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Prediction of flexural cracking in eccentrically loaded reinforced concrete members

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ABSTRACT

This paper presents a method for evaluating, on accurate and reliable bases, the cracking moment capacity of reinforced concrete members subjected to eccentric loading. The method takes into consideration the actual contribution of the concrete in the tension and compression zones of the cross section as well as the contribution of the steel reinforcements in resisting the initiation of the first flexural cracks. The proposed approach is general and applies equally to all common shapes of cross sections, all types of flexural reinforcement, and all modes of eccentric loading that could initiate flexural cracking. The derivation of the relevant equations are shown and described in detail. A digital computer program, intended to permit the designer to solve the problem at a small fraction of the time needed when resorting to laborious longhand calculations, has also been developed. Appendix A presents the flowchart and the program BASIC listing. Appendix B provides recommended values of the analysis-involved mechanical properties pertaining to the concrete and the steel reinforcement. In addition, a pattern example is worked out in Appendix C, and a summary of the notation used is given in Appendix D.

INTRODUCTION

The prediction and control of cracking in the reinforced concrete structural members are essential for the sake of insuring adequate functional performance, reliable durability and aesthetic appearance of the structures at service. The assessment of the flexural cracking behaviour of the structures at the service loads is usually connected with the study of two stages: (1) incipient-cracking stage which signifies the initiation of the first flexural cracks, and (2) postcracking stage where the structural member develops controlled cracking.

The prediction of the initiation of cracking may be as important as the control of crack width. There are instances where cracks could seriously damage or invalidate the service function of particular structures such as reservoirs, liquid containers and pressure water pipes. In a water tank for example, the walls and the floor, being under hydrostatic pressure, are mainly subjected to bending moment and axial tension that may result in critical tensile stresses on the liquid face of these structural elements. The tensile stresses induced are apt to develop cracks through which the water can easily penetrate and attack the reinforcing steel. Unless special design and construction provisions are taken to avoid the occurrence of such cracks, the serviceability of the tank will most probably be impaired. For these reasons, liquid-retaining structures have to be designed watertight (Hilal 1971; Yerlici 1975; CEB-FIB 1978; ACI Committee 313 1978; ACI Committee 224 1980; Baikov & Sigalov 1981). This means that the designer should make certain that the cracking moment-capacity of a liquid-retaining concrete member (proportioned and reinforced for a required design strength) is not less than the external effect induced under service loading.

As a result of the recent trend toward the adoption of ultimate strength design coupled with the better understanding of the properties of concrete and the use of both high strength reinforcing steel and higher concrete strengths, the design of more slender and efficient structural members has become increasingly possible. Hence the deflection of the reinforced concrete flexural members under service loads is becoming a more pronounced controlling criterion. However, because of the variation in the extent of cracking along the span of the member, the deflection computations are based on an effective moment of inertia in which the cracking moment-capacity of the beam sections is incorporated (Branson 1963; Burns & Siess 1966; ACI Committee 318 1983).

It is worth noting that the least allowable amount of reinforcement for a beam (or min ρ) is controlled by the steel area that would develop a minimum flexural strength equal to the strength of the beam if it were unreinforced, that is, to the cracking capacity of the corresponding plain concrete section (ACI Committee 318 1983; Wang & Salmon 1985).

It can be added that the cracking moment-capacity is also involved in the evaluation of the nominal shear strength provided by the concrete for prestressed members when diagonal cracking results from a combined shear and bending moment (ACI Committee 318 1983).

It is important to point out that the ACI Code (ACI Committee 318 1983) recommends the evaluation of the cracking moment-capacity for flexural reinforced concrete members on the basis of a maximum tensile stress equal to the modulus of rupture of the concrete, and a moment of inertia related to the gross concrete section about its centroidal axis, neglecting the contribution of the transformed area of the reinforcement. It is well known that this procedure does not enable an accurate prediction of the cracking moment-capacity even for the case of simple bending wherein it leads to underestimated values for the following reasons:

1. The inclusion of reinforcement increases significantly the moment of inertia of the uncracked section, and consequently, the cracking moment could be increased by as much as 37% (Park & Paulay 1975). More typically, however, this increase in the value of the cracking moment is about 20% (Nawy 1985).
2. The value specified for the modulus of rupture for normal weight concrete is $0{\cdot}62\sqrt{f'_c}$ N/mm^2, where f'_c is the standard cylinder compressive strength. However, available experimental data show a large variability and randomness in the relationship between the modulus of rupture and the cylinder compressive strength (Mirza *et al.* 1979).
3. The modulus of rupture, computed from the classical flexural formula, gives higher values for the concrete tensile strength than the split cylinder test. The difference is primarily due to the stress distribution in the concrete of flexural beam being non-linear when tensile failure is imminent (Park & Paulay 1975; Wang & Salmon 1985).

4. The modulus of rupture is based on testing to failure plain concrete beams 152 by 152 mm in cross section, having a test span of 456 mm and loaded at the third points (ASTM Standards 1986). It is believed that inclusion of reinforcement, changes in the size and relative proportions of the beam, and variations in the pattern, type and rate of the applied loading will conspicuously affect the test results (Murashev *et al.* 1968; Houk *et al.* 1970; Houghton 1976).

By comparison, the objective of this work is to develop an analytical approach for predicting the formation of the flexural cracks on the basis of the true behaviour of the reinforced concrete section under eccentric loading. Hence the proposed method aims at: (1) accounting for the elastoplastic properties of the concrete in tension at first cracking, and (2) allowing for the actual cracking resistance contributed by the constituent materials of the critical cross section through using strain compatibility, applying real stress–strain relations, and satisfying the conditions of equilibrium.

COMPUTATION OF CRACKING LOAD OR CRACKING MOMENT

It has to be emphasized at the outset that a large number of tests on reinforced concrete members have verified the following basic assumptions:

1. At all stages of loading up to flexural failure the distribution of the longitudinal strain across a section is linear (Hognestad 1951; Park & Paulay 1975; ACI Committee 318 1983).
2. At the end of the precracking stage the compressed concrete continues to behave elastically (a linear stress–strain relationship), while the tension-zone concrete, being exhausted by inelastic tensile strains, follows a non-linear relation up to its ultimate tensile strength (Murashev *et al.* 1968; Park & Paulay 1975; Nawy 1985).
3. The tension steel and the compression steel are stressed and strained no matter how small or large the load is, thereby they both contribute to the cracking strength of the section (Branson 1963; Murashev *et al.* 1968).

Based on the above, it is sufficiently accurate for analysis purposes at just prior to cracking of the concrete to assume a triangular compressive stress distribution, and to adopt a rectangular (uniform) tensile stress distribution instead of the pronouncedly curved distribution.

As a general case, an asymmetrical *I*-section doubly reinforced with a tension steel, A_s, and a compression steel, A'_s, will be considered. The section is subjected to a tensile or compressive eccentric force, N_{cr}, with an eccentricity, e', measured from the compressed extreme fiber of the section as shown in Fig. 1. The figure also illustrates the dimensions, the stress- and strain-distributions, and the internal forces on the section when the flexural cracking strength is reached.

Fig. 2 presents a typical stress–strain curve for concrete loaded up to failure in both axial compression and tension.

When a concrete specimen is subjected to a short-duration axial loading, it undergoes a strain composed of elastic and plastic components. The initial tangent modulus of elasticity of concrete, E_c, refers only to the elastic strain produced by instantaneously applied loads. The secant modulus of elasticity of concrete, E'_c, describes the total strain in compression, and may be related to E_c as follows (see Fig. 2):

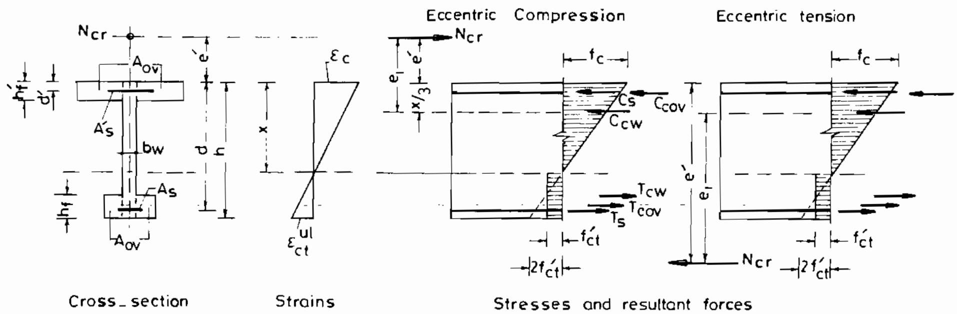

Fig. 1. Doubly reinforced flexural member with eccentric loading at cracking.

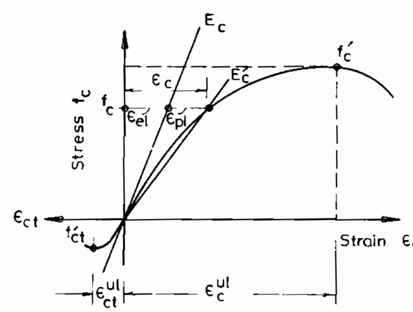

Fig. 2. Typical stress–strain diagram for concrete.

$$f_c = \varepsilon_c E'_c = \varepsilon_{el} E_c.$$

Thus,

$$E'_c = (\varepsilon_{el}/\varepsilon_c)E_c = \upsilon E_c. \quad (1)$$

The coefficient υ is equal to unity for elastic behaviour, and decreases with the increase of the stress–strength ratio and duration of loading. Similarly, under tension

$$E'_{ct} = \upsilon_t E_c. \quad (2)$$

When tensile stress approaches the direct tensile strength, f'_{ct}, the average experimental value of υ_t, as obtained from tests conducted at NIIZhB, is 0·5 (Murashev *et al.* 1968). Hence, the ultimate tensile strain in the concrete

$$\varepsilon_{ct}^{ul} = f'_{ct}/E'_{ct} = 2f'_{ct}/E_c. \quad (3)$$

Referring to the member of Fig. 1, and denoting the depth of the compression zone by x and the overall depth of the section by h, and making use of the similar triangles of the strain diagram, the strain in the concrete at the extreme compression fiber, ε_c, is given by

$$\varepsilon_c = \varepsilon_{ct}^{ul} x/(h - x).$$

By substituting from Eqn (3), the above expression for ε_c becomes

$$\varepsilon_c = (2f'_{ct}/E_c)x/(h - x). \quad (4)$$

Recalling that the concrete strain in the compression zone is mostly elastic at cracking (hence $E'_c = E_c$) and that $f_c = \varepsilon_c E'_c$, then Eqn (4) can be rewritten as

$$f_c = 2f'_{ct} x/(h - x). \quad (5)$$

Eqn (5) reveals that the line marking the stress distribution in the compression zone (Fig. 1), if produced downward, will meet the extreme fiber in tension at an imaginary stress of $2f'_{ct}$. Consequently, the steel stresses and the concrete compressive stresses at various points across the section immediately before the onset of cracking can always be expressed in terms of twice the concrete tensile strength, f'_{ct}. In effect, the stress in a steel bar (after allowance for the displaced concrete) may be obtained by the multiplication of $2(n-1)f'_{ct}$ by the ratio of the distance of the bar from the neutral axis to the distance $(h-x)$.

Based on the above conclusion, the cracking eccentric load, or cracking moment, for the general case can be found by the application of the equilibrium conditions in the manner outlined below. Taking moments of the forces in Fig. 1 about the line of action of the internal compressive force C_{cw} (contribution of the concrete of web in compression) gives

$$\begin{aligned} N_{cr}(e' + x/3) = f'_{ct}\{&b_w(h-x)(h/2 + x/6) + A_{ov}(h - h_f/2 - x/3) \\ &+ 2(n-1)A_s(d - x/3) + 2(n-1)A'_s(x-d')(x/3 - d')/(h-x) \\ &+ 2A'_{ov}(x - h'_f/2)(x/3 - h'_f/2)/(h-x)\} \end{aligned} \tag{6}$$

The appropriate sign for e' (distance between the eccentric load and the extreme compressed fiber of the section) has to be substituted into the above equation when it is used. The sign of the eccentricity, e', should be considered positive if the load lies above the section (nearer to the extreme fiber in compression), and considered negative if the load lies below the section (nearer to the extreme fiber in tension), or within the section. Equilibrium of the external and internal forces of Fig. 1 requires that

$$\begin{aligned} N_{cr} = {} & f'_{ct}b_w x^2/(h-x) + 2f'_{ct}A'_{ov}(x - h'_f/2)/(h-x) \\ & + 2(n-1)f'_{ct}A'_s(x-d')/(h-x) - f'_{ct}b_w(h-x) - f'_{ct}A_{ov} - 2(n-1)f'_{ct}A_s \end{aligned} \tag{7}$$

Equations (6) and (7) involve two unknowns, namely the cracking eccentric load, N_{cr}, and the depth of the compression zone of the section, x. Thus, in theory, N_{cr} is determined by simultaneous solution of the two equations. For cases of eccentric loading, an equivalent cracking moment, M_{cr}, may be computed as

$$M_{cr} = N_{cr}e \tag{8}$$

where e is the load eccentricity measured with respect to the geometric centroid of the cross section. The foregoing analytical procedure as demonstrated by Eqns (6) and (7) can also be used to suit prestressed concrete members. In such a case, all the terms (in Eqns (6) and (7)) reading $2(n-1)f'_{ct}A_s$ have to be replaced by $\{2(n-1)f'_{ct} + f_{se}\}A_{ps}$, where A_{ps} is the area of prestressed reinforcement in tension zone, and f_{se} is the effective prestress in the prestressed reinforcement (after allowance for prestress losses).

COMPUTER-AIDED ANALYSIS

In fact, a direct simultaneous solution of Eqns (6) and (7) for the unknowns N_{cr} and x is extremely difficult, if not almost always impossible. The rigorous mathematical difficulty can partially be eased by the adoption of successive approximations, that is, a trial-and-adjustment procedure, in which a convenient value for x is first selected and substituted in Eqn (7) to find N_{cr}, then Eqn (6) is checked accordingly, and the trial is repeated with modified x-values till Eqn (6) is finally satisfied. However, such a computational process is even laborious, time-consuming and impracticable without the aid of a computer. To fit more efficiently a computer-aided analysis, the variable x, appearing in Eqns (6) and (7), would have to be replaced with kh (i.e. $k = x/h$), and then the trial cycles conducted for electronically adjusted values of the introduced coefficient k (between 0·1 and 0·9).

To achieve the aforementioned purposes, a program that suits handheld or desktop personal computers is developed. The flowchart and the program steps are presented in Appendix A. By inputting the values of the materials properties f'_{ct} and n, of the geometrical properties A_{ov}, A'_{ov}, b_w, d, d', e', h, h_f, h'_f, and of the steel areas A_s and A'_s (see Fig. 1 and Notations (Appendix D)) for any given cross section subjected to eccentric loading, the program computes coefficient k and the cracking eccentric load, N_{cr}, and also prints out the latter. In case of simple bending, however, the program first computes the value of k that corresponds to $N_{cr} = 0$ (Eqn (7)). It then proceeds to calculate the cracking moment, M_{cr}, applying the right-hand side of Eqn (6). Note in this instance that Eqn (8) become inapplicable as N_{cr} is zero, and e is infinite.

The N_{cr} output will be positive when the section transmits eccentric compression, negative for an eccentric tension, and zero for a bending moment only.

Flexural cracks will not develop in a structural element, if, in case of eccentric loading (numerically)

$$N_{cr} \geqslant N, \tag{9}$$

or if, in case of simple (pure) bending,

$$M_{cr} \geqslant M. \tag{10}$$

It is understood that the previously outlined crack-prediction approach is valid for structural members that are likely to undergo tension-controlled failure. In this context, the program offers also the advantage of excluding from treatment eccentrically loaded members liable to fail in compression.

It is important to note at the end that a member under the combined action of an axial load, N, and a bending moment, M, is statically equivalent to the member loaded eccentrically by N. This concept of replacing the axial load and bending moment by a single eccentric load should be adhered to by the user of the program in order to input the appropriate value of e' (for example, $e' = M/N - h/2$ for a rectangular section). However, when N is a tension, its value is to be considered negative.

SUMMARY AND CONCLUSIONS

Reinforced concrete and prestressed concrete structural members subjected to eccentric loading and proportioned for adequate strength should be checked for the formation of flexural cracks, if they are destined to be water-tight. A method for predicting, on reasonable bases, the development of cracks in eccentrically loaded flexural members has been proposed. It applies to members having a wide variety of concrete cross-sectional shapes, reinforcement types and loading patterns. The method, in effect, implies a simultaneous solution of two complex, yet general, formulas (Eqns (6) and (7)). However, in almost all instances, the solution is only possible by using a computer program like the one developed and presented in Appendix A. The predicted cracking moment is shown to be mainly dependent on the concrete cross-sectional area, the ultimate tensile strain of concrete and its initial modulus of elasticity, the overall depth of the member, and the amounts of the tension and compression reinforcements.

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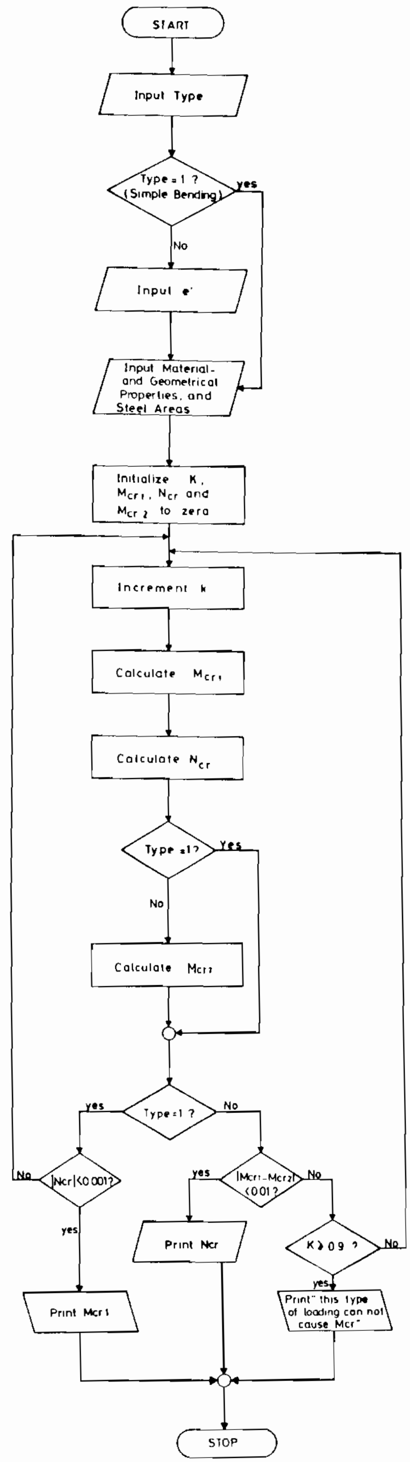
APPENDIX A—FLOWCHART AND PROGRAM FOR COMPUTING N_{cr} OR M_{cr}



```
10 REM ******************  MCRC.BAS ****************************
20 DIM C(15),C2(15):CLS:KEY OFF
30 PRINT "-INPUT-":INPUT "SIMPLE BENDING (Yes/No) ",DESI$:TYPE$=LEFT$(DESI$,1)
40 IF (TYPE$="N") OR (TYPE$="n") THEN TYPE=0:INPUT "e'  =",E:GOTO 60
50 IF (TYPE$="Y") OR (TYPE$="y") THEN TYPE=1
60 INPUT "h   =",H:INPUT "d   =",D:INPUT "hf' =",HFP:INPUT "hf  =",HF:INPUT "bw
 =",BW
70 INPUT "Aov'=",AOVP:INPUT "Aov =",AOV:INPUT "d'  =",DP:INPUT "As  =",AS
80 INPUT "As' =",ASP:INPUT "fc' =",FC:INPUT "fct'=",FCT:INPUT "n   =",NS
90 REM ***** Ist LOOP   K FROM 0.1 TO 0.9 WITH INCREMENT 0.1 *****
100 K=0:M=0:Q=0:MCR1=0:MCR2=0:NCR=0
110 FOR I=1  TO 9
120 K=K+.1
130 MCR1 =.000001*(FCT*(BW*(H-K*H)*(H/2+K*H/6)+AOV*(H-HF/2-K*H/3)+2*(NS-1)*AS*(D
-K*H/3)+2*(NS-1)*ASP*(K*H-DP)*(K*H/3-DP)/(H-K*H)+2*AOVP*(K*H-HFP/2)*(K*H/3-HFP/2
)/(H-K*H)))
140 NCR =.001*(FCT*BW*(K^2)*(H^2)/(H-K*H)+2*FCT*AOVP*(K*H-HFP/2)/(H-K*H)+2*(NS-1
)*FCT*ASP*(K*H-DP)/(H-K*H)-FCT*BW*(H-K*H)-FCT*AOV-2*(NS-1)*FCT*AS)
150 IF TYPE=1 THEN 180
160 MCR2 =.001*(NCR *(E+K*H/3))
170 IF MCR1 > MCR2    THEN C(I)=1 ELSE C(I) =2:GOTO 190
180 IF NCR  > 0       THEN C2(I)=1 ELSE C2(I)=2
190 IF I=1  THEN 230
200 IF TYPE=1 THEN 220
210 IF C(I) <> C (I-1)  THEN M=K*100:GOTO 230
220 IF C2(I) <> C2 (I-1)  THEN Q=K*100
230 NEXT I
240 IF TYPE=1 THEN 420
250 IF M=0 THEN 410
260 REM ************* CALCULATION OF K FOR Mcr1 = Mcr2 **************
270 PIT=.01:PITSH=10:INC=1:RO=0
280 K=((M-PITSH)/100)-PIT
290 FOR IJ=M-PITSH TO M STEP INC
300 RO=RO+1 :K=K+PIT
310 MCR1 =.000001*(FCT*(BW*(H-K*H)*(H/2+K*H/6)+AOV*(H-HF/2-K*H/3)+2*(NS-1)*AS*(D
-K*H/3)+2*(NS-1)*ASP*(K*H-DP)*(K*H/3-DP)/(H-K*H)+2*AOVP*(K*H-HFP/2)*(K*H/3-HFP/2
)/(H-K*H)))
320 NCR =.001*(FCT*BW*(K^2)*(H^2)/(H-K*H)+2*FCT*AOVP*(K*H-HFP/2)/(H-K*H)+2*(NS-1
)*FCT*ASP*(K*H-DP)/(H-K*H)-FCT*BW*(H-K*H)-FCT*AOV-2*(NS-1)*FCT*AS)
330 MCR2 =.001*(NCR *(E+K*H/3))
340 IF  ABS(MCR1-MCR2) < .01  THEN MCRI=MCR1:KI=K:NI=NCR:GOTO 400
350 IF MCR1 > MCR2  THEN C(RO) =1 ELSE C(RO) =2
360 IF IJ=M-PITSH THEN 380
370 IF C(RO) <> C(RO-1)          THEN GOTO 390
380 NEXT IJ
390 M=IJ:PITSH=PITSH*.1 :PIT=PIT*.1 :INC=INC*.1:RO=0:GOTO 280
400 PRINT:PRINT "-OUTPUT-":PRINT USING" Ncr = £££££.£££";NI;:PRINT" kN":END
410 PRINT "THIS TYPE OF LOADING CAN NOT CAUSE Mcr " :END
420 REM *************** CALCULATION OF K FOR SIMPLE BENDING (Ncr=0) ***********
430 PIT=.01:PITSH=10:INC=1:RO=0 :NCR=0:MCR1=0
440 K=((Q-PITSH)/100)-PIT
450 FOR IK=Q-PITSH TO Q STEP INC
460 RO=RO+1 :K=K+PIT
470 NCR=.001*(FCT*BW*(K^2)*(H^2)/(H-K*H)+2*FCT*AOVP*(K*H-HFP/2)/(H-K*H)+2*(NS-1)
*FCT*ASP*(K*H-DP)/(H-K*H)-FCT*BW*(H-K*H)-FCT*AOV-2*(NS-1)*FCT*AS)
480 IF ABS(NCR)< .001 THEN KJ=K:NJ=NCR  :GOTO 540
490 IF NCR > 0 THEN C2(RO) =1 ELSE C2(RO)=2
500 IF IK=Q-PITSH THEN 520
510 IF C2(RO)<> C2(RO-1) THEN GOTO 530
520 NEXT IK
530 Q=IK:PITSH=PITSH*.1:PIT=PIT*.1:INC=INC*.1:RO=0:GOTO 440
540 MCRJ=.000001*(FCT*(BW*(H-K*H)*(H/2+K*H/6)+AOV*(H-HF/2-K*H/3)+2*(NS-1)*AS*(D-
K*H/3)+2*(NS-1)*ASP*(K*H-DP)*(K*H/3-DP)/(H-K*H)+2*AOVP*(K*H-HFP/2)*(K*H/3-HFP/2)
/(H-K*H)))
550 PRINT:PRINT "-OUTPUT-":PRINT USING" Mcr = £££££.£££";MCRJ;:PRINT" kN.m":END
```

APPENDIX B—RECOMMENDED PROVISIONS FOR E_c, E_s, f'_{ct} AND n

According to experimental data available from different European sources, the initial tangent modulus of elasticity for moist-cured concrete may be given by the empirical formula

$$E_c = w_c^{1 \cdot 5} 0{\cdot}86 f'_c/(27 + 1{\cdot}75 f'_c) \quad \text{N/mm}^2$$

where w_c is the density of concrete in kg/m^3, and f'_c is the specified cylinder compressive strength in N/mm^2. For normal-density concrete,

$$E_c = 95000 f'_c/(27 + 1{\cdot}75 f'_c).$$

The modulus of elasticity for all nonprestressed steel may be taken as

$$E_s = 200000 \quad N/mm^2.$$

The direct tensile strength for normal-weight concrete may be considered as

$$f'_{ct} = 0{\cdot}48\sqrt{f'_c} \quad \text{N/mm}^2.$$

Detailed values of these parameters in the usual strength range of $f'_c = 15$ to 40 N/mm^2 are given (slightly modified) in Table 1.

Table 1. Concrete compressive strength and the corresponding values of tensile strength, initial modulus of elasticity and modular ratio

f'_c (N/mm^2)	f'_{ct} (N/mm^2)	E_c (N/mm^2)	$n = E_s/E_c$
15	1·86	26700	7·49
20	2·15	30600	6·54
25	2·40	33500	5·97
30	2·63	35800	5·59
35	2·84	37600	5·32
40	3·04	39000	5·12

APPENDIX C—WORKED EXAMPLE

A doubly reinforced concrete beam with the cross-section shown in Fig. 3 has concrete of $f'_c = 25$ N/mm^2.

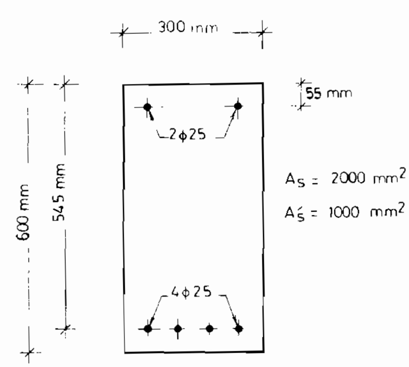


Fig. 3. Doubly reinforced beam cross section.

(1) Assuming the beam to carry only a bending moment, calculate the cracking moment, and compare the results with those obtained by using the ACI method and the transformed-section method.
(2) If the beam is to carry at service a bending moment of 80 kN·m and an axial compression of 160 kN, predict the crack-control situation.
(3) Re-work (2) if instead of an axial compression the beam transmits an axial tension.

Solution 1:

(a) Using the ACI method

$$f_r = 0{\cdot}62\sqrt{f'_c} = 0{\cdot}62\sqrt{25} = 3{\cdot}1 \quad \text{N/mm}^2,$$

$$\text{cross section } I_g = \tfrac{1}{12}(300)(600)^3 = 5{\cdot}4 \times 10^9 \quad \text{mm}^4,$$

$$y_t = \frac{h}{2} = 300 \quad \text{mm},$$

$$M_{cr} = \frac{f_r I_g}{y_t} = 55{\cdot}8 \quad \text{kN·m}.$$

(b) Using the transformed-section method

$$n = E_s/E'_c = 200000/5000\sqrt{25} = 8 \quad \text{(ACI 318-83M)},$$

$$y_t = \frac{300(600)(300) + (8-1)(1000)(545) + (8-1)(2000)(55)}{300(600) + (8-1)(2000+1000)} = 291{\cdot}47 \quad \text{mm},$$

$$\text{transformed section } I_{gt} = \tfrac{1}{12}(300)(600)^3 + 300(600)(8{\cdot}53)^2 + (8-1)(1000)(253{\cdot}53)^2$$
$$+ (8-1)(2000)(236{\cdot}47)^2$$
$$= 6645{\cdot}9 \times 10^6 \quad \text{mm}^4,$$

$$M_{cr} = \frac{f_r I_{gt}}{y_t} = 70{\cdot}684 \quad \text{kN·m}.$$

(c) Using the computer-aided method

```
-INPUT-
SIMPLE BENDING (Yes/No) yes
h    =600
d    =545
hf´  =0
hf   =0
bw   =300
Aov´=0
Aov  =0
d´   =55
As   =2000
As´  =1000
fc´  =25
fct´=2.4
n    =5.97
```
(see Table 1).

```
-OUTPUT-
 Mcr =     95.723 kN.m
```

Comparing the three values obtained for M_{cr}, the latter value exceeds the ACI- and the transformed section-value by about 71% and 35%, respectively.

Solution 2:

```
-INPUT-
SIMPLE BENDING (Yes/No) no
e´   =200 *
h    =600
d    =545
hf´  =0
hf   =0
bw   =300
Aov´=0
Aov  =0
d´   =55
As   =2000
As´  =1000
fc´  =25
fct´=2.4
n    =5.97

-OUTPUT-
 Ncr =    253.355 kN
```

Since N_{cr}(253·355 kN) > N(160 kN), cracks will not develop due to service loading.

Solution 3:

```
-INPUT-
SIMPLE BENDING (Yes/No) no
e´   =-800 †
h    =600
d    =545
hf´  =0
hf   =0
bw   =300
Aov´=0
Aov  =0
d´   =55
As   =2000
As´  =1000
fc´  =25
fct´=2.4
n    =5.97

-OUTPUT-
 Ncr =   -150.673 kN
```

Here, as N_{cr} is numerically smaller than N, cracking will occur at service.

---

* $\left(e' = \frac{M}{N} - \frac{h}{2} = \frac{80(1000)}{160} - \frac{600}{2} = 200\right)$.

† $\left\{e' = \frac{M}{N} - \frac{h}{2} = \frac{80(1000)}{(-160)} - \frac{600}{2} = -800\right\}$.

APPENDIX D—NOTATION

A_{ov} = cross-sectional area of flange overhangs in the tension zone
A'_{ov} = cross-sectional area of flange overhangs in the compression zone
A_{ps} = area of prestressed reinforcement in the tension zone
A_s = area of nonprestressed tension reinforcement
A'_s = area of compression reinforcement
b_w = width of web
C_{cw} = internal compressive force resisted by the web
d = distance from extreme compression fiber to centroid of tension reinforcement
d' = distance from extreme compression fiber to centroid of compression reinforcement
e = eccentricity of eccentric load from geometric centroid of the cross-section
e' = distance from applied eccentric load, N, to extreme compression fiber
E_c = initial tangent modulus of elasticity of concrete in compression and tension
E'_c = secant modulus of elasticity of concrete in compression
E'_{ct} = secant modulus of elasticity of concrete in tension
E_s = modulus of elasticity of steel
f_c = stress in concrete at extreme compression fiber
f'_c = specified cylinder compressive strength of concrete
f'_{ct} = direct tensile strength of concrete
f_{se} = effective prestress in prestressed reinforcement (after allowance for prestress losses)
h = overall depth of member
h_f = thickness of tension flange
h'_f = thickness of compression flange
k = ratio of depth of compression zone to overall depth of member $= x/h$
M = service moment on a section not subjected to axial load
M_{cr} = cracking moment capacity of cross-section
n = modular ratio of elasticity $= E_s/E_c$
N = service eccentric load normal to the cross-section
N_{cr} = cracking eccentric load normal to the cross-section
w_c = unit mass of concrete = 2300 kg/m^3 for normal weight concrete
x = depth of compression zone before the advent of cracking
ε_c = strain in concrete at extreme compression fiber
ε_{ct}^{ul} = ultimate tensile strain in concrete
υ_c = modular ratio of elasticity of concrete in compression $= E'_c/E_c$
υ_t = modular ratio of elasticity of concrete in tension $= E'_{ct}/E_c$

التنبؤ بالتشقق المنعطف
في الأعضاء الخرسانية المسلحة المحمّلة لامركزيا

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الصفاة ١٣٠٦٠ ، الكويت

خلاصة

يقدم هذا البحث على نحو مضبوط طريقة لحساب طاقة العزم للتشقق في الأعضاء الخرسانية المسلحة الخاضعة لتحميل لامركزي . تأخذ الطريقة في الاعتبار كلا من مساهمة الخرسانة الفعلية في منطقتي الشد والضغط للمقطع ومساهمة حديد التسليح في مقاومة انطلاق أولى الشقوق المنعطفة .

ان الطريقة المقترحة طريقة عامة اذ تصلح لجميع أشكال المقاطع المألوفة ولجميع أشكال التسليح والتحميل اللامركزي التي بامكانها أحداث الشقوق المنعطفة . ويصف البحث بالتفصيل كيفية اشتقاق المعادلات المتعلقة بالموضوع ، كما يقدم برنامج حاسوب الغرض منه تمكين المصمم من حل المسألة في جزء صغير من الوقت اللازم لحلها فيما لو لجأ الى الحساب المباشر الطويل .

يضم ملحق أ مخطط التدفق وخطوات البرنامج ، ويبيّن ملحق ب أنسب القيم للخواص الميكانيكية الداخلة في التحليل والعائدة للخرسانة وحديد التسليح ، وفي ملحق ج أعطي مثال نموذجي وأما ملحق د فهو تدوين وشرح للرموز المستخدمة .