J. Univ. Kuwait (Sci.) **12,** 1985

Methods of determination of the proper regional gravity from a Bouguer anomaly profile

E. ABDELRAHMAN*, A. YEHIA† AND Y. AMIN*

Departments of Geophysics and Mathematics†, Faculty of Science, Cairo University, Giza, Egypt*

ABSTRACT

The present paper confirms the basic idea of several previous authors on the usefulness of polynomials and least squares cracovian computation in estimating the regional gravity field from Bouguer anomaly profiles. Inverses for construction of formulas for the regional effect, computed on the assumption that the regional field may be expressed either by a straight line or by a quadratic curve, are given.

In gravimetric interpretation, it is important to know whether the least squares quadratic curve fits the observed gravity data better than the least squares line. For this reason, two tests are introduced. These are: (1) the coefficient of determination test, and (2) the *F*-test. The coefficient of determination test and the test-function have been applied to three Bouguer anomaly profiles from the northern region of the Western Desert of Egypt to study the behaviour of the regional field along them.

INTRODUCTION

The least squares method was used in estimating the regional gravity field from Bouguer anomaly maps by many authors, e.g. Agocs (1951), Simpson (1954), Fajklewicz (1959) and Abdelrahman & Amin (1982). However, it is usually required to obtain the regional field in one dimension along a profile (x-axis). When the value of the regional data is subtracted from the observed data at any point on the profile, the residual gravity anomaly value at that point is obtained.

According to Fajklewicz (1959), Paul (1967) and Abdelrahman & Amin (1982), using polynomials of higher order than the second for fitting the observed data may lead to incorrect mapping of the regional and residual fields. This is because any higher order representation of the regional field cannot simply be considered in order to avoid any possible inclusion of parts of the residual field into the regional picture (Paul 1967). For this reason, the regional field would be either a straight line or a quadratic curve when we fit the observed data on a profile by the least squares method.

In gravity prospecting, it is very important to know whether the least squares quadratic curve fits the observed data better than the least squares line. It is the aim of the present paper to choose the proper order of the regional field by the use of simple statistical methods. In this work, the method of least squares in cracovian form is used to yield the results very rapidly.

FITTING A STRAIGHT LINE BY LEAST SQUARES

Assume n pairs of observations (x_0, g_0), (x_1, g_1), . . . (x_{n-1}, g_{n-1}), where g_n are the observed gravity values interpolated at equidistant points x_i (starting from $x = 0$) along a profile from the Bouguer anomaly map. Our problem is to choose values for a_1 and a_0 so that the line $z = a_1x + a_0$ fits these observed gravity data, where z is defined as the regional gravity field.

By using differential calculus, it can be shown that the values of a_1 and a_0 can be obtained by solving the normal equations, which can be written in the cracovian form (see Fajklewicz 1959) as follows:

$$\begin{bmatrix} a_1 \\ a_0 \end{bmatrix} t \begin{bmatrix} \sum x^2 & \sum x \\ \sum x & \sum x^0 \end{bmatrix} = \begin{bmatrix} \sum gx \\ \sum g \end{bmatrix} , \qquad (1)$$

where $\sum gx$ and $\sum g$ are called the free terms and denoted by L. $\sum x^2$, $\sum x$ and $\sum x^0$ are called the elements of coefficients a_1 and a_0. t denotes the so-called unit cracovian.

Also, Equation 1 may be written in matrix notation as follows:

$$a = L.W^{-1} . \qquad (2)$$

Equation 2 is used to obtain the unknowns a_1 and a_0 immediately when the inverse of the cracovian of coefficients is computed. Tables 1–10 give the inverses W^{-1} computed for several profiles (5, 10, 15, . . . 45 and 50) by the Gaussian elimination method with full pivoting after computing the elements of the coefficients for each profile using an ICL computer, Type 1905E.

Once the inverse of W and the cracovian free terms have been obtained, the construction of the formula describing the distribution and values of the regional effect along a profile takes only minutes.

If IW^{-1} and IIW^{-1} denote the various columns of these elements in the respective inverse, the unknowns a_1 and a_0 are rapidly obtained by forming the sum of the products

$$a_1 = L.IW^{-1} \quad \text{and} \quad a_0 = L.IIW^{-1} . \qquad (3)$$

The last column of the respective inverse, denoted by C, is also of use in checking the numerical value of the coefficients a_1 and a_0, since the sum of the unknown

Table 1. Inverse of W for $N = 5$

I	II	C
0·099999999999	−0·199999999997	−0·099999999998
−0·199999999997	0·599999999991	0·399999999994

Table 2. Inverse of W for $N = 10$

I	II	C
0·012121212121	−0·054545454544	−0·042424242424
−0·054545454544	0·345454545452	0·290909090905

Table 3. Inverse of W for $N = 15$

I	II	C
0·003571428571	−0·025000000000	−0·021428571429
−0·025000000000	0·241666666669	0·216666666667

Table 4. Inverse of W for $N = 20$

I	II	C
0·001503759399	−0·014285714286	−0·012781954887
−0·014285714286	0·185714285719	0·171428571432

Table 5. Inverse of W for $N = 25$

I	II	C
0·000769230769	−0·009230769231	−0·008461538462
−0·009230769231	0·150769230773	0·141538461541

Table 6. Inverse of W for $N = 30$

I	II	C
0·000444938821	−0·006451612903	−0·006006674082
−0·006451612903	0·126881720429	0·120430107526

Table 7. Inverse of W for $N = 35$

I	II	C
0·000280112045	−0·004761904762	−0·004481792717
−0·004761904762	0·109523809522	0·104761904760

Table 8. Inverse of W for $N = 40$

I	II	C
0·000187617261	−0·003658536585	−0·003470919325
−0·003658536585	0·096341463417	0·092682926831

Table 9. Inverse of W for $N = 45$

I	II	C
0·000131752306	−0·002898550725	−0·002766798419
−0·002898550725	0·085990338165	0·083091787440

Table 10. Inverse of W for $N = 50$

I	II	C
0·000096038415	−0·002352941176	−0·002256902761
−0·002352941176	0·077647058823	0·075294117646

coefficients and the cracovian C and L are related as follows:

$$\sum a = L.C \quad . \tag{4}$$

FITTING A QUADRATIC CURVE BY LEAST SQUARES

A convenient type of curve to use is the quadratic (also called parabolic) curve which has the formula

$$z = a_2x^2 + a_1x + a_0 \quad . \tag{5}$$

Again, by using differential calculus and putting in the cracovian form, it can be shown that the coefficients a_2, a_1 and a_0 are obtained by solving the equations

$$\begin{bmatrix} a_2 \\ a_1 \\ a_0 \end{bmatrix} t \begin{bmatrix} \sum x^4 & \sum x^3 & \sum x^2 \\ \sum x^3 & \sum x^2 & \sum x \\ \sum x^2 & \sum x & \sum x^0 \end{bmatrix} = \begin{bmatrix} \sum gx^2 \\ \sum gx \\ \sum g \end{bmatrix} \quad . \tag{6}$$

In matrix notation, Equation 6 is similar to Equation 2.

The inverses are computed for several profiles and the results are shown in Tables 11–20. The coefficients can be obtained directly from the following equations:

$$a_2 = L.IW^{-1} \qquad a_1 = L.IIW^{-1} \qquad a_0 = L.IIIW^{-1} \tag{7}$$

and the last column is also of use for checking the numerical values of the coefficients from the equation

$$\sum a = L.C \quad . \tag{8}$$

Table 11. Inverse of W for $N = 5$

I	II	III	C
0·071428571414	−0·285714285656	0·142857142826	−0·071428571417
−0·285714285652	1·242857142614	−0·771428571301	0·185714285653
0·142857142822	−0·771428571301	0·885714285658	0·257142857183

Table 12. Inverse of W for $N = 10$

I	II	III	C
0·001893939394	−0·017045454549	0·022727272733	0·007575757578
−0·017045454549	0·165530303067	−0·259090909152	−0·110606060634
0·022727272733	−0·259090909152	0·618181818281	0·381818181861

Table 13. Inverse of W for $N = 15$

I	II	III	C
0·000242404654	−0·003393665158	0·007352941175	0·004201680672
−0·003393665158	0·051082740781	−0·127941176457	−0·080252100835
0·007352941175	−0·127941176457	0·464705882332	0·344117647048

Table 14. Inverse of W for $N = 20$

I	II	III	C
0·000056960583	−0·001082251082	0·003246753247	0·002221462748
−0·001082251082	0·022066529966	−0·075974025989	−0·054989747106
0·003246753247	−0·075974025989	0·370779220833	0·298051948092

Table 15. Inverse of W for $N = 25$

I	II	III	C
0·000018580453	−0·000445930881	0·001709401709	0·001282051281
−0·000445930881	0·011471571922	−0·050256410237	−0·039230769212
0·001709401708	−0·050256410233	0·308034187939	0·259487179410

Table 16. Inverse of W for $N = 30$

I	II	III	C
0·000007448753	−0·000216013825	0·001008064517	0·000799499444
−0·000216013825	0·006709339746	−0·035685483890	−0·029192157969
0·001008064517	−0·035685483890	0·263306451710	0·228629032344

Table 17. Inverse of W for $N = 35$

I	II	III	C
0·000003441180	−0·000117000117	0·000643500644	0·000529941706
−0·000117000117	0·004258116023	−0·026640926641	−0·022499810736
0·000643500643	−0·026640926641	0·229858429862	0·203861003864

Table 18. Inverse of W for $N = 40$

I	II	III	C
0·000001763320	−0·000068769485	0·000435540070	0·000368533905
−0·000068769485	0·002869627164	−0·020644599307	−0·017843741628
0·000435540070	−0·020644599307	0·203919860658	0·183710801421

Table 19. Inverse of W for $N = 45$

I	II	III	C
0·000000977875	−0·000043026483	0·000308356460	0·000266307852
−0·000043026483	0·002024917550	−0·016466234973	−0·014484343907
0·000308356460	−0·016466234973	0·183225408620	0·167067530107

Table 20. Inverse of W for $N = 50$

I	II	III	C
0·000000577154	−0·000028280543	0·000226244344	0·000198540955
−0·000028280543	0·001481785022	−0·013438914031	−0·011985409551
0·000226244344	−0·013438914030	0·166334841655	0·153122171967

ASSESSING THE FIT BY THE COEFFICIENTS OF DETERMINATION

In gravity prospecting, we usually wish to know whether the least squares quadratic curve fits the observed data better than the least squares line. This may be done as follows:

We call $\Sigma\,(g_i - \bar{g})^2$ the total variations in the gravity data along the considered profile, whose g_i are the Bouguer values and $\bar{g}$ is as usual, the mean gravity value (Fig. 1), i.e.

$$\bar{g} = \frac{1}{n}(g_0 + g_1 + \ldots + g_{n-1}) \quad . \tag{9}$$

It can be proved that with least squares lines or least squares curves (of whatever type),

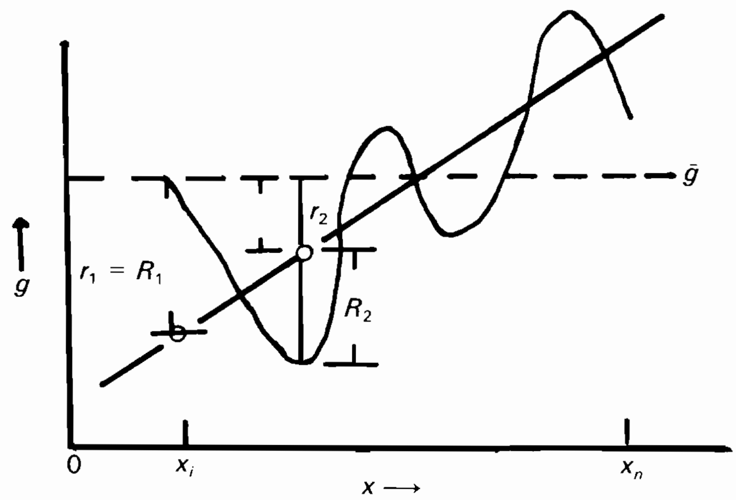

Fig. 1. Diagram showing the residuals $R_1, R_2, \ldots$ and the definition of $r_1, r_2, \ldots$ when a line is fitted to gravity data; $\bar{g}$ represents the mean gravity value.

$$\begin{aligned}\sum (g_i - \bar{g})^2 &= \sum (g_i - z_i)^2 + \sum (z_i - \bar{g})^2 \\ &= \sum R_i^2 + \sum r_i^2 \end{aligned} \tag{10}$$

where R_i is the residual anomaly value at the point x_i; $\Sigma\ R^2$ is called the unexplained variation, r_i the difference between the regional least squares value and the average gravity at any point x_i along the profile, and $\Sigma\ r^2$ the explained variation. The ratio of the explained variation to the total variation, i.e. $\Sigma\ (z_i - \bar{g})^2/\Sigma(g_i - \bar{g})^2$ is called the coefficient of determination and it acts as a measure of the goodness of fit.

If with a given set of gravity data, the coefficient of determination for the least squares quadratic curve is high whereas that for the least squares line is much lower, we consider that the quadratic curve fits the observed gravity data better than the straight line. However, if the two coefficients are not very dissimilar, a more accurate test is needed. This we now discuss.

TEST FOR IMPROVEMENT OF A QUADRATIC CURVE OVER A LINE

Another test may be used to find out whether the observed variances of two samples differ significantly. It has a wide range of application in normal sampling theory and is known as the test-function or *F*-test. This *F*-test can also be applied to detect whether the quadratic curve fits the observed data better. This can be done as follows:

We calculate the ratio

$$F = (2S_1^2 - s_1^2)/S_2^2 \tag{11}$$

where: $S_1^2 = \Sigma\ (Z_i - \bar{g})^2/2$ is the explained variation computed from fitting a quadratic curve; $s_1^2 = \Sigma\ (z_i - \bar{g})^2$ is the explained variation computed from fitting a straight line; $S_2^2 = \Sigma\ (g_i - Z_i)^2/(n - 3)$ is the unexplained variation computed from fitting a quadratic curve divided by $n - 3$; n is the number of the observed gravity points used in calculation along a considered profile; Z_i is the least squares regional gravity value obtained from fitting a quadratic curve; and z_i is the least squares regional gravity value obtained from fitting a straight line.

We then compare the computed F values with the critical F_c values (see Mack 1966). If the computed F is greater than the critical F_c ($v_1 = 1$, $v_2 = n - 3$) we are confident that the quadratic curve fits better. We should know that the critical values for F_c depend on (Mack 1966): (a) the probability level (5% or 1%), (2) the degrees of freedom v_1 with which s_1^2 is calculated (in our case $v_1 = 1$) and (3) the degrees of freedom v_2 of the smaller variance s_2^2 (in our case $v_2 = n - 3$).

EXAMPLES

The coefficient of determination test and the F-test were applied to three parallel gravity profiles interpolated from the Bouguer anomaly map of the northern part of the Western Desert of Egypt at a scale 1 : 500,000 and a contour interval 1 mgal. These profiles (AA', BB' and CC') coincide with longitudes 26°00′E, 27°00′E and 28°00′E respectively as shown in Fig. 2. The length of each profile is 220 km with 45 points of calculations. Results of computations are shown in Figs 3, 4 & 5 for AA', BB', and CC' profiles respectively.

Results indicate that both of the coefficients of determination computed for each

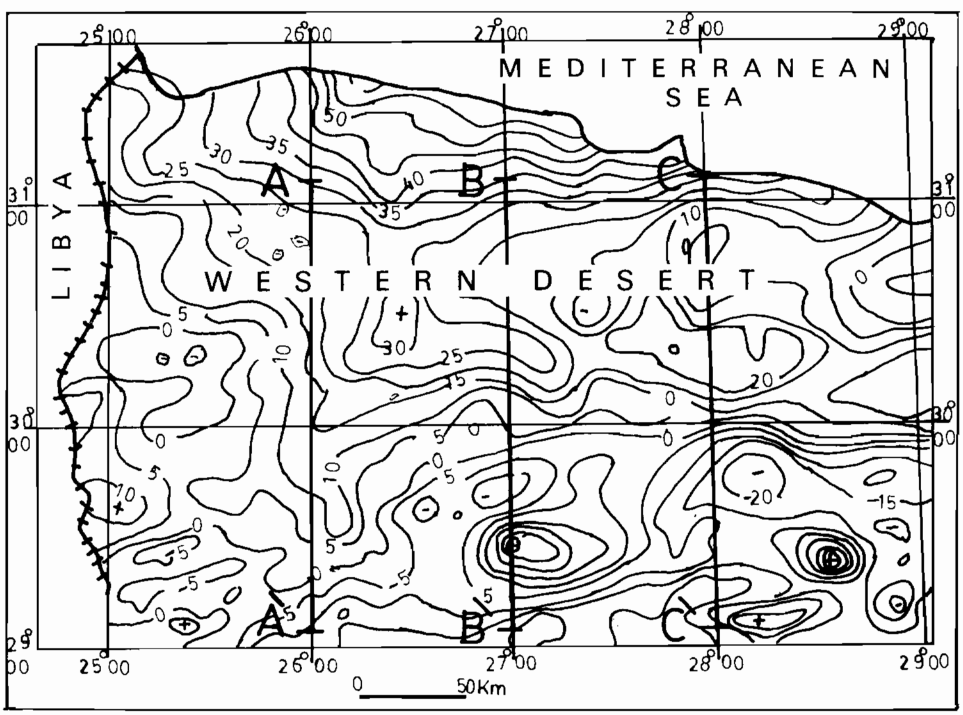

Fig. 2. Bouguer anomaly map of the northern region of the Western Desert of Egypt showing the location of AA', BB', and CC' profiles. Because of technical difficulties, the present map is replotted with 5 mgal contour interval.

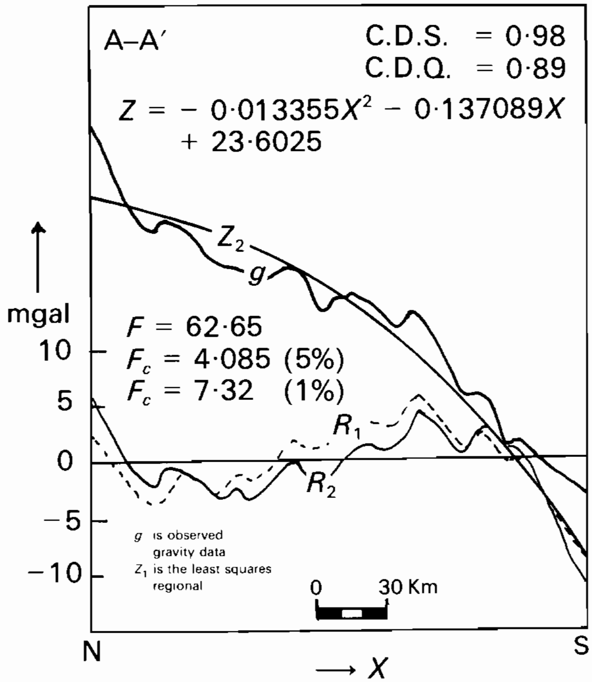

Fig. 3. Schematic curves illustrating the least squares fit technique along the profile *AA′*. C.D.S. and C.D.Q. are the coefficients of determination for least squares line and quadratic curve respectively. R_1 is the first residual while R_2 is the proper residual. The value of F (test-function) indicates that the least squares quadratic curve fits the observed data better than the least squares line.

profile are not much dissimilar and for this reason, the results of the F-test will be only taken into consideration. The analysis of the results along these three profiles indicates the following points which may be of interest:

(1) The gravity data along profile *AA′* (Fig. 3) fits the quadratic curve much better than the straight line at both probability levels 5% and 1%. Accordingly, the second order residuals only must be used for interpreting the subsurface geological conditions along this profile. Clearly, there are differences in both magnitudes and the centres of the anomalies between the residuals from first order and second order fitting which are computed along the profile *AA′*. Using the residuals obtained from first order fitting for interpretation, may cause serious errors.

(2) The gravity data along the profiles *BB′* and *CC′* (Figs 4 & 5) fits the straight line much better than the quadratic curve, at both probability levels 5% and 1%. Accordingly, the first order residual anomalies are the most suitable ones for interpretation.

(3) The relief of any anomaly computed directly from the Bouguer anomaly profile is either smaller or larger than that obtained from the proper residual anomaly profile. This result is very important, since it affects the quantitative interpretation of a subsurface structure along a profile. Only the relief obtained from the proper residual profile should be considered.

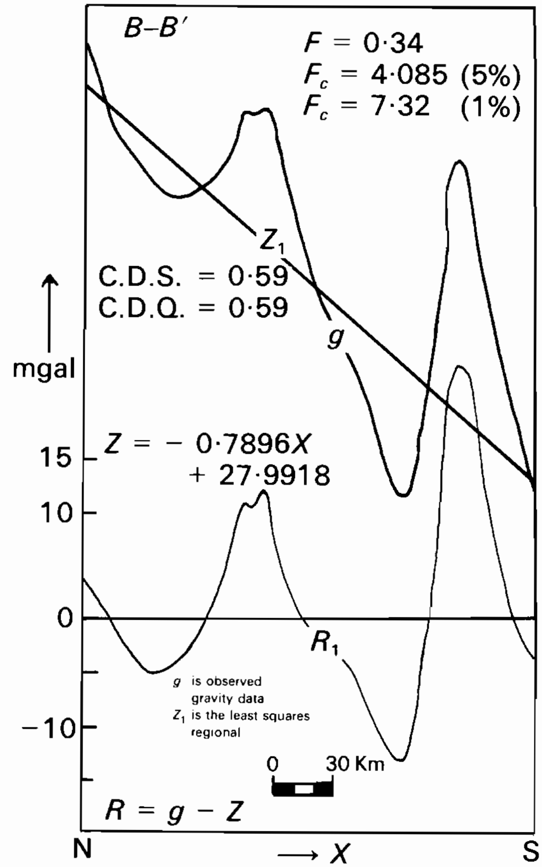

Fig. 4. Schematic curves illustrating the least squares fit technique along the profile *BB′*. C.D.S. and C.D.Q. are the coefficients of determination for least squares line and quadratic curve respectively. The value of *F* (test-function) indicates that the least squares straight line fits the observed data better than the least squares quadratic curve.

(4) Comparing the residual gravity anomalies of Figs 4 & 5 along *BB′* and *CC′* profiles with those of Fig. 3 along profile *AA′*, it can be stated that the subsurface geological conditions seem to be very complicated from east to west in the northern part of the Western Desert of Egypt both in type and magnitude.

CONCLUSIONS

From the present study the following may be concluded:

(1) The present paper confirms the basic idea of several authors on the usefulness of polynomials and least squares cracovian computation in estimating the regional gravity field from Bouguer anomaly profiles. Inverses for construction of the formulas for the regional effect, computed on the assumption that the regional field may be expressed either by a straight line or a quadratic curve are given with accuracy of 10^{-11}. However, in gravimetric interpretation, it is important to know

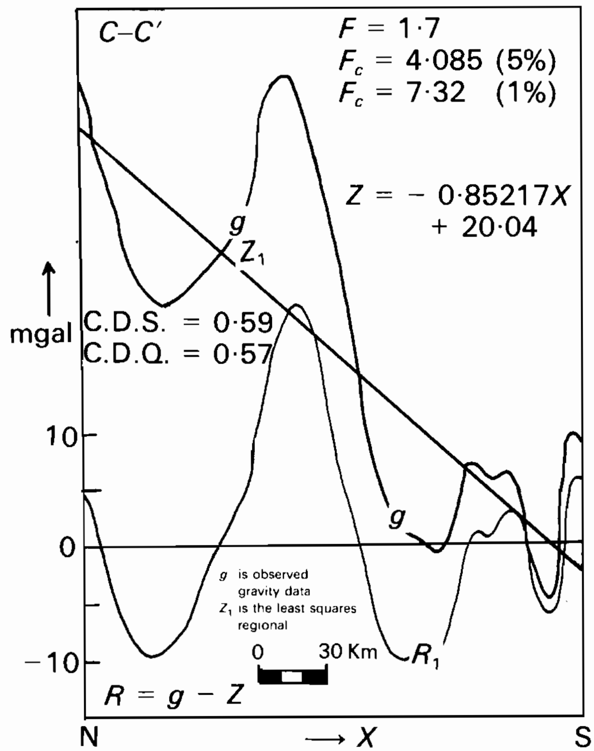

Fig. 5. Schematic curves illustrating the least squares fit technique along the profile *CC′*. C.D.S. and C.D.Q. are the coefficients of determination for least squares line and quadratic curve respectively. The value of *F* (test-function) indicates that the least squares line fits the observed data better than the least squares quadratic curve.

whether or not the least squares quadratic curve fits the observed gravity data better than the least squares line. For this reason, two tests are introduced. These are: (1) the coefficients of determination, and (2) the *F*-test. The *F*-test is more accurate.

(2) Experience shows that the computation of the numerical value of *F* depends mainly on: (a) personal error of interpolating the gravity data from maps, and (b) the errors involved in the different terms required to obtain the value of *F*. For these reasons and in order to obtain accurate result, it is recommended to: (1) use the actual corrected field values of the measured gravity or when these are not available, to graphically carry out the interpolation directly from the gravity profile for minimizing personal errors, and (2) use the values given in the inverses with the given accuracy and to carry out all other steps of computation with care so as to minimize the other errors.

ACKNOWLEDGEMENT

The authors would like to thank the authorities of the Faculty of Science, and the Scientific and Statistical Computer Centre, Cairo University, for their valuable aid in computing the inverses. We wish also to express our thanks to the Egyptian General Petroleum Corporation authorities for supplying the observed gravity map.

REFERENCES

Abdelrahman, E.M. & Amin, Y.A. 1982. On the use of cracovian computation in estimating regional gravity. Twentieth Annual Meeting. Geol. Soc. Egypt, Cairo.

Agocs, W. 1951. Least squares residual anomaly determination. Geophysics **16:** 686–96.

Fajklewicz, Z. 1959. The use of cracovian computation in estimating the regional gravity. Geophysics **15:** 465–78.

Mack, C. 1966. Essentials of statistics for scientists and technologists. Plenum, New York.

Paul, M. 1967. A method of computing residual anomalies from Bouguer map by applying relaxation technique. Geophysics **32:** 706–19.

Simpson, S.M. 1954. Least squares polynomial fitting to gravitational data and density plotting by digital computers. Geophysics **19:** 255–69.

(Received 5 April 1983, revised 7 November 1984)

طرق تعيين المجال التثاقلي الاقليمي المناسب
من قطاع شذوذ بوجير

السيد محمد عبد الرحمن
قسم الجيوفيزياء بكلية العلوم،
جامعة القاهرة ، الجيزة ،
ج ٠ م ٠ ع

عبد اللطيف يحيى
قسم الرياضيات بكلية العلوم،
جامعة القاهرة ، الجيزة ،
ج ٠ م ٠ ع

يحيى على امين
قسم الجيوفيزياء بكلية العلوم،
جامعة القاهرة ، الجيزة ،
ج ٠ م ٠ ع

خلاصة

يؤكد البحث الحالي ما جاء عن العديد من الباحثين من جدوى استخدام كثيرة الحدود والطريقة الكراكوفية لاقل المربعات في تقدير المجال التثاقلي الاقليمي من قطاع شذوذ بوجير . وقد تم حساب معكوس مصفوفات الاحداثيات المكانية على أساس ان المجال التثاقلي الاقليمي يمكن تمثيله بخط مستقيم أو بمنحنى من الدرجة الثانية .

وانه لمن المهم في طرق التفسير التثاقلي معرفة ما اذا كان منحنى الدرجة الثانية أفضل من الخط المستقيم أم لا . ولهذا السبب يتعين تطبيق نوعين من الاختبارات هما اختبار معامل التحديد واختبار ف . وقد تم تطبيق ذلك على ثلاثة قطاعات من شذوذ بوجير من شمال الصحراء الغربية بمصر لدراسة المجال الاقليمي التثاقلي على هذه القطاعات .