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Ultimate strength design of cylindrical concrete shell roofs

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ABSTRACT

This paper presents a design approach for cylindrical reinforced concrete shell roofs based on a combination of the beam method of analysis and ultimate strength method of design. The formulae governing the determination of the shell's main parts of reinforcement are derived. Design aids, comprising a chart and computer program, are developed in order to expedite the computational work. It is hoped that the designer will find the proposed approach practical and reliable.

INTRODUCTION

The main objective of this work is to present a relatively simple, but realistic, design approach for reinforced concrete cylindrical shells which are frequently used to roof large, unobstructed building areas. The shells will be assumed long, uniformly loaded, and having a symmetrical circular cross section with restraining vertical edge beams. With these conditions, cylindrical shells can adequately be analysed as a beam, thereby determining the significant internal forces induced under design loads (Chinn 1959; Lundgren 1960; Shaker 1969; Scordelis 1980; Billington 1982). The beam method is an approximate method; nevertheless, it can be used with sufficient accuracy for single-barrel or multiple-barrel shells with deep beam edges if the span-radius ratio, l_1/R_y, is greater than 2 (Fig. 1) (Scordelis 1980). Close agreement with the results of the classical shell theory was obtained by the beam method even for span-radius ratios of about 1·2 in the case of cylindrical shells under the assumptions stated above (Hilal 1971).

Based on test evidence and wide experience, it is recommended that cylindrical shells with span-barrel width ratios, l_1/l_2, exceeding 1·8 (Fig. 1) be investigated for adequate strength by the beam theory (Murashev *et al.* 1968). Further, cylindrical shells with a ratio of $l_1/l_2 \geqslant 1$ can be investigated in the same way if they have at least three transverse ribs (stiffeners) with a depth of not less than $l_2/25$ (Murashev *et al.* 1968; Baikov & Sigalov 1981).

The classic beam theory is based on an ideally elastic behaviour of a presumably intact material under design loads. This is believed to be unrealistic and conservative for analysing and designing concrete shells, since it does not account for the concrete cracking in the tension zone and its inelastic behaviour in the compression zone at

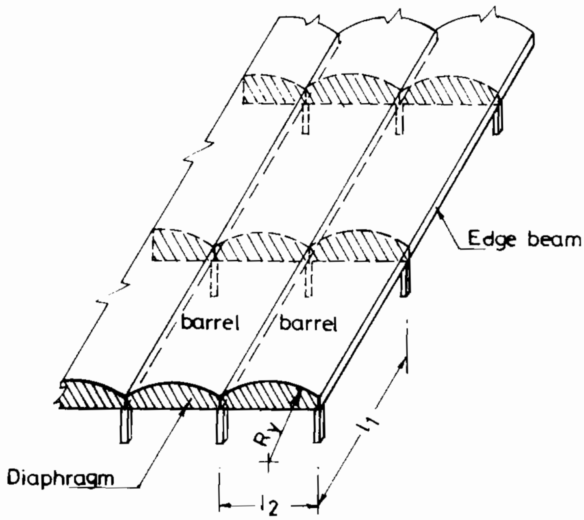

Fig. 1. Continuous multiple-barrel shell.

ultimate loading. Thus, in the present work a solely static analysis is invoked to deal with the two phenomena. In other words, the shell strength-capacity is assessed according to the state of limit equilibrium for the design loads.

The equivalent rectangular compressive stress distribution and the relating provisions of the ACI-Code (ACI Committee 318, 1983) are adopted in computing the shell's flexural strength. For cylindrical shells, the use of the average compressive stress at ultimate load as being uniformly distributed is also recommended by previous works (Baker 1950; Ernst *et al.* 1954; Murashev *et al.* 1968).

Not all the internal forces have the same significance in long shells. Of prime interest from a design standpoint are those internal forces which influence the sectional dimensions and determine the required reinforcement. Referring to Fig. 2, these are N_x for the main longitudinal steel, N_{xy} for the diagonal tension steel, and M_y for the transverse steel.

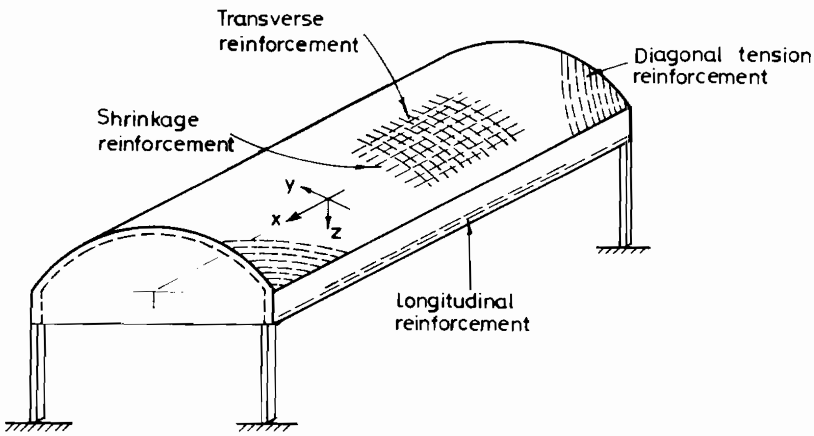

Fig. 2. Main reinforcement patterns.

Sachnowski in his book Stahlbetonkonstruktionen (Hilal 1971) recommends the following proportions for effective edge beams: h_{ed} from $0{\cdot}25h_{ov}$ to $0{\cdot}4h_{ov}$, b_{ed} from $2t$ to $4t$ (Fig. 3.1) (see also Murashev *et al.* (1968)). The minimum overall depth, h_{ov}, of a nonprestressed shell is usually $l_1/12$ and at least $l_2/6$; the arch thickness, t, ranges from $l_2/200$ to $l_2/300$ but not less than 50 to 60 mm (Murashev *et al.* 1968). The angle θ_0 is mostly not to exceed 40 to 45° to allow concrete placement without using backforms. The design of the supporting members of the shell (often called diaphragms) is not tackled in this paper.

DESIGN OF EDGE BEAM

(i) ANALYTICAL APPROACH

The function of edge beams is to stiffen the shell, add depth to section, carry loads in composite action with the shell, accommodate the longitudinal tension reinforcement, and strengthen the straight-line edges of the shell against local loads.

Fig. 3 shows a beam-edged circular cross section of a long cylindrical shell which is subjected to a maximum positive factored moment, M_u, due to external design loads, and calculated in the same manner as for an equivalent beam; the figure shows also the corresponding stress distribution for the sectional areas of concrete and steel.

Taking moments about the center of the circular part of the shell, point O, and satisfying the condition of equilibrium for the moments of the external and internal forces, yield

$$M_u/\phi = \{0{\cdot}85f_c'(2\theta_u R_y t)\}(R_y \sin\theta_u/\theta_u) - \{A_s f_y\}c_b$$

or

$$M_u/0{\cdot}9 = 1{\cdot}7f_c' t R_y^2 \sin\theta_u - c_b A_s f_y. \qquad (1)$$

Also equilibrium requires that the compressive force, C, in concrete be equal to the

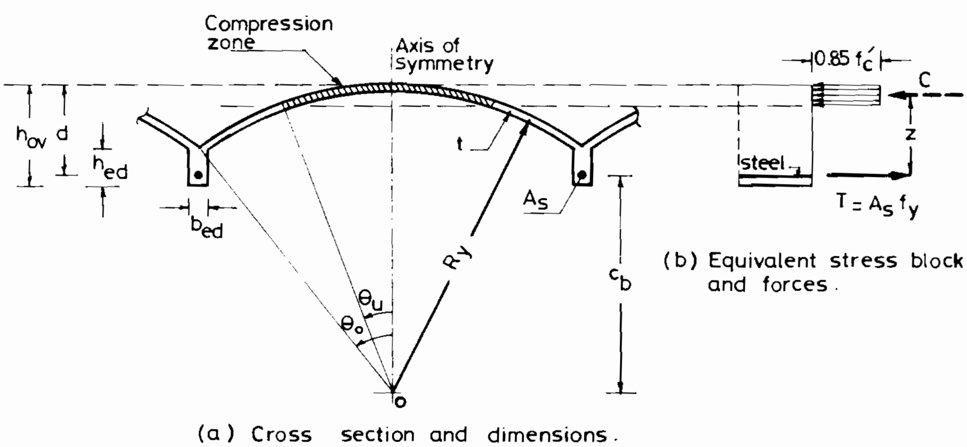

Fig. 3. Reinforced concrete cylindrical shell in state of ultimate flexural strength.

tensile force, T, in steel, from which

$$1{\cdot}7f_c' tR_y\theta_u = A_s f_y$$

or

$$A_s = 1{\cdot}7tR_y\theta_u f_c'/f_y \tag{2}$$

where θ_u is one half of the central angle of the arch marking off the concrete compression zone, R_y is the radius of curvature of the shell, t is the thickness of the shell roof, and c_b is the vertical distance from the centroid of the tension steel to a horizontal line through the center, O, of the circular arch. Other symbols are indicated in Fig. 3 and/or defined in Appendix C.

Combining Equation (1) and Equation (2) gives

$$\sin\theta_u - (c_b/R_y)\theta_u - M_u/1{\cdot}53f_c' tR_y^2 = 0. \tag{3}$$

When investigating the flexural strength (or moment capacity) of a shell with given values of t, R_y, c_b, A_s, f_c' and f_y, the value of θ_u is first found from Equation (2), and then substituted into Equation (1) to obtain M_u/ϕ (or M_u). In the design, which involves determination of the longitudinal steel area, A_s, Equation (3) is first solved for θ_u by the method of successive approximations (assuming initially $\sin\theta_u = \theta_u$) and then after A_s is determined from Equation (2).

Alternatively, A_s may be obtained graphically as illustrated in the next section.

(ii) GRAPHICAL APPROACH

For given values of the ratio c_b/R_y, Equation (3) may be considered to be composed of two variables, viz. θ_u and $M_u/f_c'tR_y^2$. The relation between these two variables is plotted in Fig. 4 for different practical values of c_b/R_y. Thus, knowing the values of $M_u/f_c' tR_y^2$, one can enter Fig. 4 from the left and proceed horizontally to meet the respective curve designated by the value given for c_b/R_y, then vertically downward to read the corresponding value of the angle θ_u in degrees. The chart offers the additional option of finding simultaneously the required steel area, A_s, by utilizing the f_c'/f_y lines and proceeding horizontally to the right to read the percentage of A_s/tR_y.

The longitudinal tension steel, A_s, is distributed in the bottom part of the edge member. Extra longitudinal steel, having a ratio of not less than 0·0035 based on the thickness of shell, should be placed in the remaining portion of the concrete tension zone to control cracking (ACI 318, 1983).

DESIGN OF DIAGONAL TENSION REINFORCEMENT

Near the diaphragms, the normal forces, N_x, become very small (about zero), whereas the membrane shearing forces, N_{xy}, reach their maximum value and give rise to the principal (diagonal) tensile forces equal, generally, to N_{xy} in magnitude and making angles of nearly 45° with the horizontal generatrix of the shell surface. The inclination could differ from 45° for longitudinally continuous shells.

In corner regions near the diaphragms, where N_{xy}/t exceeds the inclined cracking strength of concrete, that is, the permissible shear stress carried by concrete $v_c = 0{\cdot}167\sqrt{f_c'}$ MPa, diagonal reinforcement placed near the middle surface must be provided and adequately anchored in both the edge beams (members) and diaphragms

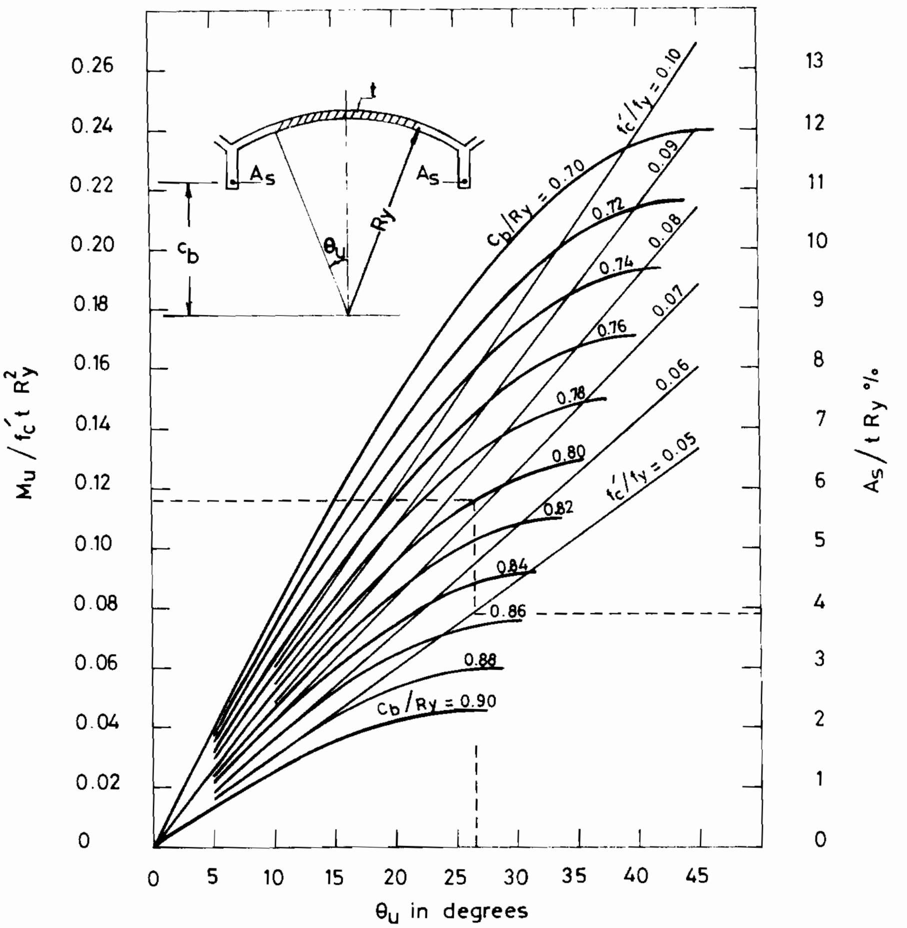

Fig. 4. Relationship between coefficient of design moment, angle θ_u, and reinforcement ratio for different values of c_b/R_y and f_c'/f_y.

(Fig. 5). In the edge beams, stirrups must be provided to resist the shear-caused diagonal tension.

The N_{xy}-forces at the ends of a simple span shell with a uniformly distributed loading may be determined as follows. Isolate a portion from the shell bounded by two vertical sections passed at the middle and end of the span, and by a longitudinal horizontal cut running anywhere in the tension zone (Fig. 6).

From consideration of equilibrium of the isolated portion of the shell, the differential horizontal compressive force is balanced by the longitudinal horizontal shear flow. The first is equal to the concrete compressive force, C, at mid-span, which corresponds to $(M_u/\phi)/z$ (see Fig. 3b).

Assuming that the horizontal shear flow, q, varies linearly as the shearing force does along the shell, i.e. from zero at mid-span to a maximum at the diaphragm, then

$$\frac{M_u/\phi}{z} = 2(q_{\max}/2)(l_1/2)$$

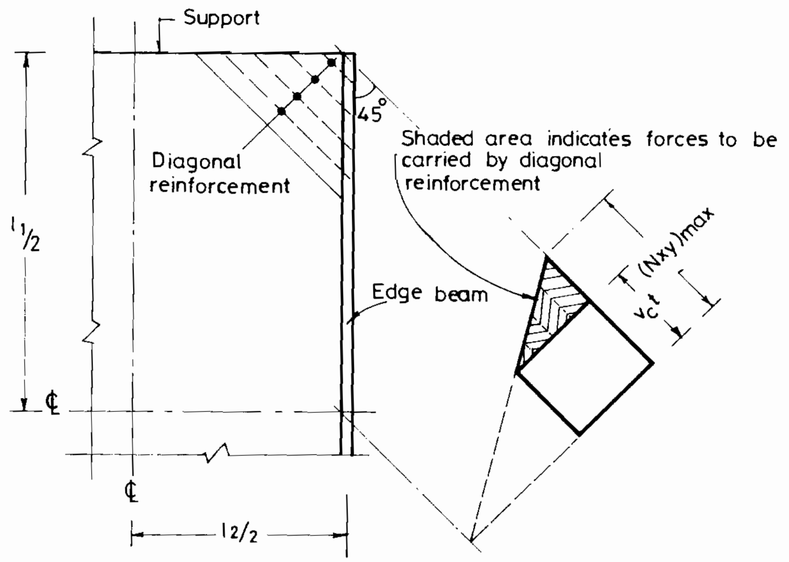

Fig. 5. Developed plan of shell diagonal tension reinforcement.

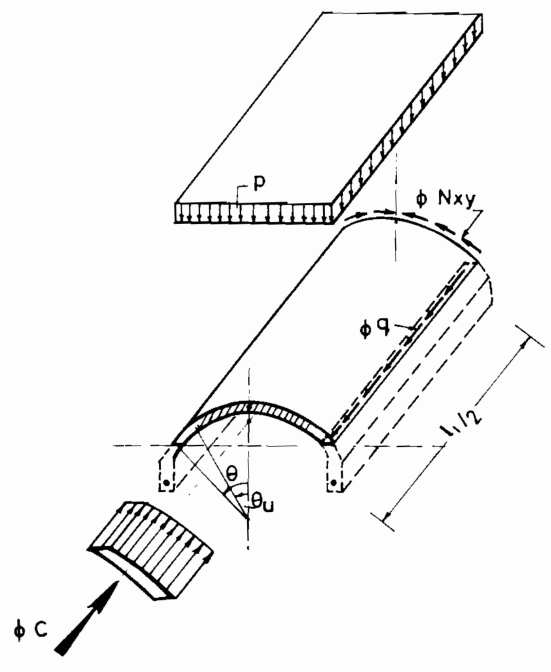

Fig. 6. A half-span shell with loads and internal forces.

or

$$A_s f_y = q_{max} l_1/2 \tag{4}$$

where A_s is the area of longitudinal steel required for the maximum positive bending moment.

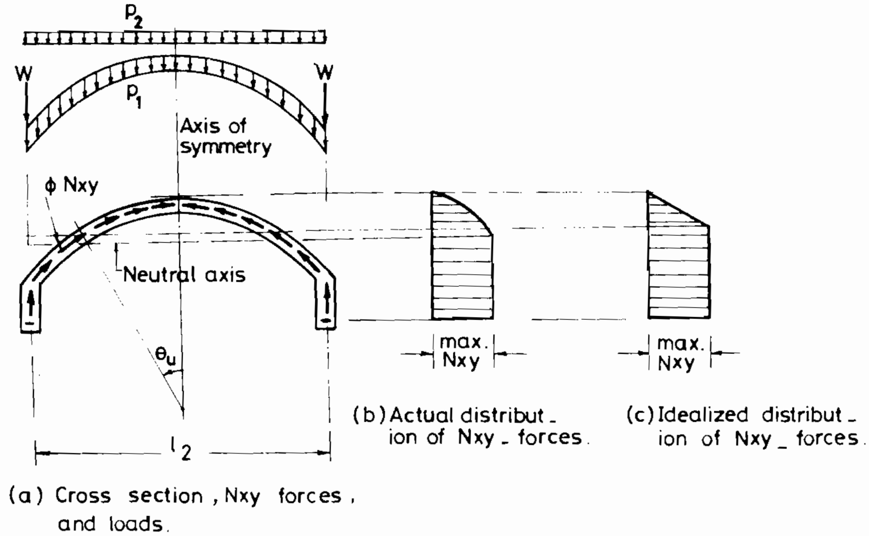

Fig. 7. Distribution of membrane shearing forces across shell barrel.

Since a horizontal shear stress acting at a point is associated with an equal vertical shear stress at the same point, then $(N_{xy})_{max} \cos\theta/t = q_{max}/(t/\cos\theta)$. Simplifying, $(N_{xy})_{max} = q_{max}$, i.e. unit shearing forces on mutually perpendicular planes are equal.

Consequently (by virtue of Equation (4)),

$$(N_{xy})_{max} = 2A_s f_y / l_1 . \tag{5}$$

In the case of multi-span shells, l_1 is replaced by the distance between the zero-moment points developed in each span. For instance, for two equal-span shells with a uniformly distributed load, this distance is about $0{\cdot}82 l_1$ according to the limit (plastic) analysis.

Referring to Fig. 5, the diagonal tension reinforcement required at the shell corners can easily be calculated and located if the shaded triangular area shown in the figure is divided into a number of sub-areas of equal base lengths (preferably a unit length each); then the diagonal reinforcement per unit length pertaining to a sub-area, ΔA_i, is equal to $\Delta A_i / f_y$.

The shear flow in the tension zone is constant; hence the N_{xy}-forces, too, remain constant. In the compression zone, N_{xy} varies nonlinearly from a maximum at the neutral axis level to zero at the extreme top fiber of the cross section (Fig. 7b). However, the nonlinear variation of N_{xy} could be idealized and replaced by a nearly equivalent linearly varying distribution. Accordingly, the N_{xy} distribution diagram takes the shape exhibited in Fig. 7c. Reference will be made to this distribution in the next section.

DESIGN OF ARCH THICKNESS AND TRANSVERSE REINFORCEMENT

To determine the transverse bending moments, M_y, it is necessary to consider first a transverse strip of unit length separated from a single-barrel shell (Fig. 8).

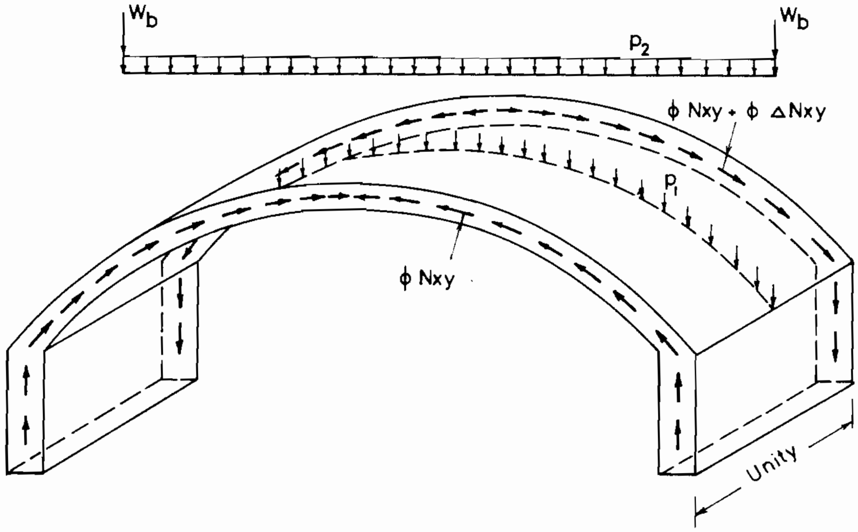

Fig. 8. Unit-length arch strip with membrane shearing forces and loads.

The strip is subjected to: the loads applied to the edge beams, W_b; the vertical distributed loads applied to the surface, p_1 and p_2; and the membrane shearing forces, ϕN_{xy} and $\phi N_{xy} + \phi \Delta N_{xy}$, acting in the planes of the sections bounding the strip.

The transverse bending moment, M_y, at any section of the strip is determined as the sum of the moments due to the applied vertical loads and the differential shearing forces (called specific shear) about the centroidal axis of the section in question. Since the vertical components of the specific shear forces, ΔN_{xy}, balance the applied vertical loads, and the ΔN_{xy}-distribution is the same as that for the N_{xy} forces, the values of ΔN_{xy} over the strip section can be computed.

In the tension zone, the specific shear is constant and given either by

$$\Delta N'_{xy} = 4A_s f_y / l_1^2 \tag{6}$$

or

$$\phi \Delta N'_{xy} = W_u / \{2d - R_y(1 - \cos \theta_u) - t/2\} \tag{7}$$

where W_u is the sum of the factored vertical loads along the strip, and d is the effective depth of the shell cross-section. The general shape of the bending moment diagram for M_y in a well-designed single-barrel shell with edge beams is depicted in Fig. 9b. As shown, M_y at the crown of the arch is at its maximum negative value. Further, along the shell arch, the transverse reinforcement, per longitudinal spacing, often consists of a relatively small-size bar curved to match the sign of the M_y-diagram. Consequently, it would be sufficient in practical cases to determine the transverse reinforcement based on M_y at the crown. Toward this aim, it is helpful to divide half of the arched cross section into a convenient number of segments, n_1, in the tension zone, and into another finite number of segments, n_2, in the compression zone.

Accordingly, the transverse bending moment at the crown, $(M_y)_c$, may be

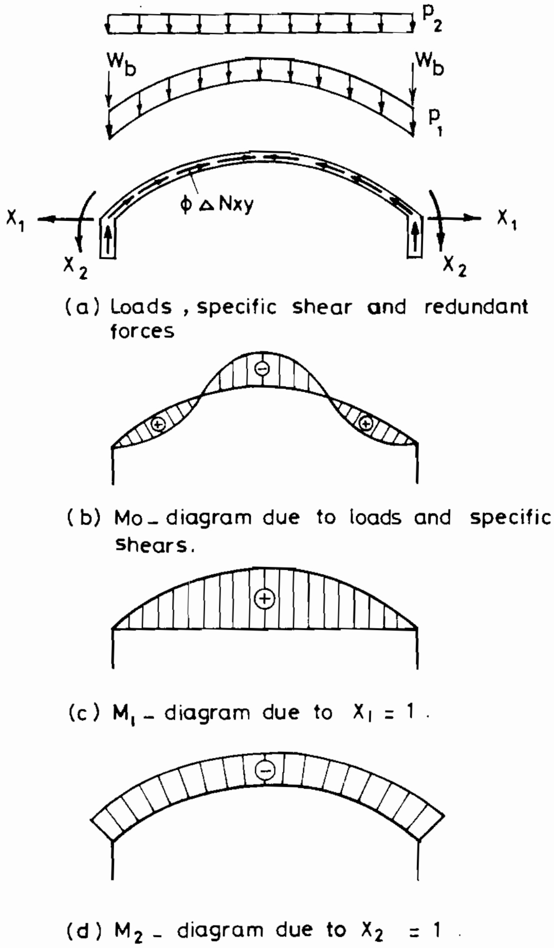

Fig. 9. Analysis of interior arch strip.

expressed as

$$(M_y)_c = (\Delta N''_{xy} d_{ed} - W_b) R_y \sin \theta_0 + \sum_{i=1}^{n_1} (\Delta N''_{xy} \Delta s_1 d_i)$$

$$+ \sum_{k=n_1+1}^{n} (\eta_k \Delta N''_{xy} \Delta s_2 d_k) - p_1 R_y^2 \theta_0 \sin(\theta_0/2) - \tfrac{1}{2} p_2 R_y^2 \sin^2 \theta_0 \qquad (8)$$

in which

$$\theta_i = \theta_u + (\theta_0 - \theta_u)(n_1 + 0{\cdot}5 - i)/n_1$$

$$\theta_k = \theta_u (n + 0{\cdot}5 - k)/n_2 .$$

Other symbols are defined below.

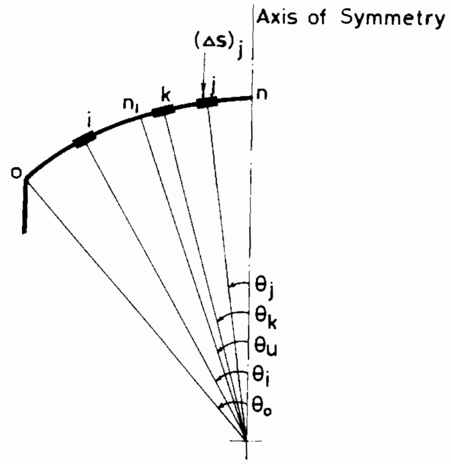

Fig. 10. Arch segments with the relevant θ-angles.

For the benefit of generalizing, the transverse bending moment at the middle of any segment, say, $(\Delta s)_j$ in Fig. 10 may be determined as

$$M_y = (\Delta N''_{xy} d_{ed} - W_b) R_y (\sin\theta_0 - \sin\theta_j) + \sum_{i=1}^{j} \Delta N''_{xy} \Delta s_1 d_i + \sum_{k=n_1+1}^{j} \eta_k \Delta N''_{xy} \Delta s_2 d_k - p_1 R_y^2 (\theta_0 - \theta_j) \sin\left(\frac{\theta_0 - \theta_j}{2}\right) - \tfrac{1}{2} p_2 R_y^2 \sin^2(\theta_0 - \theta_j) \tag{9}$$

in which

$\Delta N''_{xy} = 2(W_b + p_1 R_y \theta_0 + p_2 R_y \sin\theta_0)/\{2d - R_y(1-\cos\theta_u) - t/2\}$

θ_0 = half the angle subtended by the entire circular arch

W_b = design load applied to edge beam

p_1 = design load per unit surface area

p_2 = design load per unit horizontal area

d = effective depth of overall cross section

d_{ed} = effective depth of edge beam $= d - R_y(1-\cos\theta_0) - t/2$

$\Delta s_1 = R_y(\theta_0 - \theta_u)/n_1$

$\Delta s_2 = R_y \theta_u / n_2$

$d_i = R_y(1-\cos\theta_i)$

$d_k = R_y(1-\cos\theta_k)$

$\eta_k = (1-\cos\theta_k)/(1-\cos\theta_u)$

$i = 1, 2, \ldots, j;$ but $i \not> n_1$

$k = n_1 + 1, n_1 + 2, \ldots, j$

$n_2 = \text{Integ.}(\theta_u n/\theta_0)$

$n_1 = n - n_2.$

For $j \leqslant n_1$:

$$\theta_i = (\theta_0 - \theta_u)(j - i)/n_1$$

$$\theta_j = \theta_u + (\theta_0 - \theta_u)(n_1 + 0{\cdot}5 - j)/n_1$$

$$\theta_k = 0; \quad \text{hence } \eta_k = 0.$$

For $n_1 < j \leqslant n$:

$$\theta_i = (\theta_u - \theta_j) + (\theta_0 - \theta_u)(n_1 + 0{\cdot}5 - i)/n_1$$

$$\theta_j = \theta_u(n + 0{\cdot}5 - j)/n_2$$

$$\theta_k = \theta_u(n + 0{\cdot}5 - k)/n_2.$$

Other symbols have been defined before.

Equation (9) is a useful tool for constructing the M_y-diagram along the arched strip. However, the manual computations are tedious and time consuming. To ease substantially the computational effort, Equation (9) is programmed to suit a personal computer; the program is presented in Appendix A.

Having computed the value of M_y at the crown, the depth of the shell arch, t, as well as the required area of the transverse steel per unit strip, $(A_s)_{tr}$, can then be obtained as follows:

$$d_{ar} = \sqrt{\{(M_y)_c/k_m b\}} \tag{10}$$

$$t \approx d_{ar} + 20\ \text{mm} \tag{11}$$

$$(A_s)_{tr} = \frac{(M_y)_c/\phi}{f_y z} \tag{12}$$

where b = width of strip = unity. For selecting a suitable value for the coefficient k_m and the corresponding value of the arm z, the author's previous study may be consulted (Rayyes 1981).

Longitudinal sections of shells are subjected to membrane compressive forces, N_y, further to the bending moments, M_y. N_y is maximum at the crown and is given by $\phi(N_y)_c = -(p_1 + p_2)R_y$. Although $(N_y)_c$ has no significant effect on the design, it is, nevertheless, advisable to check the adequacy of the section strength at the crown for M_y combined with N_y by the aid of an appropriate column interaction diagram (ACI Committee 340, 1973 and 1978).

As for an interior barrel of a multi-barrel shell, the isolated unit strip (Fig. 9a) is statically indeterminate to the second degree. Its redundants X_1 and X_2 are determined on the assumption that the horizontal displacement and rotation of the arch strip at its junctions with the edge beams must each be zero. Thus according to the force (or flexibility) method

$$\delta_{11}X_1 + \delta_{12}X_2 + \delta_{10} = 0 \tag{13}$$

$$\delta_{21}X_1 + \delta_{22}X_2 + \delta_{20} = 0 \tag{14}$$

in which

$$\delta_{10} = \sum (M_0 M_1 \Delta s/EI)$$

$$\delta_{11} = \sum (M_1^2 \Delta s/EI)$$

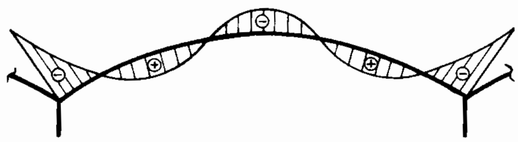

Fig. 11. Typical M_y-diagram in inner barrel.

$$\delta_{12} = \delta_{21} = \sum (M_1 M_2 \Delta s/EI)$$
$$\delta_{20} = \sum (M_0 M_2 \Delta s/EI)$$
$$\delta_{22} = \sum (M_2^2 \Delta s/EI).$$

Typical diagrams for the bending moments M_0, M_1 and M_2 are shown in Fig. 9b–d.

M_0 is the bending moment M_y as given by Equation (9), whereas M_1 and M_2 at the middle of any segment, say, $(\Delta s)_j$ in Fig. 10 are obtained as follows:

$$M_1 = R_y(\cos \theta_j - \cos \theta_0) \tag{15}$$

$$M_2 = -1. \tag{16}$$

The redundants X_1 and X_2 are determined by solving Equations (13) and (14) simultaneously; hence, the final transverse bending moment at any longitudinal section becomes

$$M_y = M_0 + X_1 M_1 + X_2 M_2. \tag{17}$$

The final transverse bending moment diagram in an interior barrel is illustrated in Fig. 11.

Here again, the transverse reinforcing steel is placed across the thickness, t, in accordance with the sign of the final M_y-diagram. Reference is made to Appendix B for further comments.

CONCLUSIONS

The following concluding remarks can be drawn from the present work:

1. Equations (1) and (2) are very useful in investigating the adequacy of the flexural strength of long cylindrical shells.
2. The chart presented in Fig. 4 provides an efficient means for obtaining the angle θ_u which is fundamental in determining the longitudinal reinforcement, specific shear, transverse bending moment, and consequently transverse reinforcement.
3. Equation (5), which gives the maximum membrane shearing force, and the explanation relevant to it dictate the diagonal reinforcement required in the barrel corners bounding the diaphragms.
4. The chart of Fig. 4 along with the computer-aided solution of Equation (9) for the transverse moments (see Appendix A) provide a convenient approach for designing long cylindrical concrete shells by the beam method, at least for single-barrel shells.

It is hoped that this work will be found comprehensive and practical for designing long cylindrical concrete shell roofs.

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APPENDIX A—PROGRAM FOR TRANSVERSE BENDING MOMENTS, M_y

```
200 INPUT W,P1,R,A0,D,U,N,T,P2,J
210 PRINT W,P1,R,A0,D,U,N,T,P2,J
220 IF J<1 OR J>N THEN PRINT" ***ERROR*** ILLEGAL SECTION" ELSE GOTO 240
230 STOP
240 N2=INT(U*N/A0)
250 N1=N-N2
260 IF J<=N1 THEN J0=U+(A0-U)*(N1+.5-J)/N1 ELSE J0=U*(N+.5-J)/N2
270 F=2*(W+P1*R*A0+P2*R*SIN(A0))/(2*D-R*(1-COS(U))-T/2)
280 D0=D-R*(1-COS(A0))-T/2
290 S1=R*(A0-U)/N1
300 S2=R*U/N2
310 L1=MIN(J,N1)
320 FOR I=1 TO L1
330 IF J<=N1 THEN I0=(A0-U)*(J-I)/N1 ELSE I0=(U-J0)+(A0-U)*(N1+.5-I)/N1
340 X1=X1+F*S1*R*(1-COS(I0))
350 NEXT I
360 IF J<=N1 THEN GOTO 420
370 FOR K=N1+1 TO J
380 K0=U*(N+.5-K)/N2
390 E=(1-COS(K0))/(1-COS(U))
400 X2=X2+E*F*S2*R*(1-COS(K0))
410 NEXT K
420 X3=(F*D0-W)*R*(SIN(A0)-SIN(J0))
430 X5=P1*R*R*(A0-J0)*SIN(.5*(A0-J0))
440 X6=.5*P2*R*R*SIN(A0-J0)**2
450 M0=X1+X2+X3-X5-X6
460 PRINT ' J = ';J;'      MY  = ';M0
470 STOP
480 END
```

The following notations used in the above program differ from those used in the text:

Program	Text	Program	Text
A0	θ_0	M0	M_y
D0	d_{ed}	R	R_y
E	η_k	S1	ΔS_1
F	$\Delta N''_{xy}$	S2	ΔS_2
I0	θ_i	U	θ_u
J0	θ_j	W	W_b
K0	θ_k		

APPENDIX B—COMMENTARY ON SHELL REINFORCEMENT

The 1983 ACI-Code (art. 19.4.9) requires the transverse reinforcement to be placed equally near both surfaces of the shell. Longitudinal reinforcement must be placed in the compression zone to account for shrinkage and temperature stresses and to serve as distributors for the transverse reinforcement. Its amount as a ratio of the gross concrete area ranges from 0·0018 to 0·002 and it is spaced not more than five times the thickness, nor 500 mm apart. All barrel reinforcement at the junction of the shell and diaphragm or edge beam must be extended through such members a distance not less than 1·2 times the development length, nor 500 mm. In multi-barrel shells, the regions near intermediate edge members are additionally reinforced with transverse bars, 6 to 10 mm in diameter and spaced 100 to 200 mm apart (Fig. 12), which are intended to resist the negative transverse moments at the supports. In the regions near end diaphragms, extra longitudinal negative reinforcement extending not less than $0{\cdot}6\sqrt{R_y t} + 300$ mm is to be provided in the shell barrel to account for the local moments due to partial fixation between the barrel and end diaphragm. In multi-span shells, longitudinal tension reinforcement matching the negative bending moment diagrams in an analogous continuous beam has to be provided, preferably in a

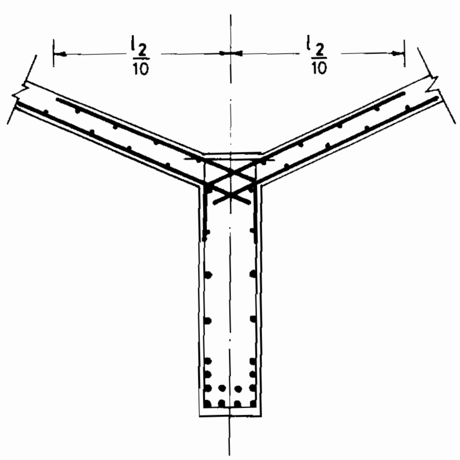

Fig. 12. Reinforcement of shell barrels above intermediate edge beam.

staggered disposition, on both sides of the intermediate diaphragms. For purposes of simplification, this reinforcement can be arranged with a uniform spacing in the middle half of the tension zone and with a varying spacing within the side quarters.

APPENDIX C—NOTATION

A_s = area of longitudinal nonprestressed positive reinforcement in an interior edge beam or two exterior edge beams.
$(A_s)_{tr}$ = area of transverse reinforcement per strip of a unit width.
c_b = vertical distance from centroid of longitudinal positive steel in edge beam to horizontal line through center of circular arch.
d = distance from top surface of shell at crown to centroid of longitudinal reinforcement in edge beam.
d_{ar} = effective depth (thickness) of shell arch.
d_{ed} = vertical distance from arch springing to centroid of reinforcement in edge beam.
f_c' = specified cylinder compressive strength of concrete, not less than 20 MPa.
f_y = specified yield strength of nonprestressed reinforcement, not greater than 400 MPa.
h_{ed} = depth of edge beam.
h_{ov} = overall depth of cross section of shell.
l_1 = length of longitudinal span of shell
= center-to-center distance between diaphragms.
l_2 = length of arch span
= distance between axes of edge beams.
M_u = design (factored) positive bending moment in a span of shell considered as a beam.
M_y = design transverse bending moment at any longitudinal section in an arch strip of unit length.
$(M_y)_c$ = design transverse bending moment at the crown of unit-length arch strip.
n = convenient number of segments into which half of arch is subdivided.
n_1 = number of segments in the tension zone of half of arch.
n_2 = number of segments in the compression zone of half of arch.
N_x = longitudinal membrane normal forces acting on unit length.
N_{xy} = nominal membrane shearing forces acting on unit length.
q_{max} = maximum nominal horizontal shear flow.
p_1 = design (factored) vertical applied load acting on unit surface area.
p_2 = design (factored) vertical applied load acting on unit horizontal area.
R_y = radius of curvature of shell.
t = thickness of arch shell.
v_c = nominal permissible shear stress carried by concrete
= $0{\cdot}167\sqrt{f_c'}$ MPa.
W_b = design (factored) vertical load applied directly to edge beam.
W_u = sum of factored vertical loads carried by a unit transverse strip of shell.
X_1 = unknown constraining horizontal force at edge of an isolated interior barrel.
X_2 = unknown constraining moment at edge of an isolated interior barrel.
z = arm of nominal flexural strength of a cross section.

ΔN_{xy} = nominal membrane differential shearing forces acting on strip of unit length (or nominal specific shear).
$\Delta N'_{xy}$ = maximum nominal specific shear.
$\Delta N''_{xy}$ = maximum factored specific shear.
Δs = length of arc segment.
Δs_1 = length of arc segment in the tension zone.
Δs_2 = length of arc segment in the compression zone.
θ = angle subtended by an arc between crown and point on the arch.
θ_0 = half of angle subtended by arch of shell.
θ_u = half of angle subtended by the arched compression zone.
ϕ = strength reduction factor = 0·9 for flexure, axial tension, and axial tension with flexure.

تصميم حدي للسقوف الخرسانية
ذوات القشرة الاسطوانية

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قسم الهندسة المدنية بجامعة الكويت ، ص . ب ٥٩٦٩ ،
الصفاة ١٣٠٦٠ ، الكويت

خلاصة

يقدم هذا البحث طريقة لتصميم السقوف الخرسانية المسلحة المشكلة على هيئة قشرة اسطوانية وذلك بربط طريقتي الجائز (الكمرة) للتحليل والحالة الحدية (الاجهاد الاقصى) للتصميم .

ويضم البحث المعادلات المستنبطة التي تحكم تعيين كميات التسليح الرئيسية للقشرة الى جانب مساعدات تصميم تتألف من رسم بياني وبرنامج للحاسب الاليكتروني . وهذان الاخيران قد قصد بهما تشهيل العمل الحسابي بشكل جذري .

نأمل ان يجد المهندس المصمم طريقة التصميم المقترحة طريقة عملية يعول عليها .