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T-graphs

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ABSTRACT

A T-graph is a graph with two disjoint maximal stable sets. The notions r-T-graph and edge-T-graph are introduced, the implications

2-T-graph → edge-T-graph → 1-T-graph → T-graph

are true but the converse of each is false.

The main result is that if in a graph G, $2 \leq \deg(V) \leq 3$ for every vertex v, then G is a 2-T-graph.

We follow the terminology of Berge (1973). A graph is called a T-graph if it has two disjoint maximal stable sets.

A graph $G = (V,E)$ is said to be an r-T-graph if for every $S \subset V$ with $|S| = r$ and with no odd cycle in G/S there exist two maximal disjoint stable sets, S_1 and S_2, with $S \subset S_1 U S_2$.

A graph $G = (V,E)$ is called an edge-T-graph if for every pair of adjacent vertices u and v there exist two maximal disjoint stable sets, S_1 and S_2, such that $u,v \in S_1 U S_2$.

It can be seen that the following implications are true:

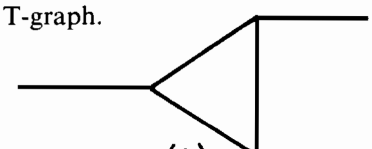
n-T-graph → (n-1)-T-graph, $n \geq 2$.

2-T-graph → edge-T-graph → 1-T-graph → T-graph.

We shall give examples to show that each one of the reverse implications is false.

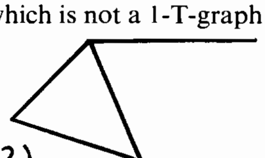
Examples

(1) The graph (1) is not a T-graph.



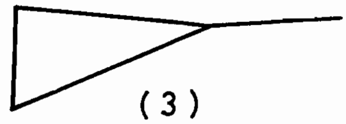
(1)

(2) The graph (2) is a T-graph which is not a 1-T-graph.



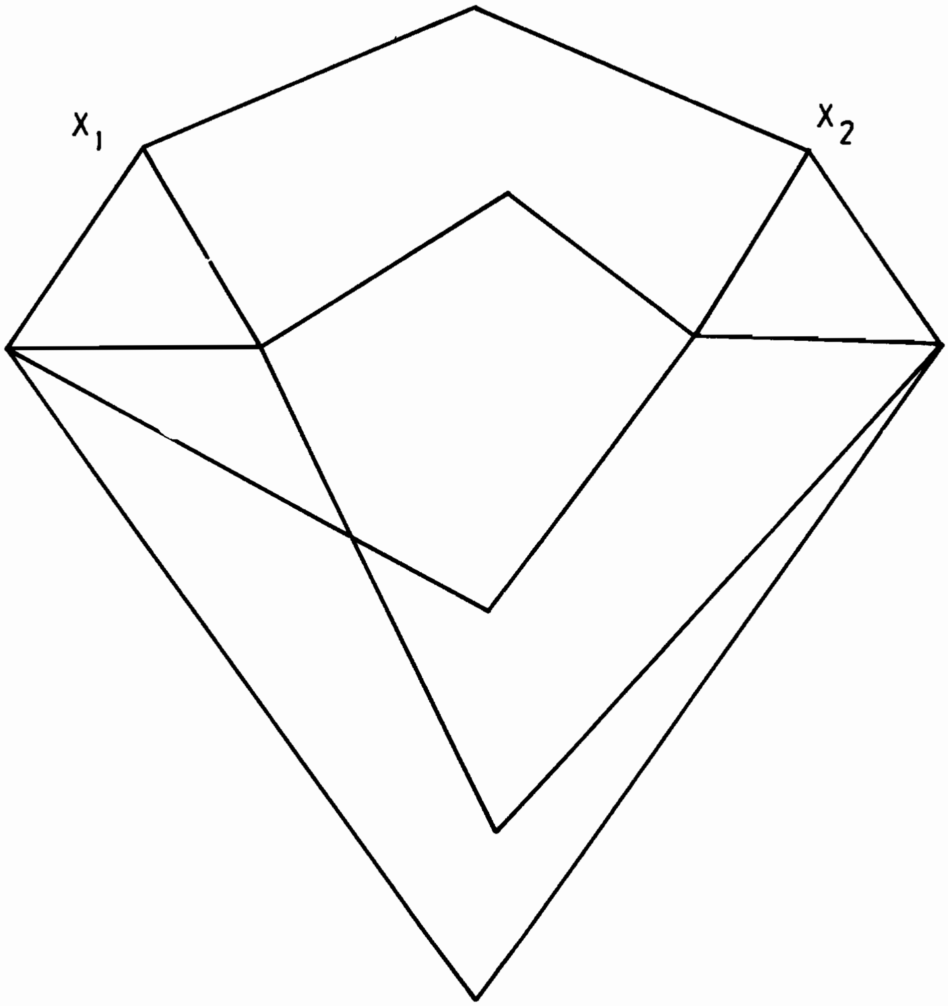
(2)

(3) The graph (3) is a 1-T-graph but it is not an edge-T-graph.



(3)

(4) The graph (4) is an edge-T-graph but it is not a 2-T-graph, since $\{x_1,x_2\}$ cannot be included in the union of two disjoint maximal stable sets.



(4)

Theorem 1. Let $G = (V,E)$ be a graph of order n with $2 \leq \deg(X) \leq 3$ for every $x \in V$, then G is an edge-T-graph.

Proof. Let u and v be two adjacent vertices of G, we will prove that there exist two disjoint maximal stable sets, 2DMSS, S_1 and S_2 with $u,v \in S_1 U S_2$. The proof is by induction on the order of G.

There are two main cases:

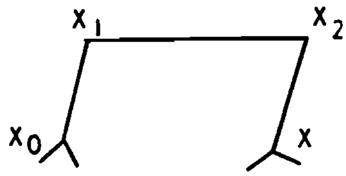
Case 1: G has a chain of length $m \geq 1$ of vertices of degree 2.

Case 2: G has no adjacent vertices of degree 2.

Case 1 can be divided into the following cases:

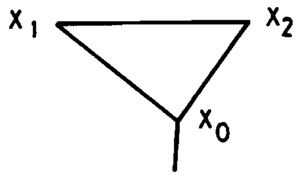
Case 1.1: $m = 1$, let x_1 and x_2 be the two adjacent vertices of degree 2.

Case 1.1.1: x_1 and x_2 are not adjacent to the same vertex.



Case 1.1.1

Case 1.1.2: x_1 and x_2 are adjacent to the same vertex x_0.



Case 1.1.2

Proof of case 1.1.

Case 1.1.1: Remove x_1 and x_2 (and of course all edges incident to them) and join x_0 to x (if they are not adjacent). We see that the reduced graph satisfies the hypothesis.

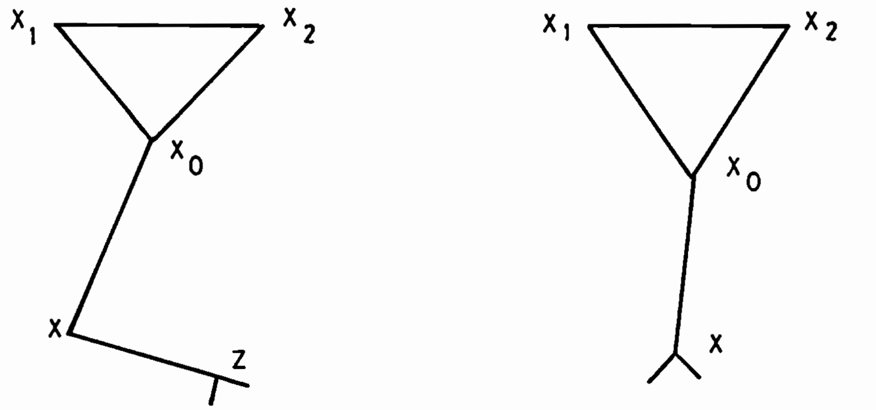
If $u,v \in V-\{x_1,x_2\}$, then by the induction hypothesis there are 2DMSS, S_1 and S_2, with $u,v \in S_1 U S_2$ in the reduced graph. Vertices of S_1 will be called circles and those of S_2 boxes. Now delete the added edge and extend S_1 and S_2 to 2DMSS of G by handling x_1 and x_2 as follows:

If x_0 or x is a box (or a circle), say x_0, we make x_1 a circle and x_2 a box (or a circle), since x cannot be a box (or a circle). And, if $x_0, x \notin S_1 U S_2$, then make x_1 a circle and x_2 a box. The remaining two cases $v,u \in \{x_1,x_2\}$ and $u \in \{x_0,x\}$, $v \in \{x_1,x_2\}$ can be handled in a similar way.

Case 1.1.2: In case (i) we remove x_1,x_2,x_0, and x (also z if degree $(z) = 2$). Now, if $u,v \in \{x_1,x_2,x_0,x\}$, then there are 2DMSS S_1 and S_2 with z a circle. Extend S_1 and S_2 to 2DMSS of G by making x a box and putting the ends of a suitable edge of the triangle $x_0x_1x_2$ in $S_1 U S_2$, so that $u,v \in S_1 U S_2$. This includes also the case $u = x$, $v = z$. The case degree $(x) = 3$ can be handled in a similar manner. Also the case in which u and v are not in $\{x_1,x_2,x_0,x\}$ can be handled similarly. Case (ii) is again similar to case (i).

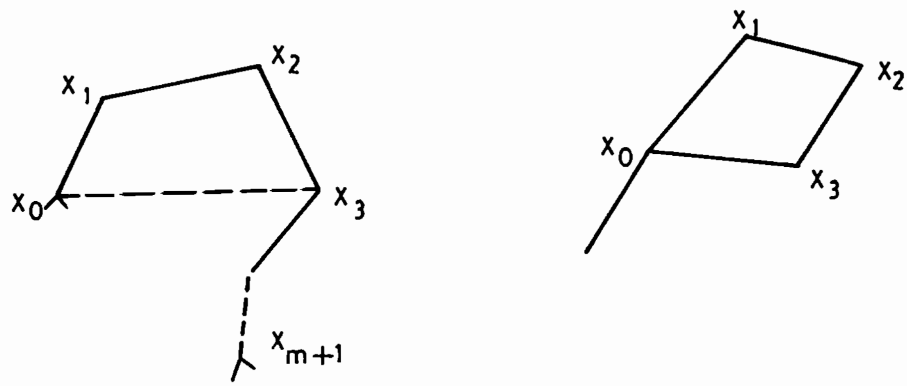
Case 1.2: G has a chain of vertices of degree 2 of order $m \geq 2$. After removing x_1 and x_2 in case (i), join x_3 and x_0 and handle it as in case 1.1.1.

Case (ii) can be handled in a way similar to 1.1. If in (ii), x_0 is of degree 2 then



(i) degree of x is 2. (ii) degree of x is 3.

Case 1·1.2



(i) (ii)

Case 1.2

x_0,x_1,x_2 and x_3 form a component of the graph, this case can be handled easily. This concludes case 1.

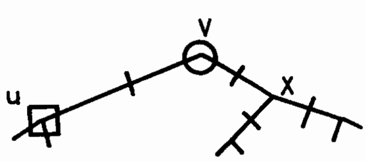
In handling the remaining cases, we remove vertices and join some vertices to others so that we get a graph with reduced order which satisfies the hypothesis.

Case 2: G does not have adjacent vertices of degree 2.

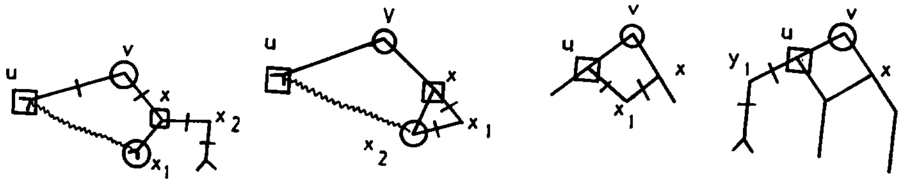
Case 2.1: degree (u) = 3, degree (v) = 2.

Case 2.1.1: u and v are not adjacent to the same vertex. The subcases of 2.1.1 depend on the other vertex x adjacent to v.

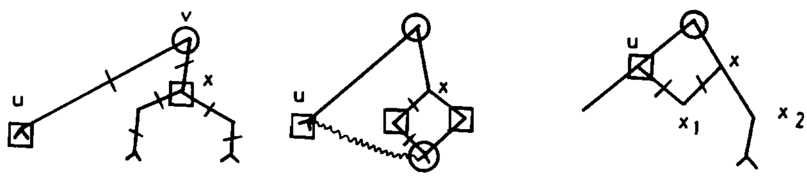
Case 2.1.1.1: x is adjacent to two vertices of degree 3.



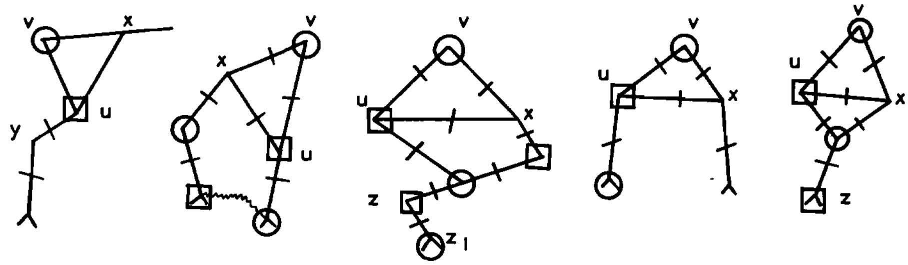
Case 2.1.1.2: x is adjacent to one vertex of degree 3.



Case 2.1.1.3: x is not adjacent to a vertex of degree 3.

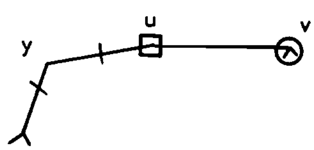


Case 2.1.2: u and v are adjacent to the same vertex x. Let y be the third vertex adjacent to u.



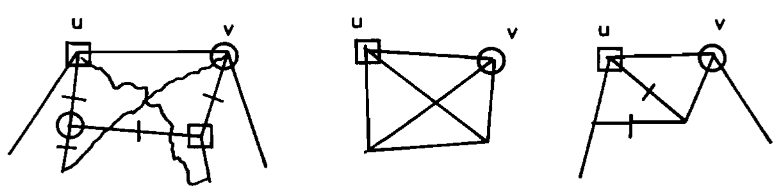
Case 2.2. degree $(u) = 3$, degree $(v) = 3$.

Case 2.2.1: One of the vertices u and v is adjacent to a vertex of degree 2.

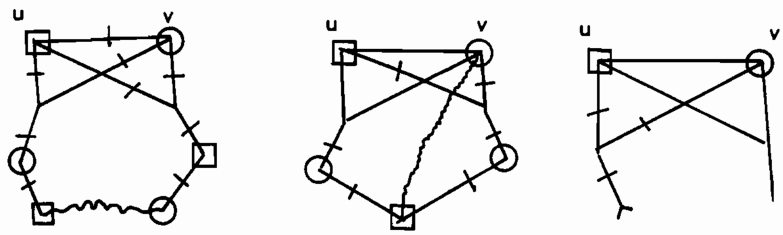


Case 2.2.2: Neither u nor v is adjacent to a vertex of degree 2. Let y_1 and y_2 be the two vertices adjacent to u and let x_1 and x_2 be adjacent to v.

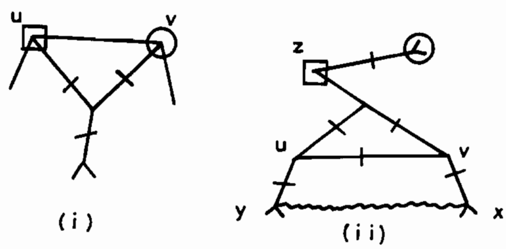
Case 2.2.2.1: x_1 is adjacent to y_2.



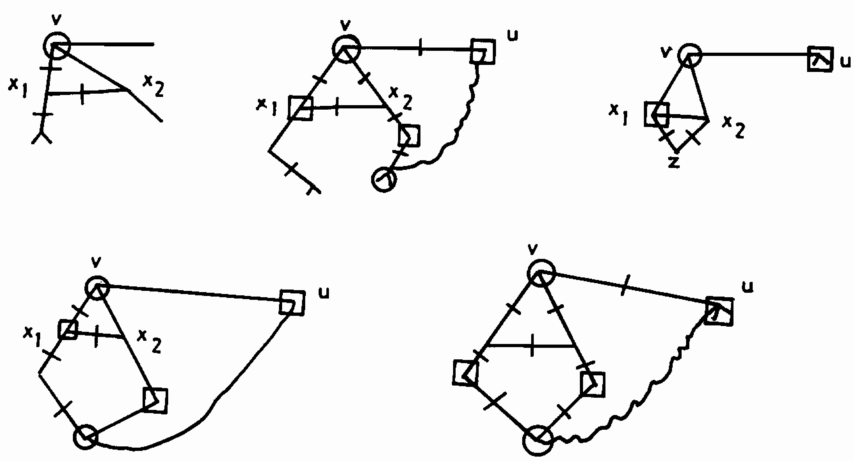
Case 2.2.2.2: u is adjacent to x_1 and v is adjacent to y_1.



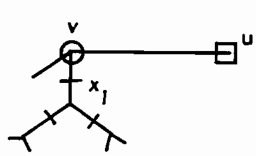
Case 2.2.2.3: u and v are adjacent to the same vertex.



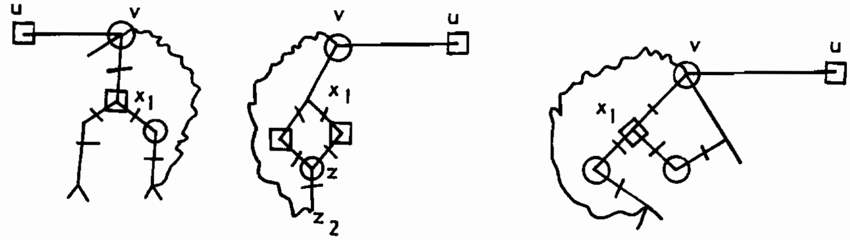
Case 2.2.2.4: x_1 is adjacent to x_2.



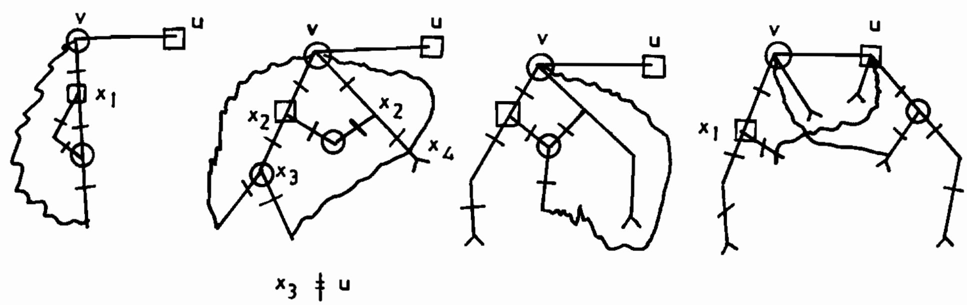
Case 2.2.2.5: x_1 and x_2 are not adjacent.
Case 2.2.2.5.1: x_1 is not adjacent to a vertex of degree 2.



Case 2.2.2.5.2: x_1 is adjacent to two vertices of degree 2.



Case 2.2.2.5.3: Either or both of x_1 and x_2 is adjacent to only one vertex of degree 2.



Now we handle each of the listed cases.

We handle each case by removing the vertices with dashed edges and joining the curly edges if they are not already existing to get a reduced order graph satisfying the hypothesis. So we find in the reduced graph two disjoint maximal stable sets with one of the vertices a box or one vertex a box and an adjacent vertex a circle. Then those sets are extended to the whole of G, by removing the extra edges (if there are any) and making boxes and circles as indicated in the figures of each case. There might remain a vertex from the removed ones which is adjacent to a box (or a circle) in the reinstalled portion of the graph and adjacent at the same time to a vertex which has not been removed. Such a vertex, we make a circle (or a box) in case it is not already adjacent to a circle (or a box). This concludes the proof.

Theorem 2: If $G = (E,V)$ is a graph of order n with $2 \leq d(x) \leq 3, x \in V(G)$, then for any two non-adjacent vertices u,v, there exist two disjoint maximal stable sets S_1 and S_2 with $u,v \in S_1 U S_2$.

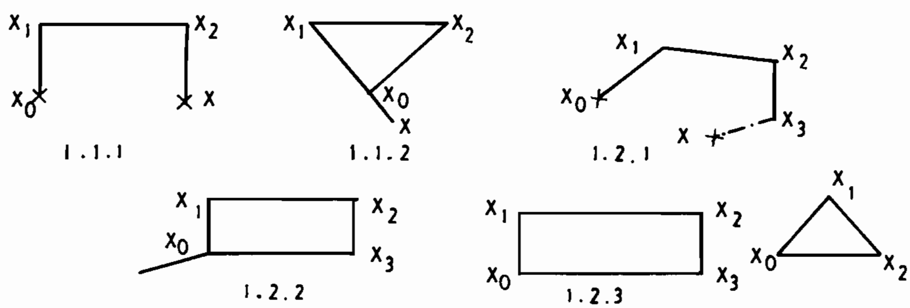
Proof: The proof is by induction on the order of G.

In the proof of the different cases the general procedure is to remove some vertices, and add some new edges and vertices to get a graph satisfying the hypothesis with order less than n for which we find 2DMSS, S_1 and S_2, either by the induction hypothesis (or by Theorem 1), and then we remove the extra vertices or edges and extend the 2DMSS to the whole graph G according to the hypothesis. Each case has three different subcases:

(i) u and v are not removed
(ii) u or v is removed
(iii) u and v are removed
We usually leave one or two of these cases because they can be handled in a way similar to the case we proved.

Case 1: There is a chain of length $m \geq 1$ of vertices of degree 2.
Case 2: G has no adjacent vertices of degree 2.

Case 1: Let $x_1 x_2 \ldots x_{m+1}$ be a chain of length $m \geq 1$ of vertices of degree 2.



Case 1.1.

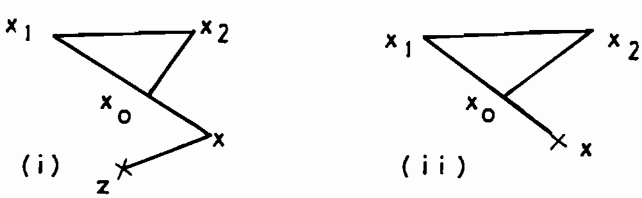
Case 1.1.1: $m = 1$ and x_1 and x_2 are not adjacent to the same vertex.

Remove x_1 and x_2 and join x_0 to x if $(x_0,x) \notin E$, then the new graph is of reduced order.

If $u,v \in V-\{x_1,x_2\}$, then by induction hypothesis there are 2DMSS, S_1 and S_2, with $u,v \in S_1 U S_2$, in the reduced graph. Then remove the edge (x,x_0) (if $(x_0,x) \notin E$), and extend S_1 and S_2 to 2DMSS of G by handling x_1 and x_2 as follows:

If x_0 or x is a box (or a circle), say x_0. Make x_1 a circle (or a box) and x_2 a box (or a circle) and if $x_0,x \notin S_1 U S_2$ make x_1 a circle and x_2 a box. The remaining two cases: $u \in \{x_1,x_2\}$ and $v \in V-\{x_1,x_2\}$ and $\{u,v\} = \{x_1,x_2\}$ can be handled in a similar way.

Case 1.1.2: If x_1 and x_2 are adjacent to the same vertex x



(i) x is of degree 2
(ii) x is of degree 3

In (i) Remove x_1,x_2,x_0 and x to get a reduced graph satisfying the hypothesis.

(a) If $u,v \notin \{x,x_0,x_1,x_2\}$, then by induction hypothesis there are 2DMSS, S_1 and S_2, with $u,v \in S_1 U S_2$. Now extend S_1 and S_2 to 2DMSS of G by making x_1 a circle and x_0 a box and then handling x (making it a circle if z is not a circle).

(b) If $u \in \{x,x_0,x_1,x_2\}$ and $v \in V-\{x,x_0,x_1,x_2\}$, then by induction hypothesis there are

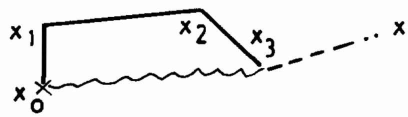
2DMSS with $v \in S_1 U S_2$ and z a circle. Now extend S_1 and S_2 by making x a box, x_0 a circle and x_1 (or x_2) a box (if $u = x_1$ (or x_2)).

(c) The case $u,v \in \{x,x_0,x_1,x_2\}$ can be handled as in case (b).

Case (ii) can be handled in a similar way.

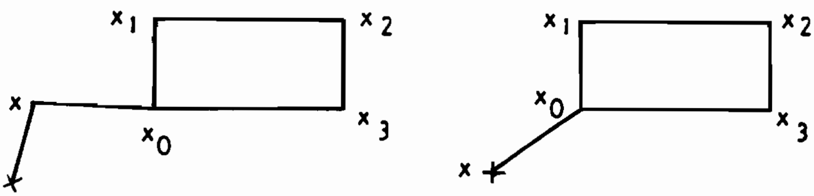
Case 1.2: $m \geq 2$.

Case 1.2.1: $m \geq 2$, x_1 and x_3 are not adjacent to the same point other than x_2.



Remove x_1 and x_2 and join x_3 to x_0, then this case is similar to case 1.1.1.

Case 1.2.2: $m = 2$, x_1 and x_3 are adjacent to the same point x_0; this case is similar to case 1.1.2.



Case 1.2.3: A cycle of order 4 is a component of the graph and it can be handled easily. The same for the triangle.

Case 2: G has no adjacent vertices of degree 2.

The different cases will be classified according to the degree of v, and the position of u.

Case 2.1: degree of $v = 2$.

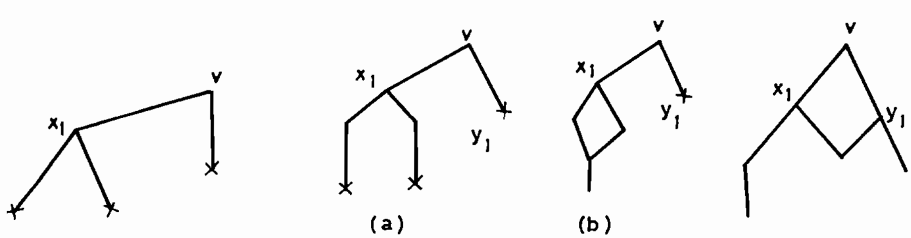
Let x_1 and y_1 be the two vertices adjacent to v.

Case 2.1.1: x_1 and y_1 are not adjacent.

Case 2.1.2: x_1 and y_1 are adjacent.

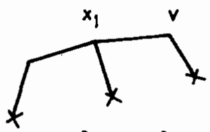
Case 2.1.1.1: One of x_1, y_1 is adjacent to two vertices of degree 3.

Case 2.1.1.2: One of x_1, y_1 is not adjacent to a vertex of degree 3.

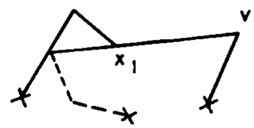


2.1 1.1 2.1.1.2.1 2.1.1.2.2

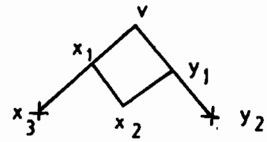
Case 2.1.1.3: Each of x_1 and y_1 is adjacent to only one vertex of degree 3.



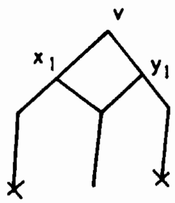
2.1.1.3.1.



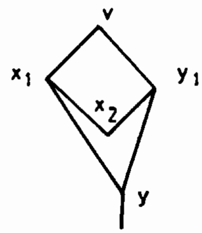
2.1.1.3.2



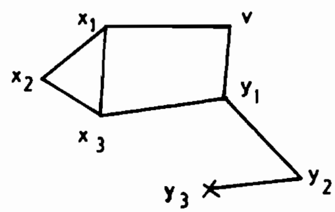
2.1.1.3.3



2.1.1.3.1

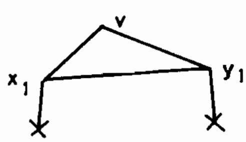


2.1.1.3.5

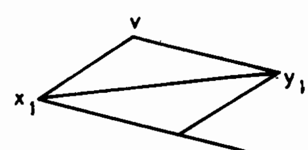


2.1.1.3.6

Case 2.1.2: x_1 and y_1 are adjacent.
Case 2.1.2.1: x_1 and y_1 are not adjacent to a vertex of degree 2 other than v.

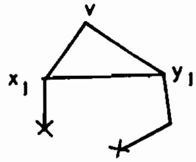


2.1.2.1.1

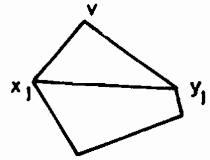


2.1.2.1.2

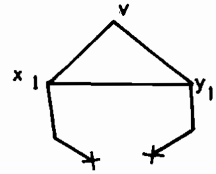
Case 2.1.2.2: One of x_1 and y_1 is adjacent to a vertex of degree 2 other than v.
Case 2.1.2.3: Both of x_1 and y_1 are adjacent to vertices of degree 2.



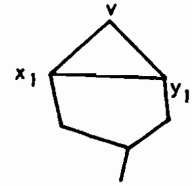
2.1.2.2.1



2.1.2.2.2



2.1.2.3.1

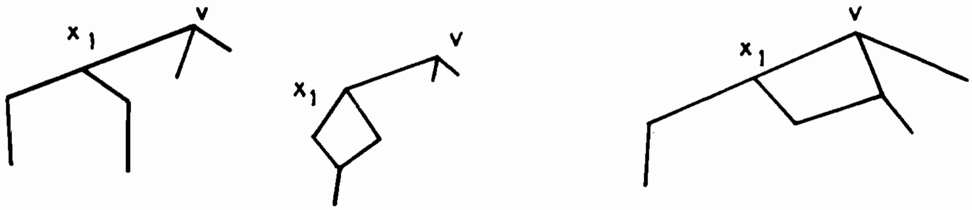


2.1.2.3.2

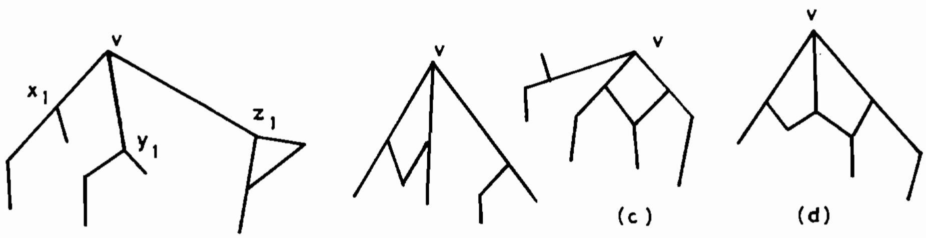
Case 2.2: Degree of $v = 3$.
Case 2.2.1: v is adjacent to a vertex of degree 2.
Case 2.2.2: v is not adjacent to a vertex of degree 2.

Case 2.2.2.1: One of the three vertices which are adjacent to v is adjacent to two vertices of degree 3.

Case 2.2.2.2: One of the three vertices which are adjacent to v is adjacent to two vertices of degree 2.

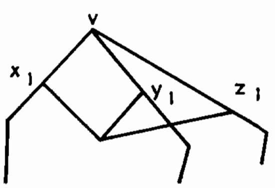


Case 2.2.2.3: Each of the vertices adjacent to v is adjacent to only one vertex of degree 3.

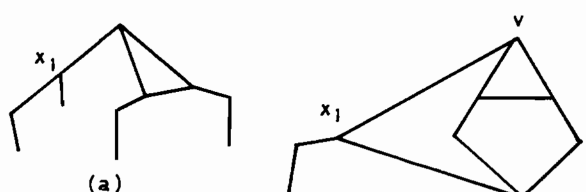


(a) (b) (c) (d)

2.2.2.3.1



2.2.2.3.2

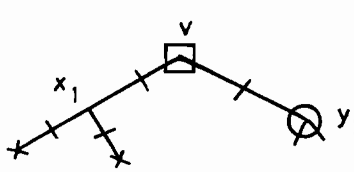


(a) (b)

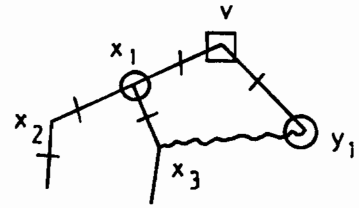
2.2.2.3.3

Now we want to handle each of the preceding listed subcases.

Case 2.1.1.1: Remove v and x_1, the order of the new graph is less than n. By the induction hypothesis, this graph has 2DMSS, S_1 and S_2, with $u \in S_1 U S_2$ and y_1 a circle. Now extend S_1 and S_2 to 2DMSS of G by making v a box and handling x_1.

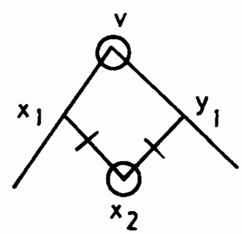


Case 2.1.1.2.1: x_3 and x_2 are not adjacent to y_1.



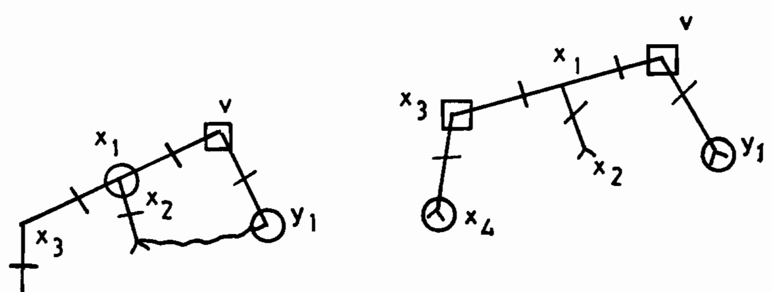
Remove v,x_1 and x_2 (or x_3) ($x_2 \neq u$) and join x_3 (or x_2) to y_1; then the new graph is of reduced order and satisfies the hypothesis. So there are 2DMSS, S_1 and S_2, with $u \in S_1 U S_2$ (if u is not removed) and y_1 a circle. Extend S_1 and S_2 to 2DMSS of G by making x_1 a circle and v a box then handling x_2 (or x_3).

Case 2.1.1.2.2: x_2 is adjacent to y_1.



Remove x_2, then the graph $G/V-\{x_2\}$ has 2DMSS, S_1 and S_2, with v a circle and $u \in S_1 U S_2$. Now extend S_1 and S_2 to 2DMSS of G by making x_2 a circle, since either x_1 or y_1 must be a box.

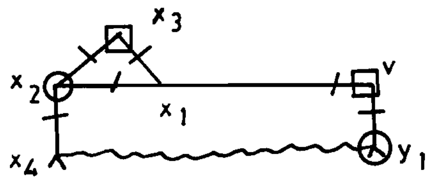
Case 2.1.1.3.1: In (i) remove v,x_1 and x_3 and join x_2 to y_1 (if $(x_2,y_1) \notin E$) to get a graph of reduced order which satisfies the hypothesis. So it has 2DMSS, S_1 and S_2, with $u \in S_1 U S_2$ and y_1 a circle. Now remove (x_2,y_1) (if $(x_2,y_1) \notin E$) and extend S_1 and S_2 to 2DMSS of G by making v a box and x_1 a circle and handling x_3.



(i) $u \notin x_3$ (ii) $u = x_3$

In (ii) remove v,x_1,x_3, so that the resulting graph has 2DMSS, S_1 and S_2, with y_1 a circle and $x_4 \in S_1 U S_2$, i.e. x_4 is a circle (or a box). Now extend S_1 and S_2 to 2DMSS of G by making v a box, x_3 a box (or a circle) and handling x_1.

Case 2.1.1.3.2: Remove v,x_1,x_2 and x_3 and join x_4 to y (if $(x_4,y_1) \notin E$), then the new graph has 2DMSS, S_1 and S_2, with $u \in S_1 U S_2$ (if $u \neq x_2,x_3$) and y_1 a circle. Now remove (x_4,y_1) if it is not originally in E and extend S_1 and S_2 to 2DMSS of G by making x_2 a circle and both of v and x_3 boxes. We see that this also solves the case $u = x_2,x_3$.



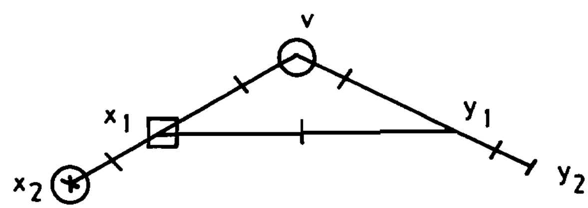
Cases 2.1.1.3.3 and 2.1.1.3.5: Can be handled in a way similar to 2.1.1.2.2.

Case 2.1.1.3.4: Was handled in 2.1.1.3.1.

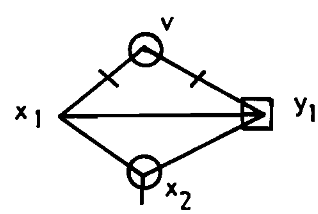
Case 2.1.1.3.6: Is handled by removing v, x_1, x_2, x_3, y_1 and y_2, then finding 2DMSS, S_1 and S_2, of the reduced graph with $u \in S_1 U S_2$, if it is not removed and y_3 a circle. The extension to G can be done easily.

Case 2.1.2.

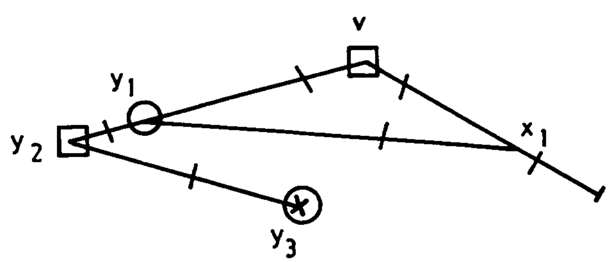
Case 2.1.2.1.1: Remove v, x_1 and y_1, then the resulting graph has 2DMSS, S_1 and S_2, with $u \in S_1 U S_2$ and x_2 a circle. Now extend S_1 and S_2 to 2DMSS of G by making x_1 a box and v a circle, and handling y_1.



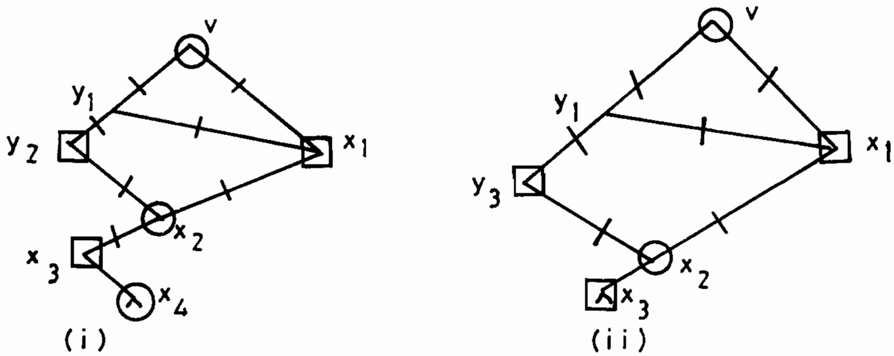
Case 2.1.2.1.2: Remove v, then $G/V-\{v\}$ has 2DMSS, S_1 and S_2, with x_2 a circle and $u \in S_1 U S_2$. Now extend S_1 and S_2 by making v a box (note that one of x_1, y_1 must be a box).



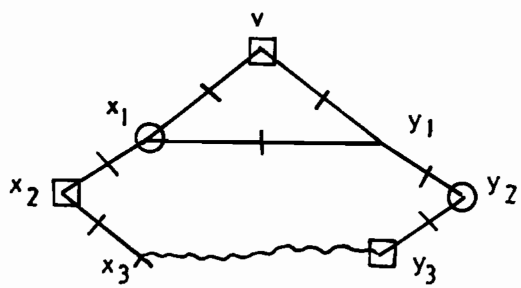
Case 2.1.2.2: Remove v, x_1, y_1 and y_2, then $G/V-\{y_1, y_2, x_1, v\}$ has 2DMSS, S_1 and S_2, with y_3 a circle and $u \in S_1 U S_2$, if u is not removed. Now extend S_1 and S_2 to 2DMSS of G by making y_2 a box, y_1 a circle and v a box, and handling x_1.



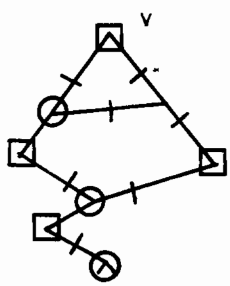
Case 2.1.2.2.2: In (i) remove v, y_1, x_1, y_2, x_2 and x_3, then the resulting graph has 2DMSS, S_1 and S_2, with x_4 a circle and $u \in S_1 U S_2$ (if u is not removed). Now extend S_1 and S_2 to 2DMSS of G by making x_3 a box, x_2 a circle, y_2 a box, v a circle and x_1 a box. Case (ii) is similar to case (i).

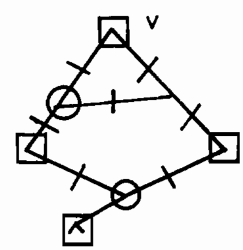


Case 2.1.2.3.1: Remove v,x_1,x_2,y_1 and y_2 and join x_3 to y_3 (if $(x_3,y_3) \notin E$), then the new graph has 2DMSS, S_1 and S_2, with $u \in S_1 U S_2$ and y_3 a box. Now remove (x_3,y_3) (if it is not in E) and extend S_1 and S_2 to 2DMSS of G by making both of x_2 and v boxes and both of x_1 and y_2 circles.

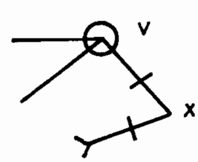


Case 2.1.2.3.2: This case can be handled similar to case 2.1.2.2.2.

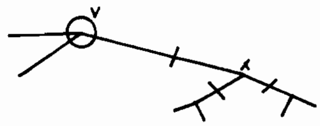




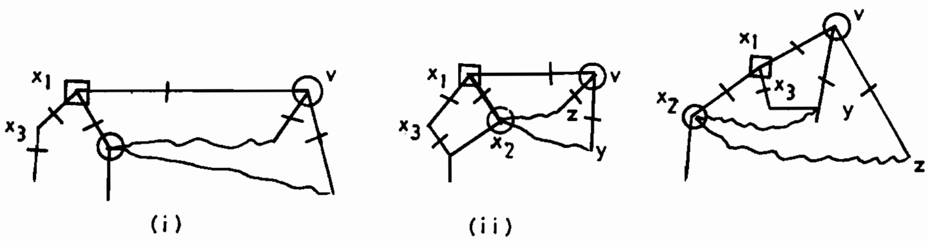
Case 2.2.1: Let x be a vertex of degree 2 which is adjacent to v; remove x, then $G/V-\{x\}$ has 2DMSS, S_1 and S_2, with $v,u \in S_1 U S_2$. Now extend S_1 and S_2 to 2DMSS of G by handling x.



Case 2.2.2.1: Can be handled in a way similar to case 2.2.1.



Case 2.2.2.2: In cases (i) and (ii) assume $x_3 \neq u$.



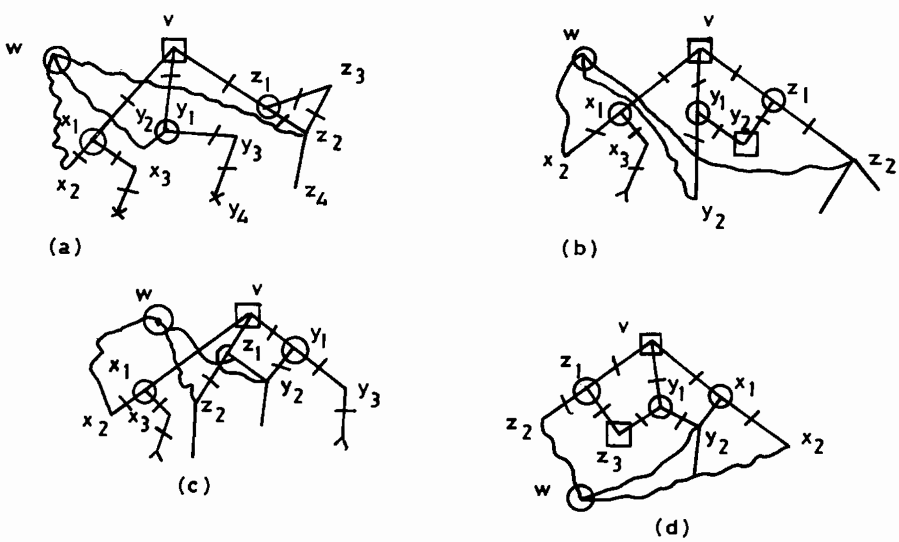
Remove v,x_1,x_3 and join y and z to x_2, then the resulting graph has 2DMSS, S_1 and S_2, with x_2 a circle and $u \in S_1 U S_2$. Now remove the additional edges and extend S_1 and S_2 to 2DMSS of G by making v a circle and x_1 a box and then handling x_3. In case (iii) x_3 is a circle.

Case 2.2.2.3.1:

(i) $u \notin \{x_3,y_3,z_3\}$ in (a)

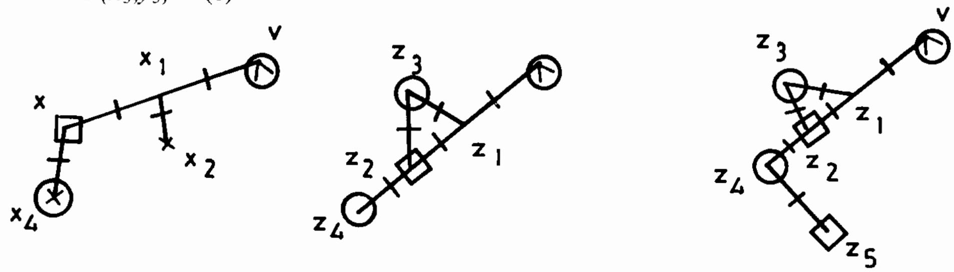
$u \neq x_3$ in (b)

$u \notin \{x_3,y_3\}$ in (c)



Remove v,x_1,y_1,z_1,x_3 and y_3 (also z_3 in (a)). Add a new vertex w and join each of x_2,y_2 and z_2 to w, then the new graph has 2DMSS, S_1 and S_2, with $u \in S_1 U S_2$ (if u is not removed) and w a circle. Now remove the vertex w and, of course, all edges incident to w, then extend S_1 and S_2 to 2DMSS of G by making each of x_1,y_1,z_1 a circle and v a box, then handling x_3 and y_3 (also z_3 in case (a)).

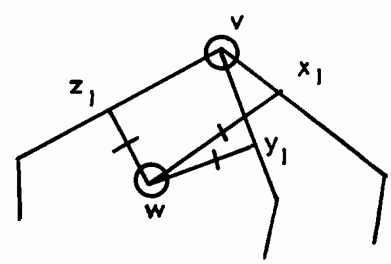
(ii) $u \in \{x_3,y_3,z_3\}$ in (a)
$u = x_3$ in (b)
$u \in \{x_3,y_3\}$ in (c)



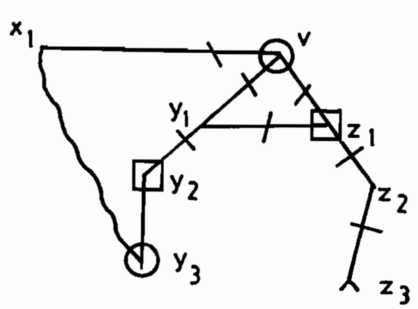
For $u = x_3$ (or y_3 in (a) and (c)), remove x_1 and x_3, then the new graph has 2DMSS, S_1 and S_2, with x_4 a circle and $v \in S_1US_2$. Now extend S_1 and S_2 to 2DMSS of G by making x_3 a box and handling x_1.

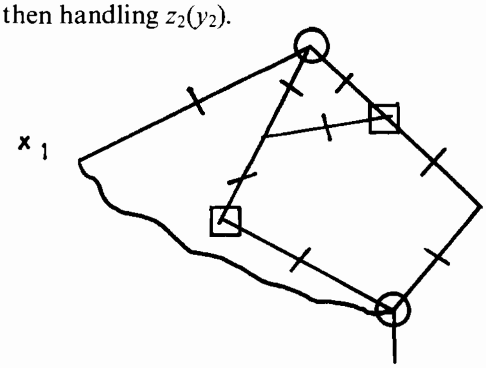
In (a) if $u = z_3$, remove z_1,z_2 and z_3 (also z_4 if $d(z_4) = 2$), then the resulting graph has 2DMSS, S_1 and S_2, with v a circle and $z_4 \in S_1US_2$ (or z_5 in S_1US_2). Now extend S_1 and S_2 to 2DMSS of G by making z_1 a box, z_3 a circle and z_4 a circle (or a box), if z_5 is a box (or a circle).

Case 2.2.2.3.2: Remove w, then $G/V-\{w\}$ has 2DMSS, S_1 and S_2, with v a circle and $u \in S_1US_2$ if $u \neq w$. Now extend S_1 and S_2 to 2DMSS of G by making w a circle; it is clear that this will solve also the case $u = w$.

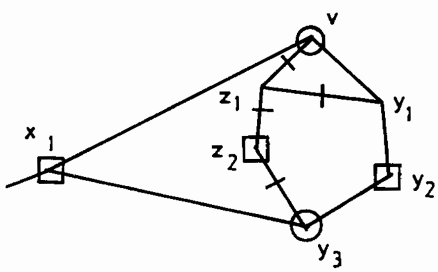


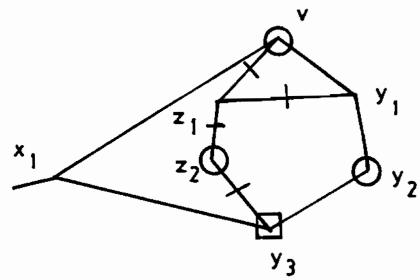
Case 2.2.2.3.2: (a) $(y_3,x_1) \notin E$. Remove v,y_1,y_2,z_1 and z_2, and join x_1 to y_3 (or z_3 if $u = z_2$), then the resulting graph has 2DMSS, S_1 and S_2, with $y_3(z_3)$ a circle and $u \in S_1US_2$ if u is not removed. Now extend S_1 and S_2 to 2DMSS of G by making both of y_2 and z_1 (or z_2 and y_1) boxes, and v a circle, then handling $z_2(y_2)$.





(b) $(y_3,x_1) \in E$. Remove z_1,z_2, then the new graph has 2DMSS, S_1 and S_2, with v a circle and $u \in S_1 U S_2$ if u is not removed. Now x_1 is either a box or adjacent to a box. If x_1 is a box then y_3 can be a circle and y_2 can be a box so we can extend S_1 and S_2 to 2DMSS of G by making z_2 a box. If x_1 is adjacent to a box, then y_3 can be a box, y_2 can be a circle and y_1 a box, so we can extend S_1 and S_2 to 2DMSS of G by making z_2 a circle.





This completes the proof of Theorem 2.

Theorem 3. If $G = (V,E)$ is a graph of order n with $2 \leq d(x) \leq 3$ $\forall x \in V$, then G is a 2-T-graph.

Proof. In Theorem 1 we have shown that for any two adjacent vertices u and v of $V(G)$, there are 2DMSS, S_1 and S_2, with $u,v \in S_1 U S_2$. Also in Theorem 2 we have proved for any two non-adjacent vertices x,y of $V(G)$, there are 2DMSS, S_1 and S_2, with $x,y \in S_1 U S_2$. Therefore for any two vertices z and w of $V(G)$ there are 2DMSS, S_1 and S_2, with $z,w \in S_1 U S_2$. G is a 2-T-graph.

Corollary. A 3-regular graph is a 2-T-graph.

We state now some conjectures.

Conjecture 1. Let $G = (V,E)$, if $2 \leq \deg(v) \leq 3$ for every r, $1 \leq r \leq R \leq |V|$, where R is the largest integer for which there is $S \subset V$, $|S| = R$ and G/S has no odd cycle.

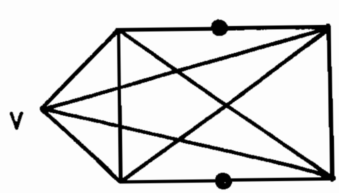
In particular, G is a 3-T-graph, if G is not a complete graph.

Conjecture 2. Every planar graph with $2 \leq \deg(v)$ for every vertex v, is a T-graph.

Conjecture 3. Hamiltonian graphs are T-graphs.

Conjecture 4. If $2 \leq \deg v \leq 4$ for every vertex of a graph G. Then G is a T-graph.

J.A. Bondy (private communication) gave the following example:



which shows that in Conjectures 3 and 4 we cannot have 1-T-graphs, since v cannot be included in the union of any 2DMSS.

REFERENCE

Berge, C. 1973. Graphs and hypergraphs. North-Holland.

(Received 15 April 1979)

بيانات ـ ت

آمنة الأسعد و فؤاد الملا
قسم الرياضيات بجامعة الكويت

خلاصة

يسمى البيان بيان ـ ت إذا احتوى على مجموعتين مستقلتين عظميين غير متقاطعتين . وفي هذا البحث نقدم مفاهيم بيان ـ ت ـ ر وبيان ـ ت ـ حد حيث ر عدد صحيح موجب . إن العبارات الآتية صحيحة :

بيان ـ ت ـ ٢ ⟵ بيان ـ ت ـ حد ⟵ ..

بيان ـ ت ـ ١ ⟵ بيان ـ ت

ولكن عكس كل من هذه العبارات غير صحيح .

إن أهم نتائج هذا البحث هي النظرية القائلة بأنه إذا كانت درجة كل رأس من رؤوس البيان بين ٢ ، ٣ فإن البيان هو بيان ـ ت ـ ٢ .