J. Univ. Kuwait (Sci.) **8**, 1981

A contribution to the methods of upward continuation of the gravitational field

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ABSTRACT

This paper deals with the problem of upward continuation of gravity anomalies due to two-dimensional body forms.

Poisson's integral was solved within a given interval and an equation for the upward continuation of the gravitational anomalous field was given in the form

$$V_z(0,-h) = \sum_{-n}^{+n} A_i\, V_z(x_i, 0) + R.$$

Values for the coefficient A_i for recalculation at different heights are given.

The given formula was checked on theoretical models in the form of a sphere and a horizontal cylinder. The results of applying the present method on gravity profile over a sphere shows that the error is comparable with that of three-dimensional methods. In all cases it is high and ranges between 30% and 90%. In the case of a horizontal cylinder it gives fairly good results. The error for some points is less than 1% and the mean error for all levels is almost always less than 10%. Besides, it was found that in all cases the error at different points along the profile was almost constant, which excludes the possibility of the formation of 'fictitious' anomalies which are usually associated with other methods.

INTRODUCTION

The transformation of the Bouguer anomaly field measured at the earth's surface into the upper and lower hemispheres is one of the important tools for gravity data processing and interpretation. The downward continuation methods are widely used for separating the local components of the gravity field, thereby studying structures near the earth's surface. On the other hand, the upward continuation technique is used for separating the regional component of the anomalous field, giving rise to attenuation of local, near-surface anomalies. In most cases when the basement with its complex lithologic composition is very near to the surface, the upward continuation is a suitable technique for excluding its complex reflection on the gravity field. Anomalies due to structural elements which may be present above the basement are usually of higher order than those due to the heterogeneities in its composition. Thus upward

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continuation will help in distinguishing between these two components and in delineating tectonic elements on the basement.

THEORY

Most of the methods of the upward continuation are based on Poisson's integral in cylindrical coordinates. They deal with three-dimensional problems (Peters 1949; Henderson & Zietz 1949; Baranov 1953). Some other methods which deal with two- and three-dimensional problems are based on Fourier series (Grant & West 1965; Morelli 1968). Yet the use of these methods is preferable when electronic computers are available.

The present method is based on solving Poisson's integral too. It is designed for anomalies due to two-dimensional bodies. Considering a coordinate system in which the h axis is positive downward to the anomalous body, then the potential at a point P with coordinate $(0, -h)$ is given in the form (Malovitchko 1960; Riad & Othman 1978).

$$V(0,-h) = -2f\mu \int ln \frac{dx}{(x^2+h^2)^{\frac{1}{2}}}. \tag{1}$$

Equation (1) is the logarithmic potential and is used for simplifying the calculations by reducing the number of variables (Grant & West 1965).

Differentiating with respect to z we get

$$V_z(0,-h) = 2f\mu \int \frac{h}{x^2+h^2} dx. \tag{2}$$

Knowing that the force of attraction of an infinite layer is given by $V_z(x,0) = 2\pi\mu f$, then Eqn (2) will take the form

$$V_z(0,-h) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{V_z(x,0)\, h}{x^2+h^2} dx.$$

This integral will be divided into three parts as follows:

$$V_z(0,-h) = \frac{1}{\pi} \int_{-\infty}^{-\Delta n} V_z(x,0) \frac{h}{x^2+h^2} dx + \frac{1}{\pi} \int_{-\Delta n}^{+\Delta n} V_z(x,0) \frac{h}{x^2+h^2} dx +$$

$$+ \frac{1}{\pi} \int_{\Delta n}^{\infty} V_z(x,0) \frac{h}{x^2+h^2} dx \tag{3}$$

$$= I_1 + I_2 + I_3$$

where Δ is the step of integration, that is the interval within which the function $V_z(x,0)$ is considered a straight line and is equal to its value at the middle of the interval. Taking $n\Delta$ large enough, then the parts of the function V_z outside the limits $-n\Delta$, $+n\Delta$ (i.e. the integrals I_1 & I_3) can be neglected.

For solving integral I_2 we may introduce the coefficient

$$A_i = \frac{1}{\pi} \int\limits_{\Delta(i-\frac{1}{2})}^{\Delta(i+\frac{1}{2})} \frac{h}{x^2+h^2}\,dx$$

$$= \frac{1}{\pi}\left[\operatorname{arctg}\frac{\Delta(i+1/2)}{h} - \operatorname{arctg}\frac{\Delta(i-1/2)}{h}\right] \tag{4}$$

where $i=0, \pm 1, \pm 2, \ldots, \pm n$. Then I_2 will take the form

$$I_2 = \sum_{-n}^{+n} A_i\, V_z(x_i,0)$$

and the general formula for the upward continuation will take the form

$$V_z(0,h) = \sum_{-n}^{+n} A_i\, V_z(x_i,0) + R \tag{5}$$

where $R = I_1 + I_3$

The coefficient A_i was calculated for different values of h/Δ and these are given in Table 1.

In order to reduce the error due to neglecting R, the coefficient A_i was integrated from $-\infty$ to $+\infty$ as follows:

$$A_i = \frac{1}{\pi} \int\limits_{-\infty}^{+\infty} \frac{h}{x^2+h^2}\,dx$$

$$= \frac{1}{\pi}\left[\operatorname{arctg}\frac{x}{h}\right]_{-\infty}^{+\infty} = 1. \tag{6}$$

Thus the last coefficients A_n in the table were calculated so as to satisfy the requirement

$$\sum_{-n}^{+n} A_i = 1.$$

This was done using the following relation

$$2A_{\pm n} = 1 - [A_0 + 2(A_1 + A_2 + \ldots + A_{n-1})].$$

In this way the error may be reduced.

Table 1. Coefficient A_i for upward continuation of gravity field

	$h/\Delta=0{\cdot}5$	$h/\Delta=1{\cdot}0$	$h/\Delta=2{\cdot}0$	$h/\Delta=4{\cdot}0$	$h/\Delta=10{\cdot}0$
*A*0	0·49994	0·29513	0·15594	0·07916	0·03180
*A*1	0·14757	0·16523	0·12684	0·07461	0·03249
*A*2	0·03958	0·06604	0·08038	0·06360	0·03058
*A*3	0·01766	0·03253	0·04952	0·05100	0·02918
*A*4	0·00994	0·01898	0·03212	0·03989	0·02743
*A*5	0·00636	0·01235	0·02211	0·03114	0·02546
*A*6	0·00442	0·00866	0·01600	0·02455	0·02340
*A*7	0·00325	0·00640	0·01206	0·01964	0·02136
*A*8	0·02124	0·00492	0·00939	0·01595	0·01941
*A*9		0·03733	0·07361	0·14005	0·27578

RESULTS AND DISCUSSION

The suggested formula was checked on theoretical models in the form of a sphere and a horizontal cylinder. Other known upward continuation methods were also used (Peters 1949; Henderson & Zietz 1949; Baranov 1953). These methods are three-dimensional. When employing our method different values for h/Δ were used so as to find the optimum condition for the ratio h/Δ. The upward continuation was calculated for three levels: 0·2, 0·4, and 0·8 km. Theoretical values for the sphere and cylinder were calculated using their usual formulae, taking the centre of mass at depth $Z=0{\cdot}6$ km, the radius $R=0{\cdot}4$ km and the density $=1{\cdot}0$ g/cm^3 (Sazhina & Grushinsky 1971). The theoretical effect of these bodies is given in Table 2. These values were used for

Table 2. Theoretical values of V_z due to sphere and horizontal cylinder; the depth of their centres is $Z=0{\cdot}6$ km

Distance (km)	V_z (sphere) (mg/l)	V_z (cylinder) (mg/l)
0·0	4·96	11·17
±0·2	4·24	10·05
±0·4	2·86	7·73
±0·6	1·75	5·58
±0·8	1·07	4·02
±1·0	0·68	2·96
±1·2	0·44	2·23
±1·4	0·30	1·73
±1·6	0·21	1·38
±1·8	0·16	1·12
±2·0	0·12	0·92
±2·2	0·09	0·77
±2·4	0·07	0·66
±2·6	0·06	0·56
±2·8	0·05	0·49
±3·0	0·04	0·43

calculating the effect at the three higher levels. The results of calculations using different methods for each level were correlated with the corresponding theoretical values. The differences between the computed and the theoretical values at each point and consequently error percent were determined (Table 3). Results of these calculations show the following:

(1) In the case of the sphere

The accuracy of all methods is low. The error of all methods varies widely from zero at some points to more than 150% at other points. The difference in the error at different points may give rise to the appearance of 'fictitious' anomalies (Baranov's method, Fig. 1). Peters' method gives the best results, but the mean error is still high (Table 3). It increases considerably from 49% to 72% when increasing the level of transformation.

Results of the present method are comparable with the results of the other

Table 3. Error resulting due to the different methods of upward continuation

Method	Level of transfor-mation	Sphere			Horizontal cylinder		
		min.	max.	mean	min.	max.	mean
Peters	0·2	2·7	162·0	49·0	2·0	43·0	17·0
(1949)	0·4	0·6	164·0	63·0	5·0	26·0	15·0
	0·8	4·5	170·0	72·0	2·5	36·0	22·0
Henderson	0·2	6·2	112·0	33·0	2·2	16·0	7·2
& Zietz	0·4	35·0	156·0	65·0	1·4	11·8	5·4
(1949)	0·8	62·0	117·0	90·0	1·1	12·5	5·7
Baranov	0·2	19·2	195·4	60·0	2·8	193·6	54·0
(1953)	0·4	2·5	157·6	61·0	0·8	36·8	18·0
	0·8	9·4	166·0	74·0	0·5	36·7	22·0
	$h/\Delta=2$						
Riad &	0·2	0·0	56·0	33·0	0·2	12·0	6·0
Othman	0·4	46·3	230·0	94·0	0·0	37·0	10·5
(1978)	0·8	90·0	141·0	106·0	0·2	2·1	1·2
	$h/\Delta=1$						
	0·2	16·6	141·0	55·0	4·4	29·9	10·5
	0·4	33·0	100·0	56·0	1·0	41·2	9·6
	0·8	62·0	129·0	91·0	0·0	14·1	7·0
	$h/\Delta=0{\cdot}5$						
	0·2	21·8	55·0	33·0	0·0	3·7	1·0
	0·4	36·0	88·7	60·0	0·5	6·6	2·4
	$h/\Delta=4$						
	0·8	94·5	270·0	148·4	2·0	42·2	14·0

three-dimensional methods. The error is also high and increases with height. More stable results are achieved when the value of $h/\Delta=0{\cdot}5$–$1{\cdot}0$. The resulting curves are smooth without considerable 'fictitious' anomalies (Fig. 2). Thus our method can equally be recommended for the upward continuation of anomalies due to three-dimensional bodies.

(2) In the case of the horizontal cylinder

Figure 3 shows the results of calculations using our method. The resulting curves are smooth with small deviations from the theoretical curves. Higher accuracy is achieved when $h/\Delta=0{\cdot}5$ (Table 3). It also shows high stability for calculations at different levels. At all points the error was less than 10%.

In the case of the horizontal cylinder, the results of the other methods show higher accuracy than in the case of the sphere (Fig. 4). This result cannot be readily explained. Of these methods, Henderson and Zietz's is the best. The maximum error in this case is always below 20%, and the mean error below 10%. It also shows a constant accuracy with respect to different levels of transformation.

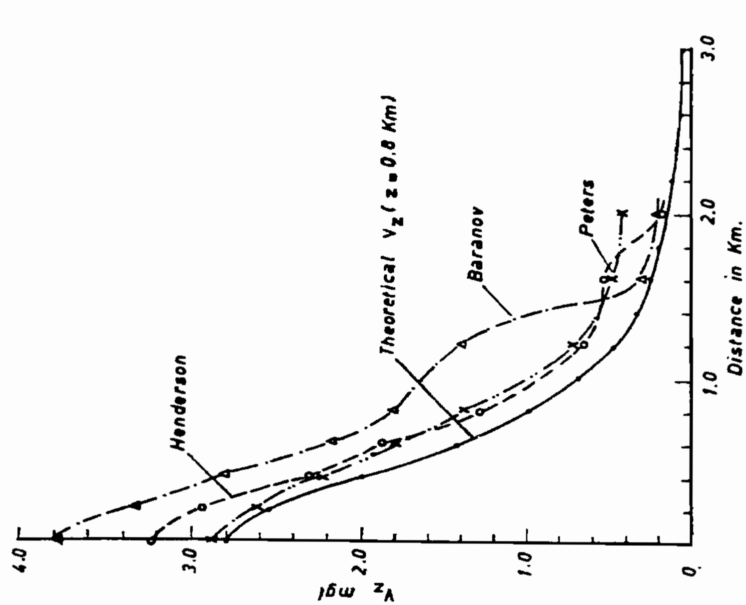

Fig. 1. Upward continuation of anomalies due to sphere using various methods.

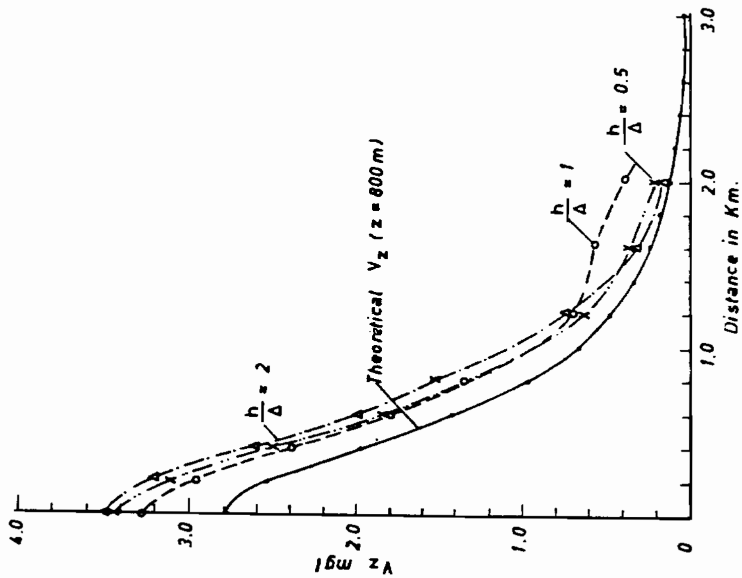

Fig. 2. Upward continuation of anomalies due to sphere using Riad & Othman's (1978) method.

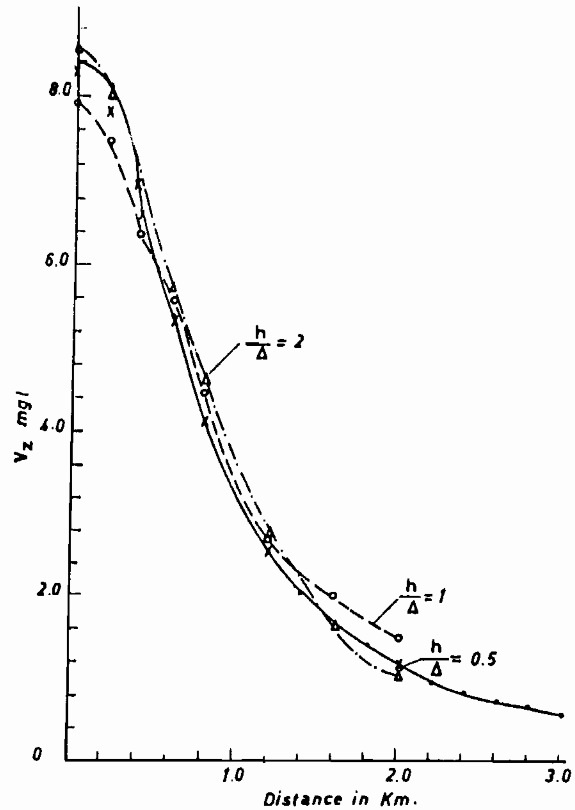

Fig. 3. Upward continuation of anomalies due to horizontal cylinder using Riad & Othman's (1978) method.

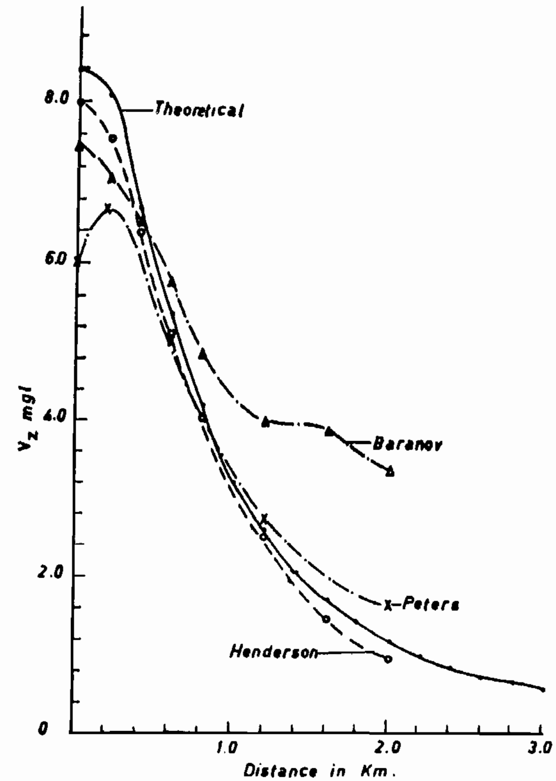

Fig. 4. Upward continuation of anomalies due to horizontal cylinder using various methods.

CONCLUSION

From the present study we see that:

(1) When applied to anomalies due to three-dimensional bodies, the suggested method gives results comparable with those of the known methods designed for three-dimensional problems.

(2) In the case of two-dimensional bodies, it gives fairly accurate results. The error at all points was below 10%.

(3) The suggested method gives smooth and stable results for calculations at different levels.

(4) It was found that better results are achieved when $h \leq \Delta$.

(5) Although the accuracy of our method is similar to that of Henderson and Zietz, our method has the advantage that it is easier to compute and its accuracy is controlled by changing the value of Δ according to the level of transfomation h.

REFERENCES

Baranov, V. 1953. Calcul du gradient vertical du champ de gravité ou du champ magnetique mesuré a la surface du sol. Geophys. Prosp. **1:** 508–34.

Grant, F.S. & West, G.F. 1965. Interpretation theory in applied geophysics. McGraw-Hill.

Henderson, R.G. & Zeitz, I. 1949. The upward continuation of anomalies in total magnetic intensity fields. Geophysics **14:** 517–34.

Malovichko, A.K. 1960. Osnovnoj kurs gravirasvedki Chast' pervaja. (Basic course in gravity prospecting. Part one). Perm' Publishing House, Moscow.

Morelli, C. 1968. Gravimetria. Del Bianco Editore, Udine.

Peters, L.J. 1949. The direct approach to magnetic interpretation and its practical application. Geophysics **14:** 250–320.

Riad, S. & Othman, A.A. 1978. A contribution to the methods of calculating the vertical gradient of the gravity field. Bollettino Di Geofisica Teoritica et Applicata **20(77):** 52–60.

Sazhina, N. & Grushinsky, N. 1971. Gravity prospecting. Mir, Moscow.

(Received 4 May 1980)

حول طرق الاستمرار الرأسي لحقل تثاقلي

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خلاصة

يتعلق هذا البحث بحالة الاستمرار الرأسي للقيم الشاذة في القياسات التثاقلية الناتجة من أجسام ذات بعدين .

وقد حصلنا على صيغة لمعادلة الاستمرار الرأسي للقيم الشاذة في مجال التثاقل ، بعد حل تفاضل بواسون في نطاق محسوب . وهذه الصيغة هي :

$$V_z(0,-h) = \sum_{-n}^{+n} A_i V_z(x_i,0) + R$$

وأعطيت قيم المعامل A_i لكي يمكن تكرار الحسابات عند المناسيب المختلفة ، وقد تم التأكد من صحة الصيغة السابقة بتطبيقها نظريا على نموذج للكرة وآخر لاسطوانة في وضع أفقي . ومن هذا التطبيق ، اتضح لنا أن الخطأ في القيم التثاقلية لقطاع على الكرة يقارب الخطأ الناتج باستعمال طرق الابعاد الثلاثة للاجسام ، وكان الخطأ كبيرا بصفة عامة ، اذ تراوحت قيمته بين ٣٠٪ و ٩٠٪ . اما في حالة الاسطوانة الافقية فقد حصلنا على نتائج أفضل حيث كان الخطأ حوالي ١٠٪ لجميع المستويات ، وقد تدنى لبعض النقط الى ١٪ .

ويمكن القول بصفة عامة أن الخطأ ثابت في جميع الحالات عند النقط المختلفة على طول القطاع ، مما يستبعد احتمال وجود قيم تخيلية شاذة كالتي تنتج عند استخدام طرق أخرى .

* العنوان الحالي : قسم الجيولوجيا بكلية العلوم بجامعة أسيوط ، أسيوط ، ج.م.ع.