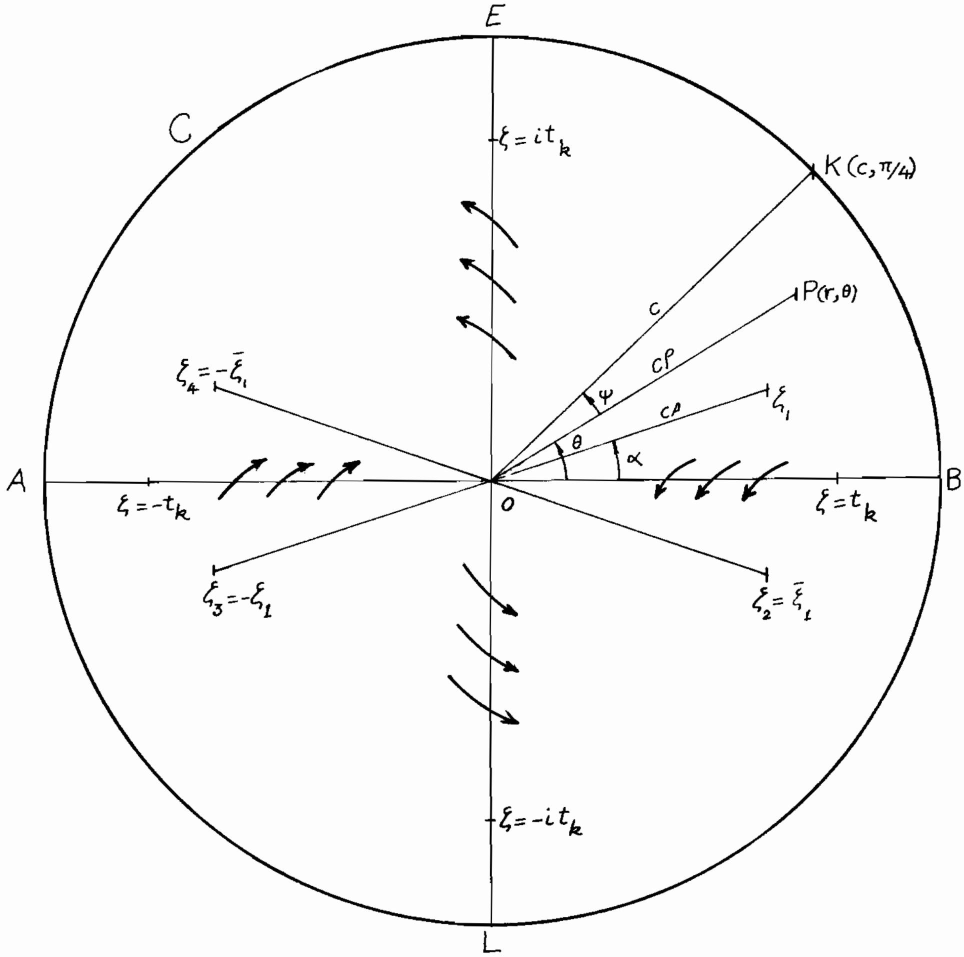

Fig. 1.

points of support are at the same horizontal level. See Figs. 1, 2. If N_k, N_k' are the values of the upward normal reactions at the points $\pm ct_k$, $\pm ict_k$, respectively and the deflection is zero at the centre O, then setting $G_s = 0$ in eqn. (45), p. 273 of Bassali (1958) shows that the deflection w, measured positively downwards, at any point (r,θ) is given by

$$2qw(\rho,\theta;s,\alpha) = S_1+S_2+S_3+S_4+8s^2\left(\frac{\rho^2}{\kappa+1}-\kappa\log s\right)+2(1-\kappa^2)Re\sum_{k=1}^{4}J(\zeta,\zeta_k)$$

$$+\sum_{k=1}^{n}\frac{N_k}{F}\Big[(\rho^2+t_k^2-2\rho t_k\cos\theta)\{\log(1+\rho^2t_k^2-2\rho t_k\cos\theta)-\kappa\log(\rho^2+t_k^2-2\rho t_k\cos\theta)\}$$

$$+(\rho^2+t_k^2+2\rho t_k\cos\theta)\{\log(1+\rho^2t_k^2+2\rho t_k\cos\theta)-\kappa\log(\rho^2+t_k^2+2\rho t_k\cos\theta)\}$$

$$+4t_k^2\left(\kappa\log t_k-\frac{\rho^2}{\kappa+1}\right)+2(\kappa^2-1)Re\{J(\zeta,t_k)+J(\zeta,-t_k)\}\Big]$$

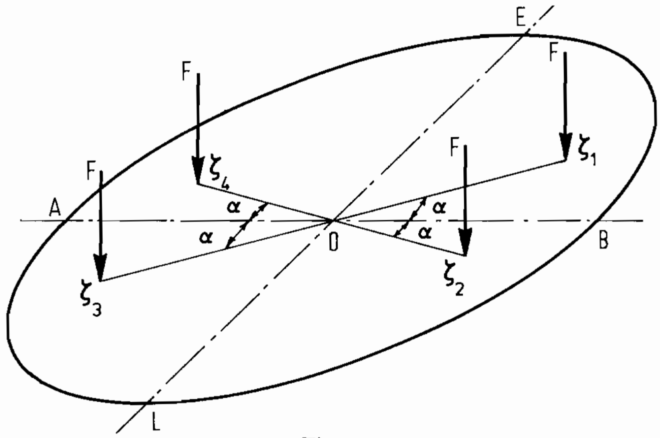

Fig. 2.

$$+\sum_{k=1}^{n}\frac{N_k'}{F}\Big[(\rho^2+t_k^2-2\rho t_k\sin\theta)\{\log(1+\rho^2t_k^2-2\rho t_k\sin\theta)-\kappa\log(\rho^2+t_k^2-2\rho t_k\sin\theta)\}$$
$$+(\rho^2+t_k^2+2\rho t_k\sin\theta)\{\log(1+\rho^2t_k^2+2\rho t_k\sin\theta)-\kappa\log(\rho^2+t_k^2+2\rho t_k\sin\theta)\}$$
$$+4t_k^2\left(\kappa\log t_k-\frac{\rho^2}{\kappa+1}\right)+2(\kappa^2-1)Re\{J(\zeta,it_k)+J(\zeta,-it_k)\}\Big], \tag{1}$$

where

$$\rho = r/c, \qquad \kappa = (3+\nu)/(\nu-1), \qquad q = 8\pi\kappa D/c^2F, \tag{2}$$

$$S_1 = (\rho^2+s^2-2\rho s\cos(\theta-\alpha))$$
$$\{\kappa\log(\rho^2+s^2-2\rho s\cos(\theta-\alpha))-\log(1+\rho^2s^2-2\rho s\cos(\theta-\alpha))\}, \tag{3a}$$

$$S_2 = (\rho^2+s^2-2\rho s\cos(\theta+\alpha))$$
$$\{\kappa\log(\rho^2+s^2-2\rho s\cos(\theta+\alpha))-\log(1+\rho^2s^2-2\rho s\cos(\theta+\alpha))\}, \tag{3b}$$

$$S_3 = (\rho^2+s^2+2\rho s\cos(\theta-\alpha))$$
$$\{\kappa\log(\rho^2+s^2+2\rho s\cos(\theta-\alpha))-\log(1+\rho^2s^2+2\rho s\cos(\theta-\alpha))\}, \tag{3c}$$

$$S_4 = (\rho^2+s^2+2\rho s\cos(\theta+\alpha))$$
$$\{\kappa\log(\rho^2+s^2+2\rho s\cos(\theta+\alpha))-\log(1+\rho^2s^2+2\rho s\cos(\theta+\alpha))\}, \tag{3d}$$

$$J(\zeta,\zeta_k) = \int_0^1 t^{-1}(1-\zeta\bar{\zeta}_k t)\log(1-\zeta\bar{\zeta}_k t)\,dt, \tag{4}$$

and Re denotes the real part. The condition of equilibrium of the plate is

$$\sum_{k=1}^{n}(N_k+N_k') = 2F. \tag{5}$$

Writing $N_k = Ff(t)\,dt$, $N_k' = Fg(t)\,dt$, replacing the summation from $k = 1$ to $k = n$ by integration from $t = 0$ to $t = 1$ and computing the real parts of the quantities involved we obtain the solution

$2qw(\rho,\theta;\, s,\alpha) =$

$$S_1+S_2+S_3+S_4+8s^2\left(\frac{\rho^2}{\kappa+1}-\kappa\log s\right)+2(1-\kappa^2)\sum_1^{\infty}\frac{(\rho s)^{2n}\cos 2n\alpha\cos 2n\theta}{n^2(2n-1)}$$

$$+\int_0^1 f(t)\Big[(\rho^2+t^2-2\rho t\cos\theta)\{\log(1+\rho^2t^2-2\rho t\cos\theta)-\kappa\log(\rho^2+t^2-2\rho t\cos\theta)\}$$
$$+(\rho^2+t^2+2\rho t\cos\theta)\{\log(1+\rho^2t^2+2\rho t\cos\theta)-\kappa\log(\rho^2+t^2+2\rho t\cos\theta)\}$$
$$+4t^2\left(\kappa\log t-\frac{\rho^2}{\kappa+1}\right)+(\kappa^2-1)\sum_1^\infty\frac{(\rho t)^{2n}\cos 2n\theta}{n^2(2n-1)}\Big]dt$$
$$+\int_0^1 g(t)\Big[(\rho^2+t^2-2\rho t\sin\theta)\{\log(1+\rho^2t^2-2\rho t\sin\theta)-\kappa\log(\rho^2+t^2-2\rho t\sin\theta)\}$$
$$+(\rho^2+t^2+2\rho t\sin\theta)\{\log(1+\rho^2t^2+2\rho t\sin\theta)-\kappa\log(\rho^2+t^2+2\rho t\sin\theta)\}$$
$$+4t^2\left(\kappa\log t-\frac{\rho^2}{\kappa+1}\right)+(\kappa^2-1)\sum_1^\infty\frac{(-1)^n(\rho t)^{2n}\cos 2n\theta}{n^2(2n-1)}\Big]dt. \qquad (6)$$

In this solution the two unknown functions $f(t)$ and $g(t)$ represent the intensities of the normal reactions of the continuous supports at the points $z = ct$ and $z = ict$, respectively. The condition of equilibrium now takes the form

$$\int_0^1 [f(t)+g(t)]\,dt = 2. \qquad (7)$$

The expression (6) for the deflection at any point P of the quadrant EOB in Fig. 1 satisfies: (i) the biharmonic equation $\nabla^4 w = 0$ at any point (r,θ) except at (cs,α), (ii) the conditions that the circular boundary $\rho = 1$ is free and (iii) the conditions of zero slope $(\partial w/\partial\theta = 0)$ on the bounding radii $\theta = 0$ and $\theta = \pi/2$. The remaining conditions that $w = 0$ on $\theta = 0$ and $\theta = \pi/2$ now lead to the two dual integral equations

$$\int_0^1 f(t)\,U(\rho,t)\,dt+\int_0^1 g(t)\,V(\rho,t)\,dt = \phi(\rho,s,\alpha), \qquad (8a)$$

$$\int_0^1 f(t)\,V(\rho,t)\,dt+\int_0^1 g(t)\,U(\rho,t)\,dt = \phi(\rho,s,\pi/2-\alpha), \qquad (8b)$$

where

$$U(\rho,t) = (\rho-t)^2\{\log(1-\rho t)-\kappa\log|\rho-t|\}+(\rho+t)^2\{\log(1+\rho t)-\kappa\log(\rho+t)\}$$
$$+2t^2\left(\kappa\log t-\frac{\rho^2}{\kappa+1}\right)+\frac{1}{2}(\kappa^2-1)\sum_1^\infty\frac{(\rho t)^{2n}}{n^2(2n-1)}, \qquad (9)$$

$$V(\rho,t) = (\rho^2+t^2)\{\log(1+\rho^2t^2)-\kappa\log(\rho^2+t^2)\}$$
$$+2t^2\left(\kappa\log t-\frac{\rho^2}{\kappa+1}\right)+\frac{1}{2}(\kappa^2-1)\sum_1^\infty\frac{(-1)^n(\rho t)^{2n}}{n^2(2n-1)}, \qquad (10)$$

$$\phi(\rho,s,\alpha) = (\rho^2+s^2-2\rho s\cos\alpha)\{\log(1+\rho^2s^2-2\rho s\cos\alpha)-\kappa\log(\rho^2+s^2-2\rho s\cos\alpha)\}$$
$$+(\rho^2+s^2+2\rho s\cos\alpha)\{\log(1+\rho^2s^2+2\rho s\cos\alpha)-\kappa\log(\rho^2+s^2+2\rho s\cos\alpha)\}$$
$$+4s^2\left(\kappa\log s-\frac{\rho^2}{\kappa+1}\right)+(\kappa^2-1)\sum_1^\infty\frac{(\rho s)^{2n}\cos 2n\alpha}{n^2(2n-1)}. \qquad (11)$$

When the concentrated force F lies on the bisector radius OK of the quadrant then $\alpha = \pi/4$ and $g(t) = f(t)$. In this case the two integral equations (8) become identical and $f(t)$ satisfies the equilibrium condition

$$\int_0^1 f(t)\,dt = 1 \qquad (12)$$

and the singular integral equation

$$\int_0^1 f(t)\,H(\rho,t)\,dt = \phi(\rho,s,\pi/4), \tag{13}$$

where

$$\begin{aligned} H(\rho,t) = {} & (\rho-t)^2\{\log(1-\rho t)-\kappa\log|\rho-t|\}+(\rho+t)^2\{\log(1+\rho t)-\kappa\log(\rho+t)\} \\ & +(\rho^2+t^2)\{\log(1+\rho^2t^2)-\kappa\log(\rho^2+t^2)\} \\ & +4t^2\left(\kappa\log t-\frac{\rho^2}{\kappa+1}\right)+\frac{\kappa^2-1}{4}\sum_1^\infty\frac{(\rho t)^{4n}}{n^2(4n-1)}, \end{aligned} \tag{14}$$

$$\begin{aligned} \phi(\rho,s,\pi/4) = {} & (\rho^2+s^2-\sqrt{2}\rho s)\{\log(1+\rho^2s^2-\sqrt{2}\rho s)-\kappa\log(\rho^2+s^2-\sqrt{2}\rho s)\} \\ & +(\rho^2+s^2+\sqrt{2}\rho s)\{\log(1+\rho^2s^2+\sqrt{2}\rho s)-\kappa\log(\rho^2+s^2+\sqrt{2}\rho s)\} \\ & +4s^2\left(\kappa\log s-\frac{\rho^2}{\kappa+1}\right)+\frac{\kappa^2-1}{4}\sum_1^\infty\frac{(-1)^n(\rho s)^{4n}}{n^2(4n-1)}. \end{aligned} \tag{15}$$

The equation of the deflection surface now takes the form

$$\begin{aligned} & 2qw(\rho,\theta;s,\pi/4) = \\ & \quad (\rho^2+s^2-2\rho s\cos\psi)\{\kappa\log(\rho^2+s^2-2\rho s\cos\psi)-\log(1+\rho^2s^2-2\rho s\cos\psi)\} \\ & \quad +(\rho^2+s^2-2\rho s\sin\psi)\{\kappa\log(\rho^2+s^2-2\rho s\sin\psi)-\log(1+\rho^2s^2-2\rho s\sin\psi)\} \\ & \quad +(\rho^2+s^2+2\rho s\cos\psi)\{\kappa\log(\rho^2+s^2+2\rho s\cos\psi)-\log(1+\rho^2s^2+2\rho s\cos\psi)\} \\ & \quad +(\rho^2+s^2+2\rho s\sin\psi)\{\kappa\log(\rho^2+s^2+2\rho s\sin\psi)-\log(1+\rho^2s^2+2\rho s\sin\psi)\} \\ & \quad +8s^2\left(\frac{\rho^2}{\kappa+1}-\kappa\log s\right)+\frac{1}{2}(\kappa^2-1)\sum_1^\infty\frac{(-1)^{n-1}(\rho s)^{4n}\cos 4n\theta}{n^2(4n-1)} \\ & \quad +\int_0^1 f(t)\Big[(\rho^2+t^2-2\rho t\cos\theta)\{\log(1+\rho^2t^2-2\rho t\cos\theta)-\kappa\log(\rho^2+t^2-2\rho t\cos\theta)\} \\ & \quad +(\rho^2+t^2-2\rho t\sin\theta)\{\log(1+\rho^2t^2-2\rho t\sin\theta)-\kappa\log(\rho^2+t^2-2\rho t\sin\theta)\} \\ & \quad +(\rho^2+t^2+2\rho t\cos\theta)\{\log(1+\rho^2t^2+2\rho t\cos\theta)-\kappa\log(\rho^2+t^2+2\rho t\cos\theta)\} \\ & \quad +(\rho^2+t^2+2\rho t\sin\theta)\{\log(1+\rho^2t^2+2\rho t\sin\theta)-\kappa\log(\rho^2+t^2+2\rho t\sin\theta)\} \\ & \quad +8t^2\left(\kappa\log t-\frac{\rho^2}{\kappa+1}\right)+\frac{1}{2}(\kappa^2-1)\sum_1^\infty\frac{(\rho t)^{4n}\cos 4n\theta}{n^2(4n-1)}\Big]\,dt, \end{aligned} \tag{16}$$

where $\psi = \pi/4-\theta$. Putting $\theta = \pi/4$ in (16) we find that the deflection profile along the radius of symmetry OK is given by

$$\begin{aligned} & qw(\rho,\pi/4;s,\pi/4) = \\ & (\rho-s)^2\{\kappa\log|\rho-s|-\log(1-\rho s)\}+(\rho+s)^2\{\kappa\log(\rho+s)-\log(1+\rho s)\} \\ & +(\rho^2+s^2)\{\kappa\log(\rho^2+s^2)-\log(1+\rho^2s^2)\}+4s^2\left(\frac{\rho^2}{\kappa+1}-\kappa\log s\right)-\frac{\kappa^2-1}{4}\sum_1^\infty\frac{(\rho s)^{4n}}{n^2(4n-1)} \\ & +\int_0^1 f(t)\Big[(\rho^2+t^2-\sqrt{2}\rho t)\{\log(1+\rho^2t^2-\sqrt{2}\rho t)-\kappa\log(\rho^2+t^2-\sqrt{2}\rho t) \\ & +(\rho^2+t^2+\sqrt{2}\rho t)\{\log(1+\rho^2t^2+\sqrt{2}\rho t)-\kappa\log(\rho^2+t^2+\sqrt{2}\rho t) \\ & +4t^2\left(\kappa\log t-\frac{\rho^2}{\kappa+1}\right)+\frac{\kappa^2-1}{4}\sum_1^\infty\frac{(-1)^n(\rho t)^{4n}}{n^2(4n-1)}\Big]\,dt, \end{aligned} \tag{17}$$

and the maximum deflection at K is determined by

$$\frac{q}{\kappa-1}w(1,\pi/4;s,\pi/4) =$$

$$(1-s)^2\log(1-s)+(1+s)^2\log(1+s)+(1+s^2)\log(1+s^2)$$
$$+4s^2\left(\frac{1}{\kappa^2-1}+\frac{\kappa}{1-\kappa}\log s\right)-\frac{\kappa+1}{4}\sum_1^\infty\frac{s^{4n}}{n^2(4n-1)}$$
$$-\int_0^1 f(t)\Big[(1+t^2-\sqrt{2}t)\log(1+t^2-\sqrt{2}t)+(1+t^2+\sqrt{2}t)\log(1+t^2+\sqrt{2}t)$$
$$+4t^2\left(\frac{1}{\kappa^2-1}+\frac{\kappa}{1-\kappa}\log t\right)-\frac{\kappa+1}{4}\sum_1^\infty\frac{(-1)^n t^{4n}}{n^2(4n-1)}\Big]dt. \tag{18}$$

The sum M_r+M_θ of the bending moments at any point (r,θ) of the quadrant is found either by using the following standard formula given by Timoshenko & Woinowsky-Krieger (1960), p. 283:

$$M_r+M_\theta = -D(1+\nu)\nabla^2 w = -D(1-\nu)\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\right), \tag{19}$$

where w is given by (16) or by applying the formula (53a), p. 275 of Bassali (1958). Setting $G = 0$ in the latter formula leads to

$$M_r+M_\theta = -\frac{(1+\nu)F}{4\pi\kappa}\Big[\kappa\log(\rho^8+s^8+2\rho^4s^4\cos 4\theta)-\log(1+\rho^8s^8+2\rho^4s^4\cos 4\theta)$$
$$+\frac{4(1-s^2)(1-\rho^8s^8)}{1+\rho^8s^8+2\rho^4s^4\cos 4\theta}-\frac{8s^2}{1+\nu}-$$
$$-\int_0^1 f(t)\Big\{\kappa\log(\rho^8+t^8-2\rho^4t^4\cos 4\theta)-\log(1+\rho^8t^8-2\rho^4t^4\cos 4\theta)$$
$$+\frac{4(1-t^2)(1-\rho^8t^8)}{1+\rho^8t^8-2\rho^4t^4\cos 4\theta}-\frac{8t^2}{1+\nu}\Big\}dt\Big]. \tag{20}$$

Putting $\theta = 0$ in (20) yields the following values for the bending moments M_x and M_y along the clamped radius OB of the quadrant:

$$\frac{(M_x)_{\theta=0}}{\nu} = (M_y)_{\theta=0} = \frac{(M_x+M_y)_{\theta=0}}{1+\nu}$$
$$= -\frac{F}{2\pi\kappa}\Big[\kappa\log(\rho^4+s^4)-\log(1+\rho^4s^4)-\frac{4s^2}{1+\nu}+\frac{2(1-s^2)(1-\rho^4s^4)}{1+\rho^4s^4}$$
$$-\int_0^1 f(t)\Big\{\kappa\log|\rho^4-t^4|-\log(1-\rho^4t^4)-\frac{4t^2}{1+\nu}+\frac{2(1-t^2)(1+\rho^4t^4)}{1-\rho^4t^4}\Big\}dt\Big]. \tag{21}$$

On the circular edge we have

$$(M_r)_{\rho=1} = 0 \tag{22a}$$

and (20) gives

$$(M_\theta)_{\rho=1} = \frac{F}{2\pi\kappa}\Big[4s^2-(1+\kappa)\log(1+s^8+2s^4\cos 4\theta)-\frac{2(1+\nu)(1-s^2)(1-s^8)}{1+s^8+2s^4\cos 4\theta}$$
$$-\int_0^1 f(t)\Big\{4t^2-(1+\kappa)\log(1+t^8-2t^4\cos 4\theta)-\frac{2(1+\nu)(1-t^2)(1-t^8)}{1+t^8-2t^4\cos 4\theta}\Big\}dt\Big]. \tag{22b}$$

This value is checked by applying the formula

$$(M_\theta)_{\rho=1} = \frac{(1-\nu^2)D}{c^2\nu}\left(\frac{\partial^2 w}{\partial\rho^2}\right)_{\rho=1}$$

given by Bassali & Halim (1963), p. 88.

B. NUMERICAL RESULTS (FIRST METHOD)

We now take $\nu = 1/3$, $\kappa = -5$ and $s = 1/2$ so that the concentrated force is applied at the mid-point of the radius of symmetry of the quadrant *EOB*. We then adopt the usual procedure (Bassali & Halim 1963; Bassali & Mahmood 1974; Bassali, Mahmood

Table 1. Values of $f(t)$ ($\nu = \frac{1}{3}$, $s = \frac{1}{2}$)

t	$f(t)$	t	$f(t)$	t	$f(t)$	t	$f(t)$	t	$f(t)$
0·02	−0·2255	0·22	0·9448	0·42	2·892	0·62	1·096	0·82	0·8713
0·06	−1·113	0·26	1·967	0·46	2·288	0·66	1·241	0·86	0·4018
0·10	−0·5208	0·30	2·082	0·50	2·380	0·70	0·7025	0·90	0·9828
0·14	−0·3771	0·34	2·819	0·54	1·670	0·74	0·9723	0·94	1·306
0·18	0·6542	0·38	2·555	0·58	1·743	0·78	0·4503	0·98	−2·783

Table 2. Values of $10^6\delta$ ($\nu = \frac{1}{3}$, $s = \frac{1}{2}$)
First Method

	θ		
ρ	15°, 75°	30°, 60°	45°
0·04	0	0	0
0·08	2	7	9
0·12	14	41	55
0·16	45	132	174
0·20	105	301	395
0·24	199	563	739
0·28	327	922	1211
0·32	484	1369	1807
0·36	659	1880	2507
0·40	839	2418	3279
0·44	1008	2939	4074
0·48	1158	3394	4813
0·52	1282	3754	5317
0·56	1381	4018	5571
0·60	1458	4208	5730
0·64	1519	4348	5845
0·68	1568	4462	5947
0·72	1609	4565	6051
0·76	1645	4668	6170
0·80	1677	4778	6309
0·84	1705	4898	6474
0·88	1726	5033	6668
0·92	1739	5182	6891
0·96	1739	5346	7144
1·00	1726	5525	7427

Table 3. Values of $(-10^6\delta_1)$
($\nu = \frac{1}{3}$, $s = \frac{1}{2}$)

ρ	$(-10^6\delta_1)$
0·04	1559
0·08	11389
0·12	30823
0·16	56942
0·20	86364
0·24	115978
0·28	142776
0·32	164367
0·36	179042
0·40	186179
0·44	186296
0·48	180695
0·52	171206
0·56	159506
0·60	147056
0·64	134752
0·68	123317
0·72	113047
0·76	103542
0·80	95197
0·84	87230
0·88	79085
0·92	68977
0·96	36365
1·00	15931

& Refai 1976) of replacing the equilibrium condition (12) and the integral equation (13) by the following system of N simultaneous linear equations:

$$\sum_{i=1}^{n} f(t_i) = N, \quad \sum_{i=1}^{n} H(\rho_j, t_i) f(t_i) = N\phi(\rho_j, \tfrac{1}{2}, \pi/4) \; (j = 1, 2, \ldots, N-1), \qquad (23a)$$

where

$$t_i = \frac{2i-1}{2N} \; (i = 1, 2, \ldots, N), \; \rho_j = \frac{j}{N} (j = 1, 2, \ldots, N-1). \qquad (23b)$$

Solving eqns (23) in the case $N = 25$ by means of an electronic computer yields Table 1 for the $f(t_i)$-values ($i = 1, 2, \ldots, 25$).

The intensity of the normal reaction at the points on either of the clamped radial edges distant ct from the centre O equals $\frac{1}{2}Ff(t)$. The negative values of $f(t)$ near $t = 0$ and $t = 1$ are to be noticed in Table 1. This is in agreement with the irregular behaviour which occurs at angular points of normally loaded thin elastic plates.

The deflection w at any point (r,θ), the clamping couple $(M_y)_{\theta=0}$ along the clamped radius OB and the bending moment $(M_\theta)_{\rho=1}$ around the circular edge may be put in the forms

$$w = \delta c^2 F/D, \qquad (M_y)_{\theta=0} = \delta_1 F, \qquad (M_\theta)_{\rho=1} = \delta_2 F, \qquad (24)$$

where δ, δ_1 and δ_2 are dimensionless quantities. Evaluating the integrals appearing in eqns (16), (21) and (22b) numerically, using the values of $f(t)$ in Table 1 we obtain Tables 2, 3 and 4.

Graphs showing the deflection profiles along the radii $\theta = 15°, 30°, 45°, 60°, 75°$ and the variation of the clamping couple along either of the clamped radial edges are plotted in Figs 3 and 4.

Table 4. Values of $10^6\delta_2$ (First Method)
($\nu = \frac{1}{3}, s = \frac{1}{2}$)

θ	15°, 75°	30°, 60°	45°
$10^6 \delta_2$	−28938	23054	47457

The maximum deflection at K is

$$w_{max} = 0{\cdot}007427\, c^2F/D \qquad (25)$$

and the maximum bending moment M_θ along the circular edge is

$$(M_\theta)_{\rho=1, \theta=\pi/4} = 0{\cdot}047457\, F. \qquad (26)$$

3. SECOND METHOD

A. QUADRANT WITH FREE CIRCULAR EDGE AND SIMPLY SUPPORTED RADIAL EDGES UNDER A POINT LOAD

We assume that the circular plate with horizontal mid-plane is loaded vertically downwards by the four forces F, $-F$, F and $-F$ acting at the four points ζ_1, ζ_2, ζ_3

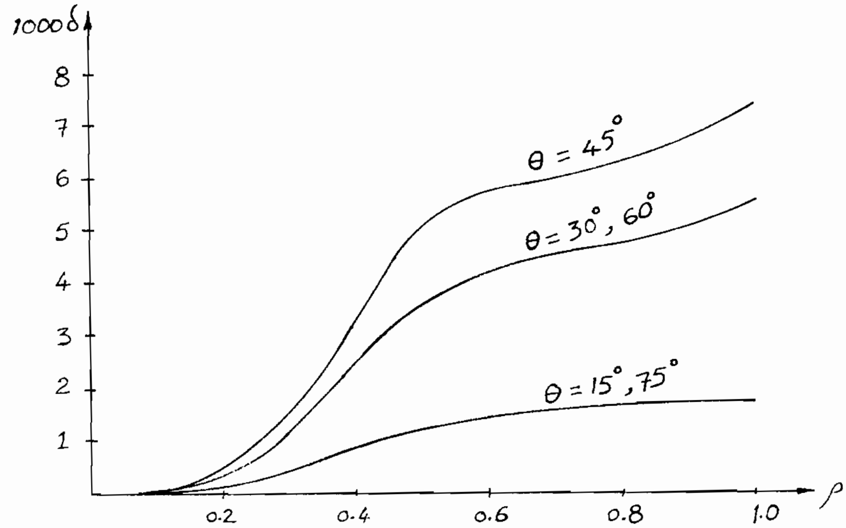

Fig. 3. Deflection profiles along radii of the quadrant

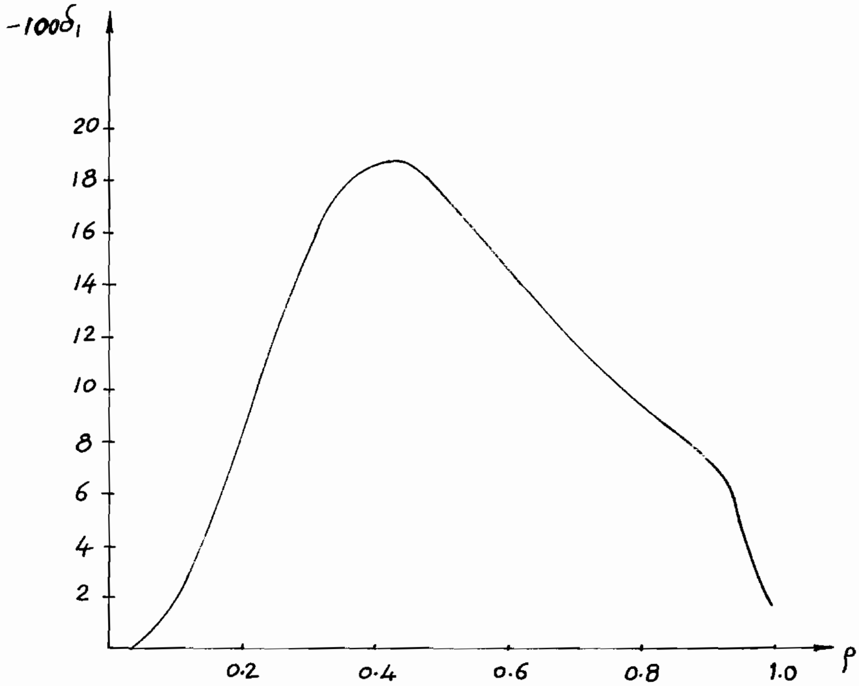

Fig. 4. Variation of the clamping couple along clamped radii of the quadrant

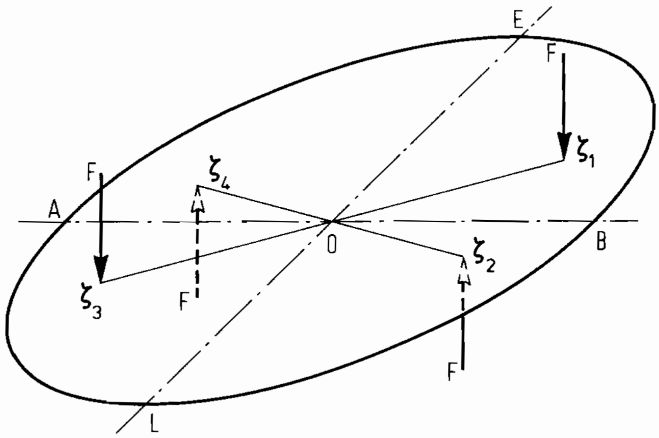

Fig. 5.

and ζ_4, respectively. See Fig. 5. This system of forces keeps the plate in equilibrium. Applying eqn. (45), p. 273 of Bassali (1958) and taking the deflection zero at the centre O we find that

$$2qw(\rho,\theta;s,\alpha) = S_1 - S_2 + S_3 - S_4 + 2(1-\kappa^2)\,Re[J(\zeta,\zeta_1) - J(\zeta,\zeta_2) + J(\zeta,\zeta_3) - J(\zeta,\zeta_4)], \tag{27}$$

where S_1, S_2, S_3, S_4 and $J(\zeta,\zeta_k)$ are defined by (3) and (4). Substituting from (4) in (27) and simplifying we get

$$2qw(\rho,\theta;s,\alpha) = S_1 - S_2 + S_3 - S_4 + 2(1-\kappa^2)\sum_{n=1}^{\infty}\frac{(\rho s)^{2n}\sin 2n\alpha\sin 2n\theta}{n^2(2n-1)}. \tag{28}$$

For $\alpha = \pi/4$ the deflection profile along the radius of symmetry of the quadrant is given by

$$\begin{aligned} qw(\rho,\pi/4;s,\pi/4) = {} & (\rho-s)^2[\kappa\log|\rho-s| - \log(1-\rho s)] \\ & + (\rho+s)^2[\kappa\log(\rho+s) - \log(1+\rho s)] \\ & - (\rho^2+s^2)[\kappa\log(\rho^2+s^2) - \log(1+\rho^2 s^2)] \\ & + (1-\kappa^2)\sum_{n=1,3,5,\ldots}^{\infty}\frac{(\rho s)^{2n}}{n^2(2n-1)}. \end{aligned} \tag{29}$$

It is easily shown that

$$\begin{aligned} \sum_{n=1,3,5,\ldots}^{\infty}\frac{z^{2n}}{n^2(2n-1)} = {} & 2z\tan^{-1}z + z\log\frac{1+z}{1-z} - \log\frac{1+z^2}{1-z^2} \\ & + \tfrac{1}{2}\Psi(-z^2) - \tfrac{1}{2}\Psi(z^2)\,(|z| \leqslant 1), \end{aligned} \tag{30}$$

where

$$\Psi(z) = -\int_0^1 \frac{\log(1-tz)}{t}\,dt = \sum_{n=1}^{\infty}\frac{z^n}{n^2}. \tag{31}$$

Substituting from (30) in (29) we get

$$\begin{aligned} qw(\rho,\pi/4;s,\pi/4) = {} & \kappa(\rho-s)^2\log|\rho-s| + \kappa(\rho+s)^2\log(\rho+s) - \kappa(\rho^2+s^2)\log(\rho^2+s^2) \\ & + (\rho^2+s^2+\kappa^2-1)\log\frac{1+\rho^2 s^2}{1-\rho^2 s^2} - (\kappa^2+1)\,\rho s\log\frac{1+\rho s}{1-\rho s} \\ & - 2(\kappa^2-1)\,\rho s\tan^{-1}(\rho s) + \tfrac{1}{2}(\kappa^2-1)[\Psi(\rho^2 s^2) - \Psi(-\rho^2 s^2)]. \end{aligned} \tag{32}$$

The maximum deflection is given by

$$\frac{q}{\kappa-1}w(1,\pi/4;s,\pi/4) = (s+\kappa)(s-1)\log(1-s)+(s-\kappa)(s+1)\log(1+s)$$
$$+(\kappa-s^2)\log(1+s^2)-2(\kappa+1)s\tan^{-1}s$$
$$+\tfrac{1}{2}(\kappa+1)[\Psi(s^2)-\Psi(-s^2)]. \quad (33)$$

The formula (53a), p. 275 of Bassali (1958) yields the expression

$$M_r+M_\theta = -\frac{(1+\nu)F}{4\pi\kappa}\left[\log\frac{1+\rho^4s^4-2\rho^2s^2\cos 2(\theta+\alpha)}{1+\rho^4s^4-2\rho^2s^2\cos 2(\theta-\alpha)}\right.$$
$$+\kappa\log\frac{\rho^4+s^4-2\rho^2s^2\cos 2(\theta-\alpha)}{\rho^4+s^4-2\rho^2s^2\cos 2(\theta+\alpha)}$$
$$\left.+\frac{8\rho^2s^2(1-s^2)(1-\rho^4s^4)\sin 2\alpha\sin 2\theta}{(1+\rho^4s^4-2\rho^2s^2\cos 2(\theta+\alpha))(1+\rho^4s^4-2\rho^2s^2\cos 2(\theta-\alpha))}\right], \quad (34)$$

from which we deduce that

$$(M_\theta)_{\rho=1} = -\frac{(1+\nu)F}{\pi\kappa}\left[\frac{1}{1-\nu}\log\frac{1+s^4-2s^2\cos 2(\theta+\alpha)}{1+s^4-2s^2\cos 2(\theta-\alpha)}\right.$$
$$\left.+\frac{2s^2(1-s^2)(1-s^4)\sin 2\alpha\sin 2\theta}{(1+s^4-2s^2\cos 2(\theta+\alpha))(1+s^4-2s^2\cos 2(\theta-\alpha))}\right]. \quad (35)$$

For $\alpha = \pi/4$ we have

$$(M_\theta)_{\rho=1} = -\frac{(1+\nu)F}{\pi\kappa}\left[\frac{1}{1-\nu}\log\frac{1+s^4+2s^2\sin 2\theta}{1+s^4-2s^2\sin 2\theta}+\frac{2s^2(1-s^2)(1-s^4)\sin 2\theta}{1+s^8+2s^4\cos 4\theta}\right] \quad (36)$$

$$(M_\theta)_{\rho=1,\theta=\pi/4} = -\frac{2(1+\nu)F}{\pi\kappa}\left[\frac{1}{1-\nu}\log\frac{1+s^2}{1-s^2}+\frac{s^2}{1+s^2}\right]. \quad (37)$$

B. CIRCULAR PLATE BENT BY TWO TWISTING COUPLES AT TWO POINTS ON A DIAMETER EQUIDISTANT FROM THE CENTRE

The solution for a thin circular plate with a free boundary and acted upon by a general system of concentrated couples was given by eqn. (47), p. 274 of Bassali (1958). As a special case of this solution we assume that the circular plate is kept in equilibrium by the two twisting couples defined by

$$G_1 = G,\ \beta_1 = -\pi/2,\ \lambda_1 = -i;\ G_2 = G,\ \beta_2 = \pi/2,\ \lambda_2 = i$$

and operating at the two points $z_1 = ct$, $z_2 = -ct$, respectively. See Fig. 6. It is easily shown that the deflection w taken as zero at the centre O is given by

$$w = \frac{cG}{8\pi\kappa D}\left[\rho\sin\theta\left\{\log\frac{1+\rho^2t^2+2\rho t\cos\theta}{1+\rho^2t^2-2\rho t\cos\theta}-\kappa\log\frac{\rho^2+t^2+2\rho t\cos\theta}{\rho^2+t^2-2\rho t\cos\theta}\right\}\right.$$
$$\left.+\frac{1-\kappa^2}{t}Re\, i\{(1-t\zeta)\log(1-t\zeta)+(1+t\zeta)\log(1+t\zeta)\}+2t(1-\rho^2)Re\frac{i(1-\zeta^2)}{1-t^2\zeta^2}\right].$$

Evaluating the real parts of the quantities involved we get

$$w = \frac{cG}{8\pi\kappa D}\left[\rho\sin\theta\left\{\log\frac{1+\rho^2t^2+2\rho t\cos\theta}{1+\rho^2t^2-2\rho t\cos\theta} - \kappa\log\frac{\rho^2+t^2+2\rho t\cos\theta}{\rho^2+t^2-2\rho t\cos\theta}\right\}\right.$$
$$\left. + \frac{\kappa^2-1}{t}\sum_1^\infty\frac{(\rho t)^{2n}\sin 2n\theta}{n(2n-1)} + \frac{2t\rho^2(1-\rho^2)(1-t^2)\sin 2\theta}{1+\rho^4t^4-2\rho^2t^2\cos 2\theta}\right]. \tag{38}$$

Putting $\pi/2-\theta$ instead of θ in (38) we obtain the solution

$$w = \frac{cG}{8\pi\kappa D}\left[\rho\cos\theta\left\{\log\frac{1+\rho^2t^2+2\rho t\sin\theta}{1+\rho^2t^2-2\rho t\sin\theta} - \kappa\log\frac{\rho^2+t^2+2\rho t\sin\theta}{\rho^2+t^2-2\rho t\sin\theta}\right\}\right.$$
$$\left. + \frac{\kappa^2-1}{t}\sum_1^\infty\frac{(-1)^{n-1}(\rho t)^{2n}\sin 2n\theta}{n(2n-1)} + \frac{2t\rho^2(1-\rho^2)(1-t^2)\sin 2\theta}{1+\rho^4t^4+2\rho^2t^2\cos 2\theta}\right], \tag{39}$$

which corresponds to the case in which the circular plate is acted upon by the two twisting couples

$$G_1 = G,\ \beta_1 = \pi,\ \lambda_1 = -1;\ G_2 = G,\ \beta_2 = 0,\ \lambda_2 = 1$$

acting at the two points $z_1 = ict$, $z_2 = -ict$, respectively. See Fig. 6.

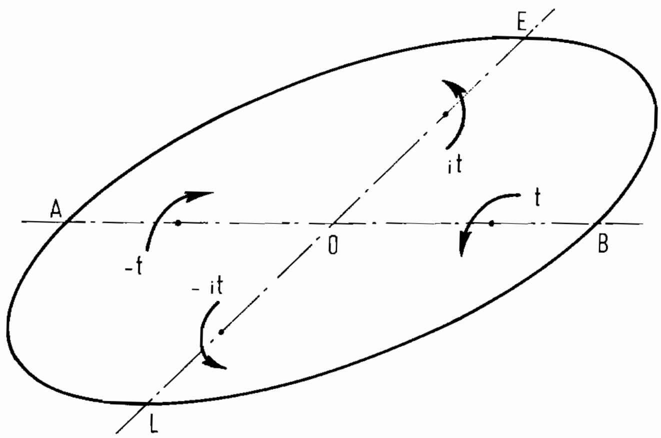

Fig. 6.

C. FINAL SOLUTION (SECOND METHOD)

On the solution (28) we now superpose solutions corresponding to line distributions of twisting couples extending along the two perpendicular diameters AOB and EOL of the circular plate. See Fig. 1. Putting $cFf(t)dt$ instead of G in (38) and $cFg(t)dt$ instead of G in (39) and applying the principle of superposition we find that the solution

$$2qw(\rho,\theta;s,\alpha) =$$
$$S_1 - S_2 + S_3 - S_4 + 2(1-\kappa^2)\sum_1^\infty\frac{(\rho s)^{2n}\sin 2n\alpha\sin 2n\theta}{n^2(2n-1)}$$

$$+2\int_0^1 f(t)\Big[\rho\sin\theta\Big\{\log\frac{1+\rho^2t^2+2\rho t\cos\theta}{1+\rho^2t^2-2\rho t\cos\theta}-\kappa\log\frac{\rho^2+t^2+2\rho t\cos\theta}{\rho^2+t^2-2\rho t\cos\theta}\Big\}$$
$$+\frac{2t\rho^2(1-\rho^2)(1-t^2)\sin 2\theta}{1+\rho^4t^4-2\rho^2t^2\cos 2\theta}+\frac{\kappa^2-1}{t}\sum_1^\infty\frac{(\rho t)^{2n}\sin 2n\theta}{n(2n-1)}\Big]dt$$
$$+2\int_0^1 g(t)\Big[\rho\cos\theta\Big\{\log\frac{1+\rho^2t^2+2\rho t\sin\theta}{1+\rho^2t^2-2\rho t\sin\theta}-\kappa\log\frac{\rho^2+t^2+2\rho t\sin\theta}{\rho^2+t^2-2\rho t\sin\theta}\Big\}$$
$$+\frac{2t\rho^2(1-\rho^2)(1-t^2)\sin 2\theta}{1+\rho^4t^4+2\rho^2t^2\cos 2\theta}+\frac{\kappa^2-1}{t}\sum_1^\infty\frac{(-1)^{n-1}(\rho t)^{2n}\sin 2n\theta}{n(2n-1)}\Big]dt \tag{40}$$

satisfies: (i) the biharmonic equation $\nabla^4 w = 0$ at any point (r,θ) except at (cs,α), (ii) the conditions that the circular edge is free, and (iii) the conditions that $w = 0$ on the radii $\theta = 0$, $\theta = \pi/2$. The two unknown functions $f(t)$ and $g(t)$ in (40) are to be determined from the conditions

$$\left(\frac{1}{\rho}\frac{\partial w}{\partial\theta}\right)_{\theta=0}=0,\ \left(\frac{1}{\rho}\frac{\partial w}{\partial\theta}\right)_{\theta=\pi/2}=0\ (\rho\neq 0) \tag{41}$$

expressing the vanishing of the slope on the two bounding radii OB and OE of the quadrant. See Fig. 1. Substitution from (40) in (41) leads to the two dual integral equations (8) in which

$$U(\rho,t)=\tfrac{1}{2}(1+\kappa^2)\log\frac{1+\rho t}{1-\rho t}-\kappa\log\frac{\rho+t}{|\rho-t|}+\frac{2\rho t(1-\rho^2)(1-t^2)}{(1-\rho^2t^2)^2}, \tag{42}$$

$$V(\rho,t)=(\kappa^2-1)\tan^{-1}(\rho t)+2\rho t\left\{\frac{1}{1+\rho^2t^2}-\frac{\kappa}{\rho^2+t^2}+\frac{(1-\rho^2)(1-t^2)}{(1+\rho^2t^2)^2}\right\}, \tag{43}$$

$$\phi(\rho,s,\alpha)=s\sin\alpha\left\{\log\frac{1+\rho^2s^2+2\rho s\cos\alpha}{1+\rho^2s^2-2\rho s\cos\alpha}-\kappa\log\frac{\rho^2+s^2+2\rho s\cos\alpha}{\rho^2+s^2-2\rho s\cos\alpha}\right\}$$
$$+\frac{2\rho s(1-\rho^2)(1-s^2)\cos\alpha}{1+\rho^4s^4-2\rho^2s^2\cos 2\alpha}+\frac{\kappa^2-1}{\rho}\sum_1^\infty\frac{(\rho s)^{2n}\sin 2n\alpha}{n(2n-1)}. \tag{44}$$

The infinite series appearing in (44) can be put in closed form and we have

$$\sum_1^\infty\frac{(\rho s)^{2n}\sin 2n\alpha}{n(2n-1)}=\tfrac{1}{2}\rho s\sin\alpha\log\frac{1+\rho^2s^2+2\rho s\cos\alpha}{1+\rho^2s^2-2\rho s\cos\alpha}$$
$$+\rho s\cos\alpha\tan^{-1}\frac{2\rho s\sin\alpha}{1-\rho^2s^2}-\tan^{-1}\frac{\rho^2s^2\sin 2\alpha}{1-\rho^2s^2\cos 2\alpha}. \tag{45}$$

For $\alpha = \pi/4$ we have $g(t) = f(t)$ and $f(t)$ satisfies the singular integral equation

$$\int_0^1 f(t)\Big[\tfrac{1}{2}(1+\kappa^2)\log\frac{1+\rho t}{1-\rho t}-\kappa\log\frac{\rho+t}{|\rho-t|}+(\kappa^2-1)\tan^{-1}(\rho t)$$
$$+2\rho t\left\{\frac{1}{1+\rho^2t^2}-\frac{\kappa}{\rho^2+t^2}+\frac{2(1-\rho^2)(1-t^2)(1+\rho^4t^4)}{(1-\rho^4t^4)^2}\right\}\Big]dt$$
$$=\frac{s}{\sqrt{2}}\left\{\log\frac{1+\rho^2s^2+\sqrt{2}\rho s}{1+\rho^2s^2-\sqrt{2}\rho s}-\kappa\log\frac{\rho^2+s^2+\sqrt{2}\rho s}{\rho^2+s^2-\sqrt{2}\rho s}+\frac{2\rho(1-\rho^2)(1-s^2)}{1+\rho^4s^4}\right\}$$
$$+\frac{\kappa^2-1}{\rho}\sum_{n=1,3,5,\dots}\frac{(-1)^{(n-1)/2}(\rho s)^{2n}}{n(2n-1)}. \tag{46}$$

Setting $\alpha = \pi/4$ in (45) and inserting the result in (46) we find that the right side takes the closed form

$$\frac{s}{\sqrt{2}}\left\{\tfrac{1}{2}(\kappa^2+1)\log\frac{1+\rho^2s^2+\sqrt{2}\rho s}{1+\rho^2s^2-\sqrt{2}\rho s}-\kappa\log\frac{\rho^2+s^2+\sqrt{2}\rho s}{\rho^2+s^2-\sqrt{2}\rho s}+(\kappa^2-1)\tan^{-1}\frac{\sqrt{2}\rho s}{1-\rho^2s^2}+\frac{2\rho(1-s^2)(1-\rho^2)}{1+\rho^4s^4}\right\}-\frac{\kappa^2-1}{\rho}\tan^{-1}(\rho^2s^2). \quad (47)$$

The equation of the deflection surface now becomes

$$\begin{aligned}
2qw(\rho,\theta;s,\pi/4) = {}& (\rho^2+s^2-2\rho s\cos\psi)\{\kappa\log(\rho^2+s^2-2\rho s\cos\psi)-\log(1+\rho^2s^2-2\rho s\cos\psi)\} \\
& -(\rho^2+s^2-2\rho s\sin\psi)\{\kappa\log(\rho^2+s^2-2\rho s\sin\psi)-\log(1+\rho^2s^2-2\rho s\sin\psi)\} \\
& +(\rho^2+s^2+2\rho s\cos\psi)\{\kappa\log(\rho^2+s^2+2\rho s\cos\psi)-\log(1+\rho^2s^2+2\rho s\cos\psi)\} \\
& -(\rho^2+s^2+2\rho s\sin\psi)\{\kappa\log(\rho^2+s^2+2\rho s\sin\psi)-\log(1+\rho^2s^2+2\rho s\sin\psi)\} \\
& -2(\kappa^2-1)\sum_{n=1,3,5,\ldots}^{\infty}\frac{(-1)^{(n-1)/2}(\rho s)^{2n}\sin 2n\theta}{n^2(2n-1)} \\
& +2\int_0^1 f(t)\left[\rho\sin\theta\left\{\log\frac{1+\rho^2t^2+2\rho t\cos\theta}{1+\rho^2t^2-2\rho t\cos\theta}-\kappa\log\frac{\rho^2+t^2+2\rho t\cos\theta}{\rho^2+t^2-2\rho t\cos\theta}\right\}\right. \\
& +\rho\cos\theta\left\{\log\frac{1+\rho^2t^2+2\rho t\sin\theta}{1+\rho^2t^2-2\rho t\sin\theta}-\kappa\log\frac{\rho^2+t^2+2\rho t\sin\theta}{\rho^2+t^2-2\rho t\sin\theta}\right\} \\
& \left.+\frac{4t\rho^2(1-\rho^2)(1-t^2)(1+\rho^4t^4)\sin 2\theta}{1+\rho^8t^8-2\rho^4t^4\cos 4\theta}+\frac{2(\kappa^2-1)}{t}\sum_{n=1,3,5,\ldots}^{\infty}\frac{(\rho t)^{2n}\sin 2n\theta}{n(2n-1)}\right]dt,
\end{aligned} \quad (48)$$

where $\psi = \pi/4-\theta$. Substituting from (48) in (19) or applying the formula (53a), p. 275 of Bassali (1958) we get

$$\begin{aligned}
M_r+M_\theta = {}& -\frac{(1+\nu)F}{\pi\kappa}\left[\tfrac{1}{4}\log\frac{1+\rho^4s^4+2\rho^2s^2\sin 2\theta}{1+\rho^4s^4-2\rho^2s^2\sin 2\theta}-\tfrac{1}{4}\kappa\log\frac{\rho^4+s^4+2\rho^2s^2\sin 2\theta}{\rho^4+s^4-2\rho^2s^2\sin 2\theta}\right. \\
& +\frac{2\rho^2s^2(1-s^2)(1-\rho^4s^4)\sin 2\theta}{1+\rho^8s^8+2\rho^4s^4\cos 4\theta}-2\rho^2\sin 2\theta\int_0^1 tf(t)\left\{\frac{1+\rho^4t^4-2(1-t^2)(1-\rho^4t^4)}{1+\rho^8t^8-2\rho^4t^4\cos 4\theta}\right. \\
& \left.\left.-\frac{\kappa(\rho^4+t^4)}{\rho^8+t^8-2\rho^4t^4\cos 4\theta}+\frac{4(1-t^2)(1+\rho^4t^4)(1-\rho^8t^8)}{(1+\rho^8t^8-2\rho^4t^4\cos 4\theta)^2}\right\}dt\right],
\end{aligned} \quad (49)$$

from which we deduce that

$$\begin{aligned}
(M_\theta)_{\rho=1} = {}& -\frac{2(1+\nu)F}{\pi\kappa}\left[\frac{1}{2(1-\nu)}\log\frac{1+s^4+2s^2\sin 2\theta}{1+s^4-2s^2\sin 2\theta}+\frac{s^2(1-s^2)(1-s^4)\sin 2\theta}{1+s^8+2s^4\cos 4\theta}\right. \\
& \left.+\sin 2\theta\int_0^1 tf(t)\left\{\frac{2(1-t^2)(1-t^4)-(1-\kappa)(1+t^4)}{1+t^8-2t^4\cos 4\theta}-\frac{4(1-t^2)(1+t^4)(1-t^8)}{(1+t^8-2t^4\cos 4\theta)^2}\right\}dt\right],
\end{aligned} \quad (50)$$

$$(M_\theta)_{\rho=1,\theta=\pi/4} = -\frac{2(1+\nu)F}{\pi\kappa}\left[\frac{1}{1-\nu}\log\frac{1+s^2}{1-s^2}+\frac{s^2(1-s^2)(1-s^4)}{1+s^8}\right.$$
$$\left.-\int_0^1 tf(t)\left\{\frac{1-\kappa}{1+t^4}+\frac{2(1-t^2)(1-t^4)}{(1+t^4)^2}\right\}dt\right]. \quad (51)$$

D. NUMERICAL RESULTS (SECOND METHOD)

Taking $\nu = 1/3$, $\kappa = -5$, $s = 1/2$ and solving the singular integral equation (46) numerically we obtain Table 5 for the values of $f(t)$.

Table 5. Values of $f(t)$ ($\nu = \frac{1}{3}$, $s = \frac{1}{2}$)

t	$f(t)$	t	$f(t)$	t	$f(t)$	t	$f(t)$	t	$f(t)$
0·02	−0·0760	0·22	0·3097	0·42	0·3228	0·62	0·3941	0·82	0·0942
0·06	0·0827	0·26	0·1919	0·46	0·4992	0·66	0·1813	0·86	0·2604
0·10	−0·0672	0·30	0·4348	0·50	0·2938	0·70	0·3416	0·90	0·0539
0·14	0·1727	0·34	0·2920	0·54	0·4525	0·74	0·1337	0·94	0·2018
0·18	0·0538	0·38	0·5502	0·58	0·2379	0·78	0·2986	0·98	−0·0259

Table 6. Values of $10^6\delta$ ($\nu = \frac{1}{3}$, $s = \frac{1}{2}$)
Second Method

	θ		
ρ	15°, 75°	30°, 60°	45°
0·04	−1	−1	−1
0·08	−3	−4	−3
0·12	1	18	28
0·16	19	87	123
0·20	63	230	315
0·24	137	464	626
0·28	245	792	1064
0·32	382	1208	1626
0·36	537	1688	2293
0·40	697	2198	3033
0·44	849	2698	3798
0·48	983	3122	4508
0·52	1094	3458	4985
0·56	1182	3700	5213
0·60	1249	3869	5346
0·64	1301	3990	5437
0·68	1344	4085	5515
0·72	1379	4170	5595
0·76	1410	4254	5689
0·80	1439	4345	5804
0·84	1463	4471	5944
0·88	1482	4562	6111
0·92	1492	4691	6307
0·96	1491	4834	6531
1·00	1474	4990	6784

Table 7. Values of $10^6\delta_2$ (Second Method)
($\nu = \frac{1}{3}$, $s = \frac{1}{2}$)

θ	15°, 75°	30°, 60°	45°
$10^6\delta_2$	−29278	21170	45247

Numerical evaluation of the integrals appearing in (48) and (50) leads to Tables 6 and 7 for the dimensionless quantities δ and δ_2 defined by (24).

The maximum deflection at K obtained by the second method is

$$w_{\max} = 0{\cdot}006784\ c^2F/D \tag{52}$$

and the maximum bending moment M_θ along the circular edge is

$$(M_\theta)_{\rho = 1,\, \theta = \pi/4} = 0{\cdot}045247\ F. \tag{53}$$

4. COMPARISON OF TWO METHODS

Comparing Table 6 with Table 2, and Table 7 with Table 4 shows good agreement between the results obtained by the two different methods at points sufficiently far from the corner points of the quadrant. It is worthy of mention here that the case of a thin isotropic uniformly loaded plate having the shape of a quadrant of a circle with clamped radial edges and free circular edge was discussed by Bassali & Halim (1963) and the maximum deflection in this case is given by eqn. (52i), p. 94 of the paper:

$$\begin{aligned} w_{\max} &= 0{\cdot}010418\ p_0c^4/D \\ &= 0{\cdot}013265\ c^2F/D, \end{aligned} \tag{54}$$

where $F = \pi p_0 c^2/4$ is the total load on the plate. When the same plate is simply supported along the radial edges instead of being clamped, the maximum deflection was given by eqn. (55), p. 95 of the same paper:

$$\begin{aligned} w_{\max} &= 0{\cdot}066045\ p_0c^4/D \\ &= 0{\cdot}084091\ c^2F/D. \end{aligned} \tag{55}$$

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خلاصـــة

يعالج هذا البحث كيفية ايجاد الازاحات العمودية والقيم الحدية لعزوم الانثناء التى تحدث فى لوحة رقيقة مرنة على شكل ربع دائرة ، عندما تكون هذه اللوحة محملة تحميلا عموديا بحمل مركز عند نقطة منها وحافتها الدائرية حرة ونصفا قطريها مثبتين تثبيتا تاما .

* العنوان الحالى : كلية الهندسة بجامعة عين شمس بالعباسية ، القاهرة ، ج.م.ع.