J. Univ. Kuwait (Sci.) **12,** 1985

Graphical representation of the geometric factor for bevel gears with back shoulders

E. M. AL-SHAREEDAH

Department of Mechanical Engineering, University of Kuwait, P. O. Box 5969, Kuwait

ABSTRACT

The strength of straight teeth bevel gears with back shoulders has been shown to be greater than the conventional cut and straight bevel gears. The strength is due to the additional support which exists on the back of their teeth. The equation derived for predicting the strength of this new type of gear contains a geometric factor which involves the basic geometric quantities of these gears. Plots of the geometric factor for a wide range of gear are presented and the interrelationship of the diametral pitch with the geometric factor has been demonstrated.

NOMENCLATURE

a, b	=	Length and width of virtual plate
C	=	Cone dimension of the gear.
D	=	Pitch diameter.
f	=	Constant, defined in Equation 3.
F	=	Face width.
G	=	Constant, defined in Equation 2.
GF	=	The geometric factor defined in Equation 5.
h_p	=	Thickness of the virtual plate.
N_g	=	Number of teeth in a gear.
N_p	=	Number of teeth in a pinion.
P	=	Diametral pitch.
P_0	=	Total load in units of force.
R_1, R_2	=	Radii, defined in Fig. 1.
γ	=	Pitch angle.

INTRODUCTION

Bevel gears are the most efficient means of transmitting rotation between angularly disposed shafts. Power requirements in machines involving bevel gears may be in thousands of horsepower and in many applications they have been successfully operated at speeds exceeding 20,000 rpm.

Straight bevels are the oldest, simplest and still the most widely used. The most recent development regarding these straight bevel gears is in the area of materials and relates to the type of process used when manufacturing these gears (Spotts 1971). Forging gears is one such process, where a precise billet is cut, heated to a specific forging temperature and then formed and coined between two dies. At that temperature and with the pressure of the two dies, the material can flow from within the dies, leaving a back shoulder as a result of this forging operation (Al-Shareedah & Lehnhoff 1985). The back shoulder can also be obtained through standard gear cutting operations. Fig. 1 shows some differences between straight teeth bevel gears, with and without back shoulder.

Analysis of straight bevel gears with back shoulders was performed and an interesting plate model and plate formula were obtained (Al-Shareedah & Lehnhoff 1985). The gear tooth for this type of bevel gear was assumed to resemble a plate rigidly supported at two adjacent edges and analyzed in a similar manner to that initially performed by Lewis (Spotts 1971; Dudley 1962). The plate model was compared with a three dimensional finite element model in which a general purpose finite element program such as NASTRAN was used and a close relationship between the two models was obtained (Al-Shareedah & Lehnhoff 1985; NASA 1972). The finite element model is shown in Figs 2 and 3, where Fig. 2 shows a finite element computer-generated model and Fig. 3 shows only the surface elements of that model.

The three dimensional finite element model shown in Figs 2 and 3 indicates a lack of symmetry of the gear tooth. Since it was cumbersome to generate data for this problem manually, a mesh generation program was used. This program produces element data and coordinate data for each three dimensional element of the entire gear tooth. The quadrilateral element was chosen to be the unit of division of the gear tooth in this program, this choice being made from considerations of accuracy and generation simplicity. The generated elements are arranged in layers extending from the tip to the back of the tooth and numbered in such a way that they create less sparsenses of the matrix and, therefore, better accuracy.

The finite element model (Fig. 2) is the optimum model with respect to accuracy and the number of elements. The optimized model is achieved by reaching a point at which no appreciable changes in stresses or deflections are apparent when the number of elements is increased. The particular flexibility of the element generator program,

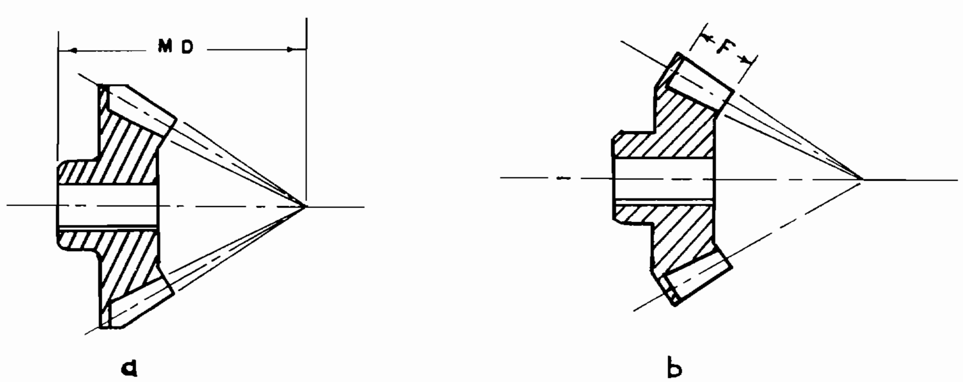

Fig. 1. Bevel gears (a) Machine cut (b) Forged or machine cut with back shoulder.

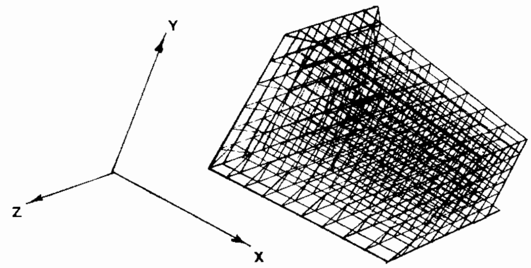

Fig. 2. Computer-generated finite element gear teeth.

where continuous subdivision and comparison of element factors are readily obtainable, made this optimized mesh easy to achieve.

It has been found, from previous investigations, that a bevel gear with a back shoulder resembles a plate supported rigidly at two adjacent edges (Al-Shareedah & Lehnhoff 1985). Due to this resemblance, the following stress design equation was obtained:

$$\sigma = \left(\frac{12P_0}{h_p^2}\right)\left(\frac{\pi}{2b}\right)^2\left(\frac{G}{f}\right) \tag{1}$$

where the dimensionless loading constant

$$G = 0{\cdot}132045 \tag{2}$$

and

$$\begin{aligned} f = {} & 3\left(\frac{\pi}{2a}\right)^4\left(\frac{a}{2}\right)\left(\frac{b}{2}\right) - 2\left(\frac{a}{2}\right)\left(\frac{\pi}{2a}\right)^4\left(\frac{2b}{\pi}\right) + 2\nu\left(\frac{\pi}{2a}\right)\left(\frac{\pi}{2b}\right) \\ & - 2\nu\left(\frac{a}{2}\right)\left(\frac{\pi}{2a}\right)^2\left(\frac{\pi}{2b}\right) - 2\nu\left(\frac{b}{2}\right)\left(\frac{\pi}{2a}\right)\left(\frac{\pi}{2b}\right)^2 \\ & + 3\left(\frac{\pi}{2b}\right)^4\left(\frac{a}{2}\right)\left(\frac{b}{2}\right) - 2\left(\frac{b}{2}\right)\left(\frac{\pi}{2b}\right)^4\left(\frac{2a}{\pi}\right) \\ & + 2\left(\frac{\pi}{2a}\right)^2\left(\frac{\pi}{2b}\right)^2\left(\frac{a}{2}\right)\left(\frac{b}{2}\right) . \end{aligned} \tag{3}$$

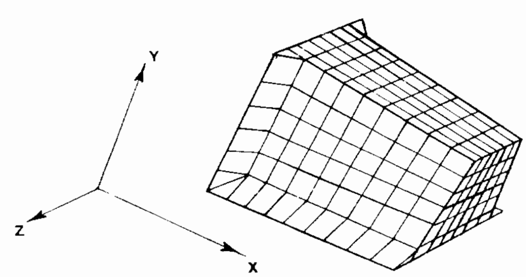

Fig. 3. Surface elements of a gear tooth.

Equation 1 is then reduced to

$$\sigma = \left(\frac{12P_0}{h_p^2}\right) GF \tag{4}$$

where the dimensionless constant GF is given by

$$GF = \left(\frac{\pi}{2b}\right)^2 \frac{G}{f} \quad . \tag{5}$$

Since the geometric factor of Equation 5 involves several variables with complicated terms, an electronic computer was used to calculate and generate tables of this geometric factor for several diametral pitch gears with different sizes (Al-Shareedah & Lehnhoff 1984). The purpose of this paper is to carry the analysis of the geometric factor further and to generalize the application of the geometric factor to include a wide range of gear sizes, so that the information obtained can be presented graphically.

ANALYSIS

In order to calculate the dimensionless geometric factor included in Equation 5, two basic quantities of the gear tooth must be obtained. These quantities, a and b, are contained in the factor and are given in terms of the dimensions of the gear by the following equation:

$$a = F \tag{6}$$

$$b = 2{\cdot}25 D_1 / N_g \tag{7}$$

where

$$D_1 = 2(C - F)/\cos\gamma \quad . \tag{8}$$

For best design practice, the face width F should not exceed 1/3 the cone distance C (Coleman 1962). Therefore

$$F = C/3 \quad . \tag{9}$$

Substituting (9) into (8), Equation 10 is then obtained

$$D_1 = 4F/\cos\gamma \tag{10}$$

where

$$\gamma = \tan^{-1}(N_g/N_p) \quad . \tag{11}$$

Substituting Equation 10 into Equation 7, the following expression is obtained:

$$b = 9a/(N_g \cos\gamma) \quad . \tag{12}$$

Since the diametral pitch P and the cone distance C are given by the following equations:

$$P = N_g/D \tag{13}$$

$$C = D/2 \sin\gamma \quad . \tag{14}$$

Thus, substituting Equations 13 and 14 into Equation 9, the following expression is obtained:

$$a = \frac{(N_g^2 + N_p^2)^{1/2}}{6P} \tag{15}$$

Also, Equation 15 can be used for substitution in Equation 12 to obtain the following:

$$b = \frac{3}{2P}\left(\frac{N_g^2 + N_p^2}{N_g N_p}\right) \tag{16}$$

The geometric factor can now be determined and substitution of Equations 15 and 16 into Expression 3, produces the geometric factor described in Equation 5 and becomes a function of several variables as follows:

$$GF = GF(N_g, \gamma, \upsilon) \quad . \tag{17}$$

Equation 17 indicates that the geometric factor GF is a function of the gear teeth number N_g, the gear pinion teeth ratio γ and Poisson's ratio υ, where these factors are basic gear parameters. It should be mentioned here that the diametral pitch P in Equation 15, when substituted to obtain the final geometric factor of Equation 17, cancels out from the entire expression, leaving that expression independent of the diametral pitch.

For such complicated expression, as seen from Equations 3 and 17, a computer was used to calculate the geometric factors for a wide range of gear ratios. A plot of Equation 17 for steel gears (Poisson's ratio = 0·3) is drawn in Fig. 4, showing the variation of the geometric factor with the gear ratios and with the number of gear teeth, N_g. These curves show a slowly decreasing slope when the number of gear teeth is increased. They cover a wide range of gear teeth numbers ranging from 15 to 40.

The shape of the curves indicated in Fig. 4 is clearly predicted by Equation 5, and therefore by Equation 3, where these equations show the geometric factor as a quadratic function of a and b. These variables, a and b in Equation 3, are described in terms of the basic gear parameters as in Equations 15 and 16. Therefore the

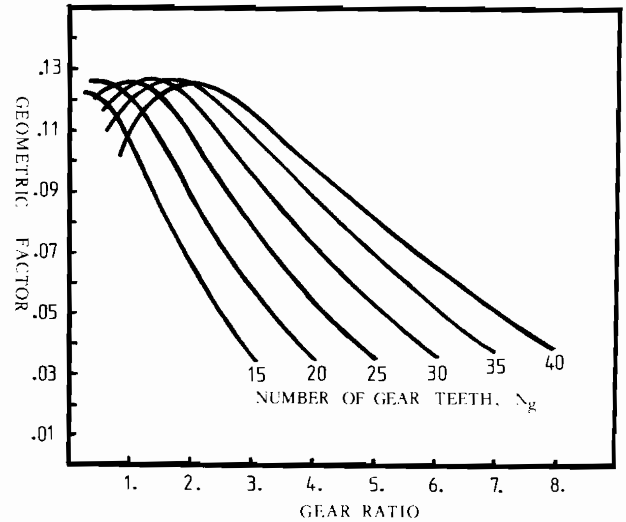

Fig. 4. The geometric factor for steel bevel gears with back shoulders.

behaviour of the geometric factor is a quadratic behaviour in terms of these basic parameters.

CONCLUSION

As seen from Equation 17, the geometric factor described in Equation 5 is an invariant function of the diametral pitch P of bevel gears with back shoulders. The geometric factor, which involves basic gear parameters, is calculated and demonstrated graphically by showing a family of curves involving the basic gear parameters. Through the graphical presentation of the geometric factor, it is now possible to design bevel gears with back shoulders by a more simple procedure.

REFERENCES

Al-Shareedah, E.M. & Lehnhoff, T.F. 1984. Calculation of the geometric factor using the plate formula for bevel gears with a back shoulder. Can. Trans. Mech. Eng. **8(3):** 138–41.

Al-Shareedah, E.M. & Lehnhoff, T.F. 1985. General plate formula for bevel gears with back shoulders. Can. Trans. Mech. Eng. **9(2):** 21–33.

Coleman, W. 1962. Design of bevel gears. McGraw-Hill, New York.

Dudley, D.W. 1962. Gear handbook. McGraw-Hill, New York.

NASA **1972.** NASTRAN, a general purpose finite element program initiated by NASA, and maintained by Langely Space Center, Va.

Spotts, M.F. 1971. Design of machine elements. Prentice Hall, N.J.

(Received 22 July 1984, revised 22 December 1984)

APPENDIX

SAMPLE CALCULATION 1

Let it be required to determine the strength of a 20 teeth, 16 diametral pitch standard bevel gear having a back shoulder, with gear pinion teeth ratio of 2:1 and an external load of 100 lb (45·4 kg). The tangential load component for standard gears will then be

$$P_0 = 100 \cos 20^\circ = 93{\cdot}97\,\text{lb} \quad (419\,\text{N})$$

From Equation 13

$$D = N_g/P = 20/16 = 1{\cdot}25'' \quad (0{\cdot}0318\,\text{m})$$

The thickness h_p in Equation 1 is given by the following:

$$h_p = 1{\cdot}4125D/(N_g \sin^2\gamma) = 0{\cdot}1113 \quad (0{\cdot}00283\,\text{m})$$

where

$$\gamma = \tan^{-1}(N_g/N_p) \quad .$$

From Fig. 4, the geometric factor for a 20 teeth gear having a gear pinion rate of 2:1 is

$$GF = 0{\cdot}09$$

Substituting into the stress Equation 5

$$\sigma = \frac{12P_0}{h_p^2} GF = 8192{\cdot}3\,\text{lb/in}^2 \quad (56{\cdot}486 \times 10^6\,\text{N/m}^2)$$

SAMPLE CALCULATION 2

For the gear described in Example 1, let the number of teeth be 26 instead of 20 with the same loading condition. Using the same procedure as above, the geometric factor *GF* can be found by linear interpolation

$$GF = 0{\cdot}111 \quad .$$

Therefore

$$\sigma = 10103{\cdot}7\,\text{lb/in}^2 \quad (69.67 \times 10^6\,\text{N/m}^2)$$

When using this criterion in design, a small safety factor should be included into the calculation procedure, thus producing a safe and more reliable design.

التمثيل البياني للعامل الهندسي للدواليب المسننة المخروطية ذات الاكتاف الخلفية

ابراهيم الشريدة
قسم الهندسة الميكانيكية بجامعة الكويت

خلاصة

اظهرت الدراسات السابقة ان قوة الدواليب المسننة المخروطية والمستقيمة ذوات الاكتاف الخلفية أكبر من قوة الدواليب المستقيمة العادية . وترجع هذه الزيادة في القوة الى الدعم الاضافي خلف هذه الدواليب .

إن المعادلة التي استخلصت لمعرفة قوى التحمل لهذا النوع من الدواليب المسننة ، تحتوي على عامل هندسي يتضمن الابعاد الهندسية الاساسية لهذه الدواليب . وفي هذه الدراسة تم عمل رسوم بيانية للعامل الهندسي لانواع متعددة من الدواليب المسننة المخروطية ، كما أوضحت الدراسة العلاقة بين الخطوة القطرية لهذه الدواليب والعامل الهندسي لها .