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Optimisation of steel transmission poles

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ABSTRACT

In this paper, optimisation techniques are applied to the design of steel transmission poles. A structure of minimum weight is the eventual goal.

INTRODUCTION

Many structural designers avoid computers in their everyday design problems either because of inexperience in electronic computation or the anticipated higher initial cost of developing a reusable design program. In general, designers select some of the initial variables and the final design is then found from a limited number of satisfactory possibilities. The design is not looked upon from a standpoint of coming close to some optimum, say minimum weight, although designers try to approach this goal as best they can.

A review of the optimisation techniques will not be undertaken in this work since many references accomplish this end. It is the purpose of this paper to show that employing optimisation techniques might not be as difficult as first imagined. Even a limited optimisation approach can produce significant savings. The goal in this work is a structure of minimum weight.

The use of tapered, hollow, steel poles in lieu of truss structures to carry transmission lines is a fairly recent phenomenon. Because there were no guidelines to follow in their design, power and fabricating companies initially were somewhat conservative. As larger poles were constructed and designers searched for competitive products, concern grew about the safety of the final product. Recognising the need for investigation into this area, the American Society of Civil Engineers appointed a task committee to make recommendations for the design of steel transmission poles.

For the transmission pole, the usual loads considered on the pole are due to various combinations of wind, ice and cable tension. These loads and overload factors are usually predetermined for the designer in a fabricating company by the power company. In addition to the stresses due to the loading, those due to the weight of the pole and the secondary moment effects are usually considered. A deflection constraint may also be imposed on the pole. Other factors to be considered are local and overall buckling, shear and torsion.

Some of the difficulties involved in obtaining an optimum design are the effects of the geometry-dependent wind loading, the many possible loading combinations, the many stress restrictions and the fabrication and material availability constraints. Because of these and possible additional complications, optimisation is usually not considered for these designs.

THE MINIMUM WEIGHT DESIGN OF STEEL TRANSMISSION POLE

Considering the pole shown in Fig. 1, and ignoring the weight of the arms because they constitute a constant term in the objective function, the weight can be expressed as

$$W = \pi\rho \int_0^L [D(z) - t(z)]\,t(z)\,dz \simeq \rho\pi \sum_{k=1}^{n} (D_k - t_k)\,t_k\,\Delta z \tag{1}$$

where ρ is the weight of steel per unit volume,

$$D(z) = D_o + (z/L)\,(D_b - D_o) \tag{2}$$

D_o and D_b are top and bottom diameters respectively, n equals the number of finite pieces taken along the pole, and z is as shown in Fig. 1.

The total stress at any section due to bending moments and axial force should be less than the yield strength of the material f_y, the constraint equation is then expressed

$$f_a + f_b \leq f_y \tag{3}$$

where at any section k, the stress due to axial forces

$$f_a = \sum_{i=1}^{j} \frac{p_{vi}}{A_k} + \frac{W_k}{A_k} \tag{4}$$

where P_{vi} is the vertical load acting at arm i, A_k is area of section k and is equal to $\pi(D_k - t_k)\,t_k$, W_k is the total weight of the part of the pole above section k, and j is the number of arms at or above the section.

The total bending stress on the section is f_b. This is due to the transverse and longitudinal loads, plus the secondary moment effect of the vertical forces including the weight of the pole above the section.

Let M_{Lk} be the total moment about the longitudinal axis at section k,

$$M_{Lk} = W_k \delta_{TT} + \sum_{i=1}^{j} P_{vi}\delta_{iT} + \sum_{i=1}^{j} P_{Ti}(z_k - z_i) \tag{5}$$

where δ_{TT} is the transverse deflection of the centroid of the weight above the section; δ_{iT} is the transverse displacement of vertical load i from section k (it is equal to the transverse deflection of point i minus the transverse deflection of point k); P_{Ti} is the transverse load acting at arm i; z_k is the location of section k from the top of the pole and z_i is the location of arm i.

Interchanging subscripts T and L in eqn (5), the total moment about the transverse axis at section k, M_{Tk}, is obtained.

Since there are forces in the transverse and longitudinal directions, the location of maximum axial stress θ_k has to be solved for at each section k where θ_k is the direction of maximum stress measured from the transverse axis as shown in Fig. 1.

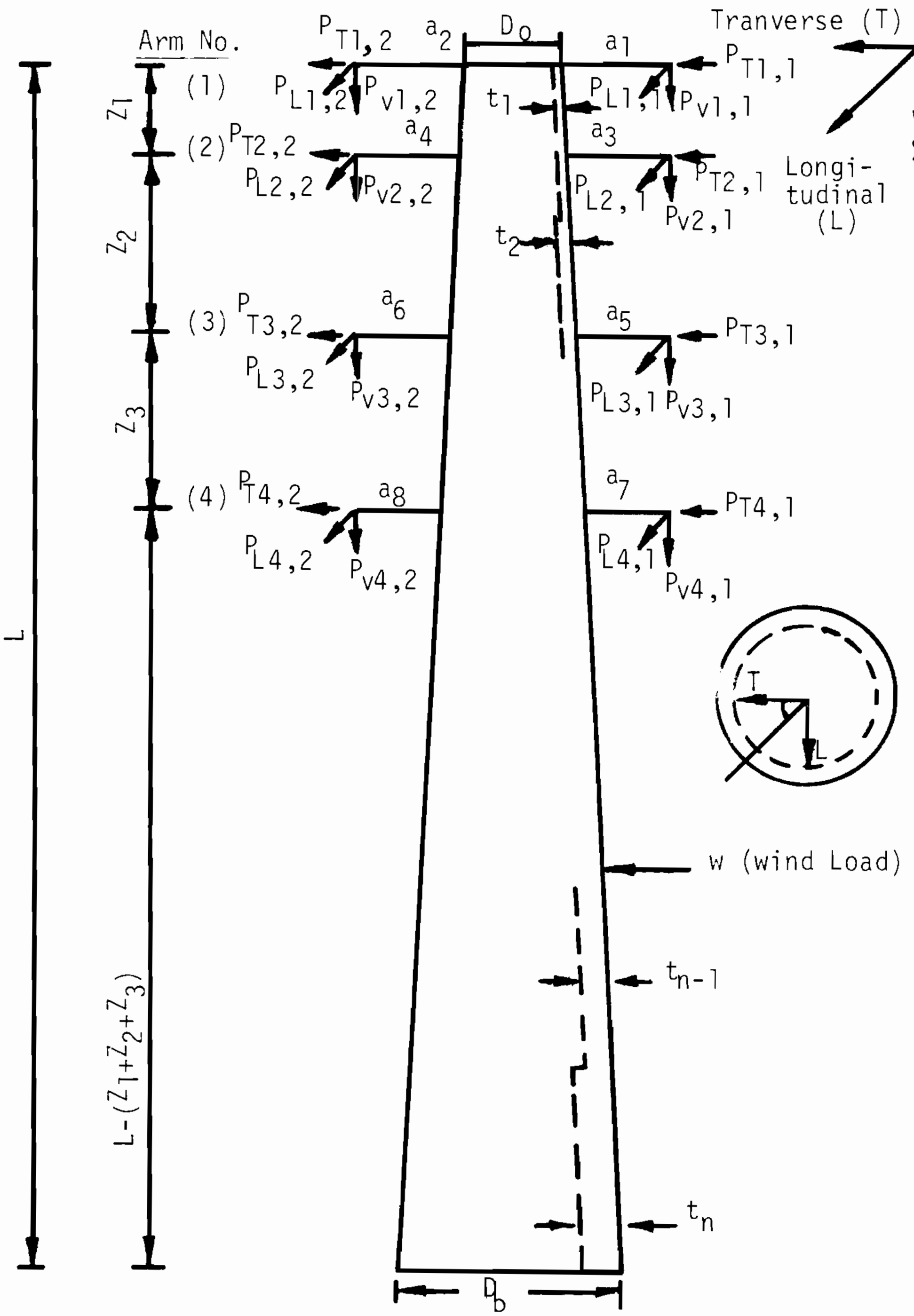

Fig. 1. Steel transmission pole.

$$\theta_k = \tan^{-1}(M_{LK}/M_{TK}) \tag{6}$$

Since wind loads on the pole are liable to act in any direction, the most critical direction is the direction of maximum stress. Superimposing these effects gives the total bending stress as

$$f_b = \frac{\left\{M_{Tk}\cos\theta_k + M_{Lk}\sin\theta_k + \int_0^z wz[D(z)]\,dz\right\}D_k/2}{I_k} \tag{7}$$

where I_k is the moment of inertia at section k. For a thin tube

$$I_k = \frac{\pi D_k^3 t_k}{8} \tag{8}$$

in which t_k is the thickness of the cross section.

Now, the local buckling constraint is taken to be

$$\frac{D_k}{t_k} \leq \frac{4800}{f_y} \tag{9}$$

This is eqn 2.10 in ASCE (1972). This, however, is not the form of the limit used by most fabricating companies. The limit that is most commonly used is

either $$\frac{D_k}{t_k} \leq 100 \text{ or } 90 \tag{10}$$

The applicable constraint limiting the maximum shearing stress due to transverse and longitudinal forces and the twisting moments is given by the equation

$$f_{st} \leq 0{\cdot}6f_y \tag{11}$$

from $f_{\text{shear}} = (VQ/It)$ and $f_{\text{torsion}} = [(TD/2)/2I]$, one obtains

$$f_{sTk} = \frac{V_k Q_k}{2I_k t_k} + \frac{T_k D_{k/2}}{2I_k} \leq 0{\cdot}6f_y \tag{12}$$

where f_{sTk} is the maximum shearing stress at a section, and

$$V_k = \sum_{i=1}^{j}(P_{Ti}\cos\phi_k + P_{Li}\sin\phi_k) + \int_0^z wD(z)\,dz \tag{13}$$

and the location of maximum shearing stress at section k,

$$\phi_k = \tan^{-1}\left[\sum_{i=1}^{j} P_{Li}\Big/\sum_{i=1}^{j} P_{Ti}\right] \tag{14}$$

where ϕ_k is measured from the transverse axis. T_k is the total twist on the section, Q_k is the first moment of the area and equal to $t_k D_k^2/2$, I_k and t_k are as defined before.

After some manipulation, eqn (12) becomes

$$f_{sTk} = \frac{2V_k}{D_k t_k} + \frac{2T_k}{D_k^2 t_k} \leq 0{\cdot}6f_y \tag{15}$$

Limiting the transverse component of deflection serves the purpose of keeping the right of way to a minimum. Limiting the longitudinal component of deflection limits the amount of sag and the clearance between the wires and the ground.

The interim report (ASCE 1972) does not limit the deflection to a certain value. As a rule of thumb, fabricating companies place a limit of 10% of the total length on the tip deflection. This limit serves as a safeguard against wind-induced vibrations and lateral torsional buckling. Upon a rare occasion, the customer specifies a deflection limit as low as 5 or 6% of the total length.

When a deflection limit is specified, it is meant then that the maximum total deflection vector should be within the specified limit. It is thought that a more logical way might be to limit both transverse and longitudinal components of the deflection and at the same time limit the total deflection vector. However, since the electric companies only specify the maximum allowable total deflection vector, this is the only limit used here. It will be designated by the symbol δ_{max}.

After the pole is designed, a check for the buckling load is made using Newmark's method of numerical integration. If the total vertical forces are less than the buckling load, the design is accepted, if not a design is then sought that satisfies the buckling criteria.

$$\Sigma P_v \leq \text{Euler buckling load} \tag{16}$$

Lateral–torsional buckling is not usually a problem with closed circular tubes because of their superior torsional stiffness (ASCE 1972).

Beside the above behavioural constraints, many additional constraints must be considered in the general formulation of this problem. These are the side constraints. They limit the top and bottom diameters within a certain range in order to make fabrication possible. They also limit the thickness to a certain minimum, as required by NESC (1948), to serve as a safeguard against manufacturing imperfections and corrosion.

In mathematical notations these side constraints can be expressed as

$$L_{D_o} \leq D_o \leq U_{D_o}$$

$$L_{D_b} \leq D_b \leq U_{D_b}$$

$$L_{t_k} \leq t_k$$

where L_{D_o}, L_{D_b}, U_{D_o} and U_{D_b} are the lower and upper limits on D_o and D_b, and L_t is the lower limit on the thickness t_k.

The two approaches to be taken in the solution of the problem are:

(1) The continuous thickness solution where the exact thickness, as required by the above constraint, is used. In mathematical notation

$$t_i \geq 0, i = l, \ldots, n.$$

(2) The discrete thickness solution, where thicknesses are not used unless they are available. This is a more practical problem since sheets economically only come in specified thicknesses. A minimum thickness of 3/16 in. or 1/4 in. is required by NESC (1948), depending on the location of the pole. A typical pole is made of sheets having thicknesses of 3/16 in., 1/4 in., 3/8 in., 7/16 in., etc. The problem is then finding the cut-off point of each segment.

METHOD OF SOLUTION

In general, the optimum solution for a structural engineering problem is controlled by one or more constraints. For this reason, the search for the optimum design usually can be confined to the constraint hypersurfaces. This can be accomplished by considering all the requirements on stresses and local buckling as in some way limiting the minimum acceptable thickness at any cross-section of the pole after first selecting the top and bottom diameters. Since the stresses due to the secondary moment effect of the vertical forces are dependent on the deflection of the pole, and the deflection of the pole is in turn dependent on these stresses plus the horizontal forces, an iterative scheme has to be employed to obtain the thickness of the final design. The total weight of the pole is then calculated. This weight is then the minimum weight design for the initial top and bottom diameters selected.

The pattern search technique is then employed to systematically look for the designs which further reduce the weight of the pole. This is the hybrid design method. This method has the same results as the method of gradient projection; however, this method has the advantage that the required thicknesses are calculated on the basis of equality constraints which causes the move to the constraints' hypersurfaces to be accomplished automatically.

If the deflection constraint becomes active, the TCSTP (ASCE 1972) recommends that the pole is to be designed for a deflection limit rather than for strength, which means that a reduction of the yield strength of the pole can be used since the local buckling criterion then becomes more lenient, i.e. larger maximum diameter to thickness ratio is allowed. When a direct limit on the maximum diameter to thickness ratio is specified by the customer, the local buckling constraint is then independent of f_y. In that case, the pole is optimised with the yield strength as a design parameter. Then, once f_y is selected, the maximum stress for the optimised pole is calculated and recommended for use.

It is worth mentioning here that the Euler buckling load constraint was never found to govern. Usually, the axial loads on the pole constitute about 20% of the Euler buckling load. On the other hand, the deflection constraint can be easily controlled, especially when a maximum allowable top deflection vector of less than 6% of the length is specified.

Using the unit load method, the deflection can be expressed as

$$\delta_{T\text{top}} = \sum_{k=1}^{n} \frac{M_{Lk} + \alpha_T\{w[D_o z_k^2/2 + (D_b - D_o) z_k^3/3L]\} z_k \Delta z_k}{E(I_k + I_{k-1})/2} \tag{17}$$

$$\delta_{L\text{top}} = \sum_{k=1}^{n} \frac{M_{Tk} + \alpha_L\{w[D_o z_k^2/2 + (D_b - D_o) z_k^3/3L]\} z_k \Delta z_k}{E(I_k + I_{k-1})/2} \tag{18}$$

$$\delta = \sqrt{\delta_{L\text{top}}^2 + \delta_{T\text{top}}^2} \leq \delta_{\max} \tag{19}$$

where δ is the total deflection, $\delta_{T\text{top}}$ and $\delta_{L\text{top}}$ are the deflection in the transverse and longitudinal directions respectively, z_k is the length of sheet k, α_T and α_L are the percentage of the full wind pressure acting in the transverse and longitudinal directions respectively. These are usually specified by the customer. If α_T and α_L are not specified, then full wind pressure is assumed to act in the direction of maximum horizontal forces.

After each design cycle, the deflection's components and their total vector of the pole are known. If these components are less than the specified limit, then the design is accepted.

Once the deflection constraint is violated, there are two directions to be followed:

(1) If the local buckling constraint is a function of the yield strength such as in eqn (9), the yield strength of the material is reduced. This in turn gives a more lenient local buckling criterion. This procedure is continued until an f_y is found where the weight is minimum and the deflection is satisfactory.

(2) If eqn (10) is used where the diameter–thickness ratio is limited by a given constant, a pole of minimum weight is designed with a satisfactory deflection. This is accomplished by increasing the top and bottom diameters until a design is found which satisfies deflection. Then a design is sought where the weight is decreasing and the deflection is satisfactory.

    If there is no such design, the incremental change in the top and bottom diameter is divided by two and the desired design is again looked for. This procedure is continued until the incremental change in the top and bottom diameters becomes less than 1/16 in. and there is no design of less weight that satisfies the deflection constraint, at which point the search is terminated and the design is accepted as a minimum weight design.

The above procedure for deflection constraint also holds for the Euler buckling constraint.

Since the pole is a beam-column, an iterative procedure must be used to obtain the final design for each cycle (i.e. for a fixed top and bottom diameter). This procedure can be summarised as follows:

(1) Calculate shearing forces, moments, and axial loads (not including the weight of the pole).
(2) Design the pole to satisfy forces as obtained in step (1).
(3) Calculate the slope and deflection along the pole by using numerical integration.
(4) Calculate the stress due to shearing forces, bending moments, axial loads, and the secondary moments of the vertical loads (including the weight of the pole).
(5) Calculate the deflection along the pole. If this deflection does not agree within prescribed limits (taken as 1 in.) with the deflection obtained in step (3), then repeat steps (4) and (5) using the deflected shape obtained in step (5), until the deflection at the top and middle of the pole agrees within 1 in. with their counterparts from the previous iteration.

Steps (1), (2), and (3) have to be executed only for the first design cycle (i.e. before the optimisation routine is started). For the consequent design cycles, the deflection shape obtained in the previous cycle is used and the calculation is started from step (4).

EXAMPLE

There will be three designs investigated. The first of these will be designated the continuous design. In this design method, a linearly tapered, circular pole is divided into a large number of segments, the thickness of each being a design variable. As the number of segments taken increases, the solution becomes closer and closer to a continuous design. This design is not very practical because of the cost of the large number of connections needed and the cost of obtaining plates of varied thickness. In the approximate design, steel sheets are provided according to their availability. The continuous design is used as the optimum and the next largest available thickness is

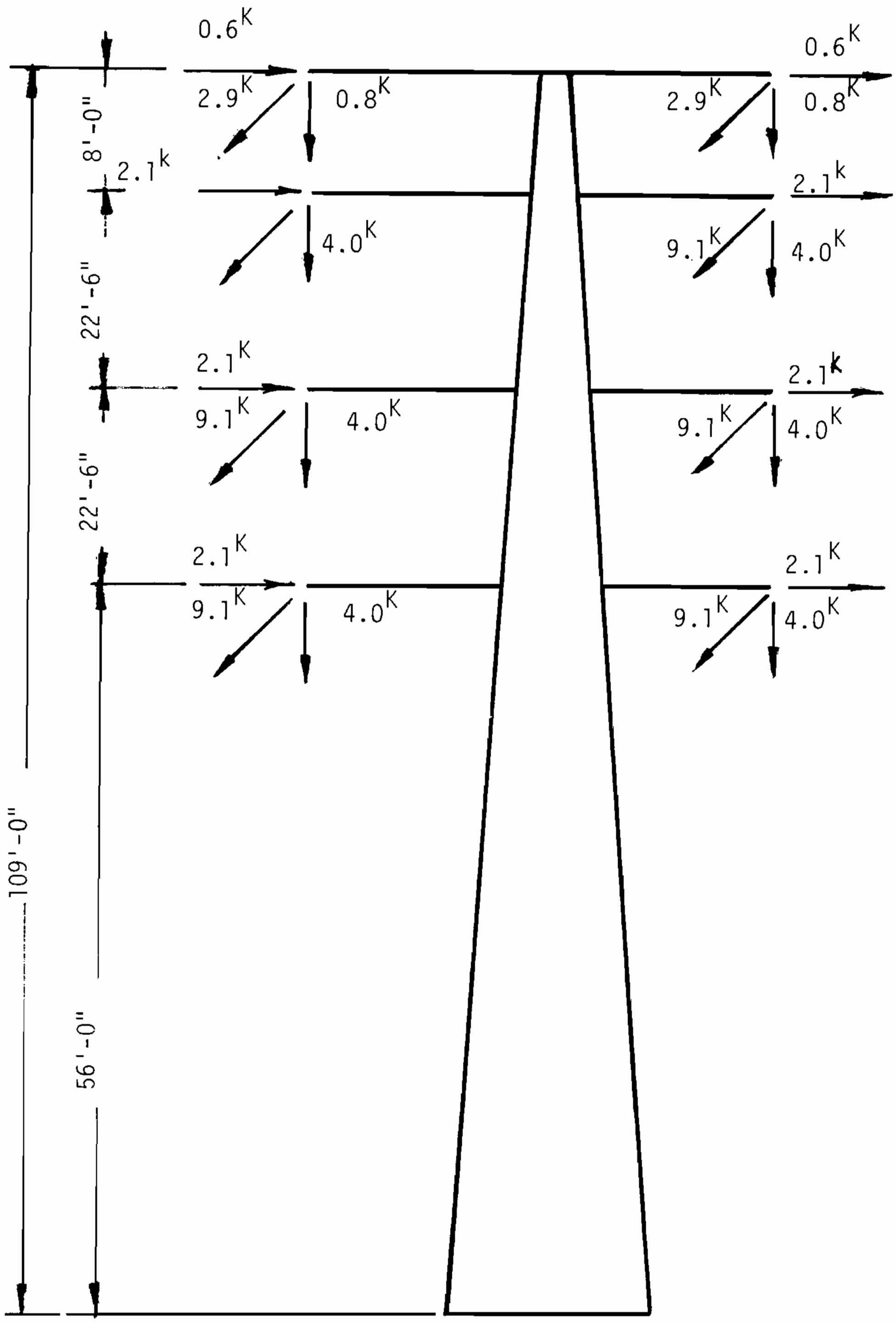

Fig. 2. Example design problem.

used for each segment. Hence, one plate thickness will span many segments. In the discrete design, the same available plate thicknesses are used as the approximate design. The difference arises in that these plates are then made part of the original optimisation. Hence, a slightly different overall geometry of the pole might arise for the minimum weight solution. In other words, the discrete design is a redesigning with available sections instead of approximating the optimum, continuous design, with these sections. The discrete design should produce a pole with less overall weight than produced by approximate design.

The pole and its loading are shown in Fig. 2. The loading corresponds to wind and cable tension acting on an angle pole. The desired yield strength of the material, f_y, was taken to be 60 ksi until the deflection constraint became active at which point the yield strength was reduced to give the least weight that satisfied all the constraints. The specifications used are those recommended by TCSTP (ASCE 1972). In particular, the limiting ratio of diameter to thickness is given by eqn. (9)

$$\frac{D}{t} \leq \frac{4800}{f_y}$$

The top and bottom diameters were optimised to the nearest 1/16 in. The allowable range for these variables was taken as

$$8 \text{ in.} \leq D_o \leq 50 \text{ in.}$$

$$8 \text{ in.} \leq D_b \leq 72 \text{ in.}$$

The maximum tip deflection (total vector) is taken to be 0·05%L.

As a check on the convergence, three initial values for D_o and D_b were arbitrarily selected. The designs obtained for all three starting points were the same.

(a) The continuous solution

The continuous solution represents a pole made of 30 equal segments. The thickness of each segment is allowed to have any value satisfying all the constraints.

Starting with $f_y = 60$ ksi, the results were

D_o (in.)	D_b (in.)	Wt. (kips)	Deflection (in.)
13·00	50·00	17·34	105·2 (=8·0%L)

Even though this deflection is considered satisfactory by most electric companies, a deflection constraint of 0·05%L was imposed here.

If f_y is kept constant, the results are

D_o (in.)	D_b (in.)	Wt. (kips)	Deflection (in.)
16·50	56·72	21·54	65·4 (=5·0%L)

The same problem is solved, this time with f_y as a design variable. The results are

D_o (in.)	D_b (in.)	Wt. (kips)	f_y (ksi)	Deflection (in.)
16·00	61·25	19·31	44·00	63·6 (=4·9%L)

where f_y is the yield strength of the material (to the nearest ksi) which produces the minimum weight of a pole satisfying all the original constraints. This indeed is the minimum weight pole for the deflection specified if the yield strength is taken as a design variable. Comparing the last result obtained to the one before it where f_y was kept as a constant, it is found that a 10·4% savings in weight can be obtained if f_y is used

Table 1. Results of example

(a) Variation of thickness

Segment	Location (inches, from top)	(a) Thickness required (in.)	(c) Thickness used (in.)
1	43·6	0·161	5/16
2	87·2	0·174	5/16
3	130·8	0·198	5/16
4	174·4	0·202	5/16
5	218·0	0·216	5/16
6	261·6	0·230	5/16
7	305·2	0·247	5/16
8	348·8	0·263	5/16
9	392·4	0·293	5/16
10	436·0	0·325	7/16
11	479·6	0·350	7/16
12	523·2	0·370	7/16
13	566·8	0·385	7/16
14	610·4	0·397	7/16
15	654·0	0·415	7/16
16	697·6	0·436	7/16
17	742·3	0·453	9/16
18	784·8	0·463	9/16
19	828·4	0·479	9/16
20	873·0	0·488	9/16
21	925·6	0·498	9/16
22	959·2	0·502	9/16
23	1003·0	0·506	9/16
24	1046·0	0·510	9/16
25	1090·0	0·512	9/16
26	1134·0	0·514	9/16
27	1177·0	0·420	9/16
28	1221·0	0·534	9/16
29	1264·0	0·548	9/16
30	1308·0	0·562	9/16

(b) Comparison of solutions

Solution type	D_o (in.)	D_b (in.)	Weight (kips)
(a) Continuous	16·00	61·25	19·31
(b) Discrete	13·25	59·00	21·19
(c) Approximate	16·00	61·25	21·88

as a design variable. The reason for this reduction is that the local buckling constraint which limits the maximum d/t ratio is a function of f_y, eqn (9). This allowable ratio increases as f_y decreases. The required thicknesses along the pole are shown for this leastweight solution in Table 1(a).

Fig. 3 shows the variation between the minimum weight obtained versus the material yield strength. Notice that the weight decreases with a decrease in f_y until no more reduction in f_y is needed to satisfy the deflection constraint. Once the deflection constraint is inactive, a further reduction in f_y produces an increase in weight because of the additional thickness required where the bending stress governs.

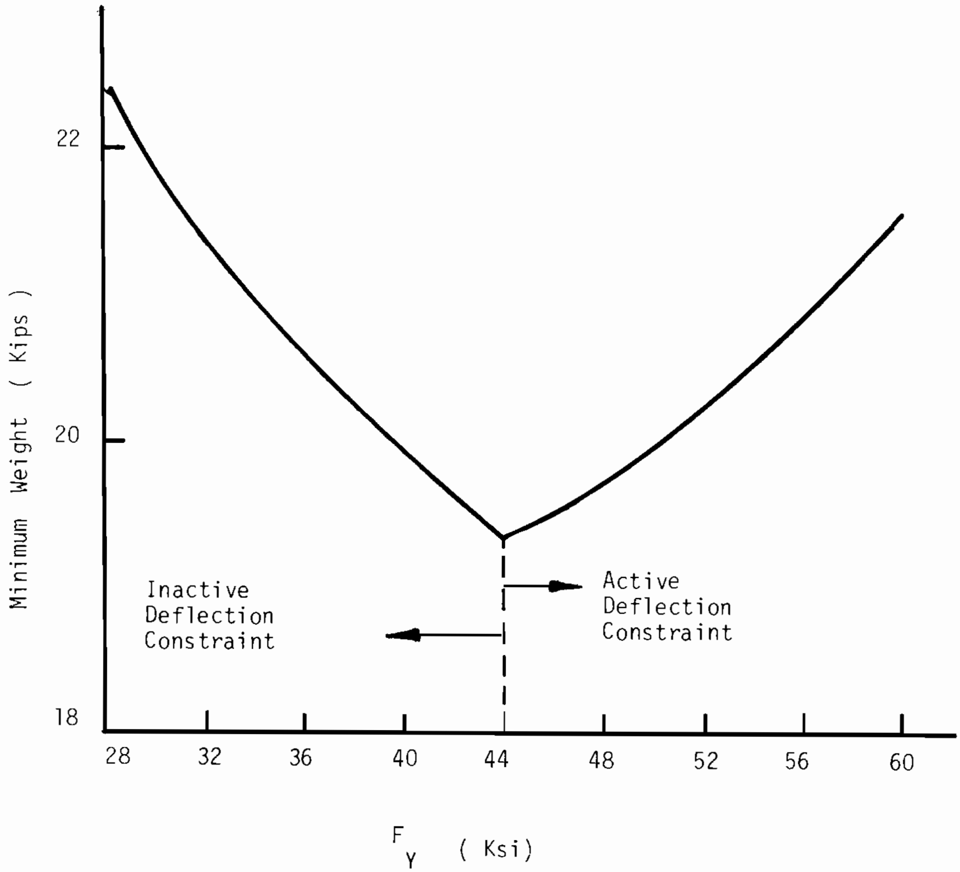

Fig. 3. Minimum weight versus yield strength.

(b) The discrete solution

For the same design criteria, the so-called discrete solution was the next case investigated.

The extra constraints imposed on the problem are:

(1) The thickness is allowed to vary in 1/8 in. increments with a minimum allowable of 3/16 in. to satisfy the NESC (1948) requirements.

(2) The length of each segment, except the last, is not allowed to be less than 15 ft; this

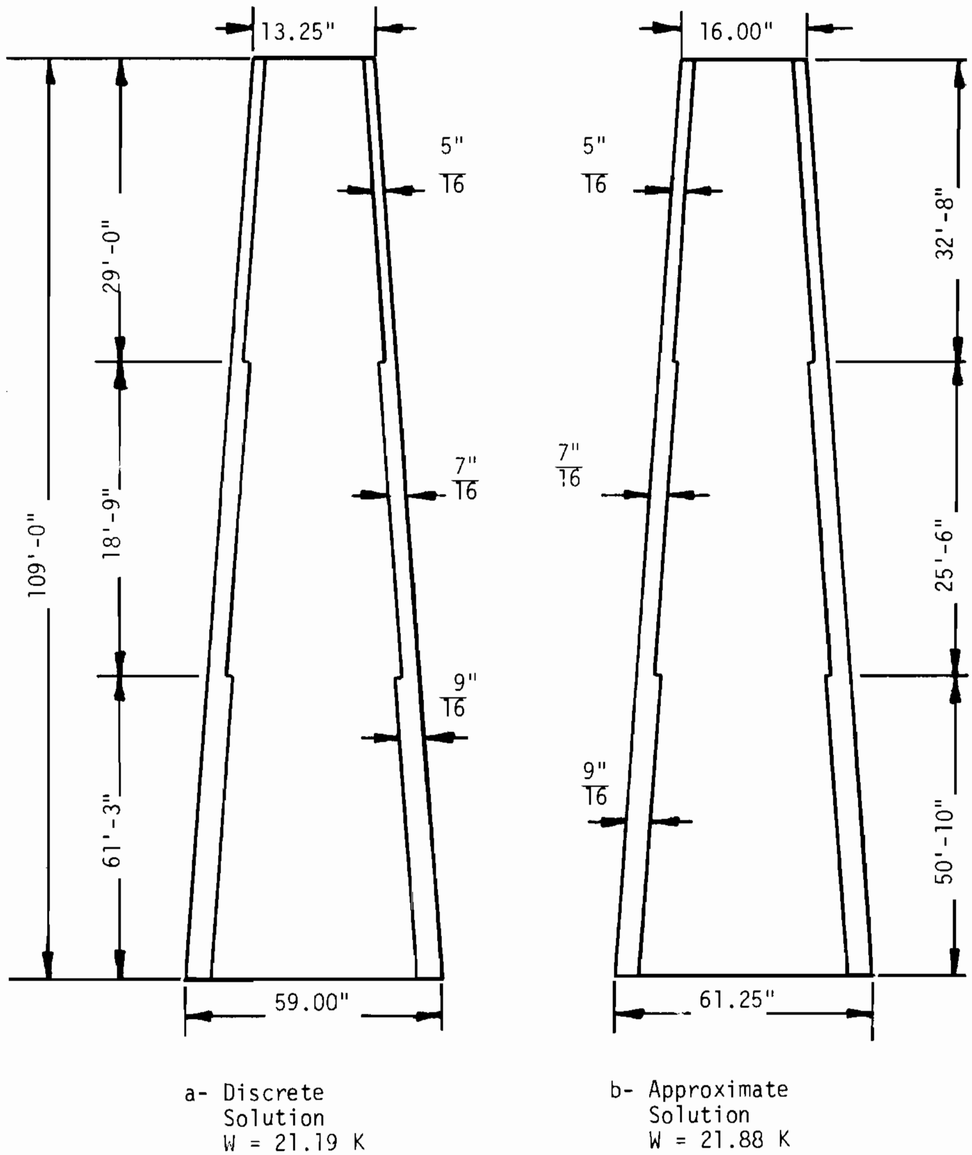

Fig. 4. Discrete and approximate solution.

constraint is for fabrication purposes to reduce the amount of welding. The last segment length is as needed to make the total equal to the length of the pole.

The results with f_y treated as a design variable are:

D_o (in.)	D_b (in.)	Wt. (kips)	f_y (ksi)	Deflection (in.)
13·25	59·00	21·19	44·00	65 (=5·0%L)

The weight of this pole is 13·3% greater than that obtained in part (a). However, the cost involved in the fabrication of the continuous solution would undoubtedly far outweigh the weight savings.

The design for this case is shown in Fig. 4a. For convenience the arms are removed and the thicknesses are exaggerated.

(c) The approximate solution

For the approximate solution, all the constraints imposed on the discrete solution are still considered. From previous discussions about the nature of the approximate solution, the design obtained is the same as the continuous solution except that the thickness is everywhere greater than or equal to that obtained. The weight is 3·6% greater than the weight obtained in part (b). The pole is shown in Fig. 4b. The variation in thickness of the pole is also given in the last column of Table 1(a).

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الطرق المثلى لتصميم الأعمدة الفولاذية

جاسم الصايغ
قسم الهندسة المدنية بجامعة الكويت

هوارد ابشتين
قسم الهندسة المدنية بجامعة كنيكتيكت ،
ستور ، كنيكتيكت ، الولايات المتحدة الأمريكية

خلاصة

في هذا البحث استعمل المؤلفان الطرق المثلى لتصميم الأعمدة الفولاذية لتوصيل الكهرباء والهدف الأساسي هو إيجاد منشأة ذات أقل وزن .