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Identification of trends in the development of Kuwait University

M. EZZ AL-DIN AND N. AL-QADDUMI

Department of Civil Engineering, University of Kuwait, P.O. Box 5969, Safat 13060, Kuwait

ABSTRACT

The number of students in a university is one of the basic indices of its development. The purpose of this paper is to determine the trends of development of students' population in Kuwait University. Two models of trends are considered: the logistic curve as well as the straight line. The parameters of these models are calculated for individual faculties (colleges) and for the whole university. The values of mean-square errors of approximation real time series by means of logistic and linear function are provided. The results are summarized in tables and are presented graphically. Suggestions for future research are given.

INTRODUCTION

Kuwait University was officially inaugurated in 1966. Although it is still a young university , it is developing dynamically. The analysis of the statistical data reflecting the historical evolution of the University shows that since its foundation, the University has, year after year, achieved a reasonable development in various aspects.

Many different factors can be used for expressing the dynamics of Kuwait University evolution. One can mention, for example, the total number of students of the University, the number of post-graduate students, the number of graduates in various disciplines, the total number of staff members and demonstrators, the total number of new students enrolled in the individual faculties, and the University budget. Observing and registering the numerical values of these factors in successive years, we obtain a collection of various time series which reflects diverse aspects of the process of the University development. For instance, the amount of expenditure of the University arranged according to yearly periods forms the time series reflecting the dynamics of financial activity of the University during the period under consideration, i.e. in the academic years 1966/67–1981/82 (Kuwait University 1976; Ezz Al-Din 1979, 1986).

Even a superficial analysis of the above-mentioned dynamic series allows us to ascertain that each of them shows a specific kind of variation representing a certain general trend. A knowledge of these trends can make it easier to manage both the whole University and its particular faculties in the current period, and to construct realistic plans for the future development of the University body. The identification of trends should not be an *ad hoc* activity, but a whole system of forecasting how the university development should be constructed. A good system of forecasts should be a complex one. This is self-evident because the University consists of many organizational sections which form a complex system with many interactions. It is a goal-oriented system. The goal of the University activity can be defined as education and graduation of specialists in various branches of knowledge, as well as research work. In order that the effect of the University activity leads to definite results, it is necessary to know, to some extent at least, how the University responds to certain actions on which the accomplishment of the goal depends.

A mathematical model obtained through identification makes it possible to determine a set of instructions according to which the development of the University will proceed. These instructions can be expressed in the form of mathematical formulae illustrating the trends of the University development. The question of trends identification is connected with a variety of statistical problems.

The purpose of this paper is to determine the trends of development of the total number of students in Kuwait University and in its individual faculties in the period 1966/67–1981/82. Two approximations of trend curves will be considered. The first one assumes that the empirical time series can be smoothed by the aid of logistic curves. In the second case, as a model of trend curves, the straight lines will be considered. In both cases the mean-square error of approximation will be calculated. The results of both approximations will be compared. Some final remarks on the possibilities of utilization of the obtained results will be given to show the process of forecasting the future development of Kuwait University.

LOGISTIC CURVES AS A MODEL OF GROWTH OF THE STUDENTS POPULATION OF KUWAIT UNIVERSITY

Assume that we shall smooth the time series representing the growth of the total number of students of Kuwait University as well as of its particular faculties during the academic years 1966/67–1979/80. For the sake of simplifying the computations, the year 1966/67 is taken as the year 0 and the subsequent years are numbered as follows: 1, 2, . . . , 15. Thus the number 15 corresponds to the academic year 1981/82.

Let us consider time series that give the total number of students of Kuwait University during the period 1966/67–1981/82. They are represented numerically in Table 1.

The selection of the trend curves on the basis of given time series makes certain hypothetical assumptions on the development of the given processes (Bos & Jenkins 1970; Cox & Hinkley 1974). It appears that in the historical evolution of Kuwait University, we can distinguish two distinct stages. The first one commences in the academic year 1966/67 when the University began. It had two faculties at that time: The Faculty of Science, Arts and Education, and the University College for Women. The total number of students enrolled in the University at its inauguration was 418. In the following years the number of faculties and the number of university students systematically increased. In 1981/82 the University comprised six faculties with 9359 students. Table 1 indicates that from 1966/67 until 1977/78 the number of students in the whole University and in individual faculties (except Law and Shari'a) increased steadily.

As a second stage of the development of Kuwait University we treat the academic years 1978/79 and 1979/80 in which the increase in the number of students in the

Table 1. Evolution of the number of students at Kuwait University

Academic year	No. of years since inaugura-tion (t_i)	University Faculty						Total no. of students
		Science	Arts and Education	Comm., Econ. & Pol. Sc.	Law & Shari'a	Eng. & Petr.	Medicine	
1966/67	0	83	335	...	...	...	...	418
1967/68	1	136	542	140	56	...	...	874
1968/69	2	194	711	327	105	...	...	1337
1969/70	3	271	845	459	138	...	...	1713
1970/71	4	313	863	641	171	...	...	1988
1971/72	5	488	961	814	216	...	...	2479
1972/73	6	703	1252	1068	264	...	...	3287
1973/74	7	1021	1467	1091	257	...	...	3836
1974/75	8	1168	1838	1165	274	...	...	4445
1975/76	9	1644	2383	1338	340	127	...	5832
1976/77	10	1884	3117	1669	498	304	56	7528
1977/78	11	2177	3892	2061	535	551	102	9318
1978/79	12	1591	3586	1998	533	646	154	8508
1979/80	13	1517	3547	2003	552	818	299	8736
1980/81	14	1623	3611	2025	622	993	285	9303
1981/82	15	1608	3354	1857	676	1000	332	9359

Comm., Econ. & Pol. Sc. = Commerce, Economics and Political Science
Eng. & Petr. = Engineering and Petroleum

above-mentioned faculties becomes slightly slower. It is possible that we observe a transient state of saturation of the University, i.e. the number of students is stabilized and decreases temporarily at a certain level. From this fact it appears that the mathematical function describing the development of the student population of Kuwait University as well as the functions illustrating the growth of the number of students in each University Faculty should at first rise at an increasing speed, then this speed should decrease and finally cease altogether. In other words, the trend curves should approach asymptotically certain straight lines parallel to the abscissae, i.e. to the axis of time (Figs 1–4).

In the time series theory, such law of growth of a population is called Robertson's law and is expressed in the form of the following differential equation:

$$\frac{1}{y_t}\frac{dy_t}{dt} = \gamma(k - y_t) \qquad (1)$$

where $\gamma > 0$ denotes the coefficient of proportionality between the speed of the growth of the function and the value of the function y_t and the factor $(k - y_t)$ (Christ 1966; Lange 1978; Tinter 1952; Theil 1971).

From this law of growth one can deduce the formula of the logistic function. It is given as a solution of Equation (1) and can be written in the following form:

$$y_t = \frac{k}{1 + b\,e^{-at}} \qquad (2)$$

where k, b and a are the parameters of the logistic function. Their numerical values are to be estimated from statistical data.

In smoothing the empirical time series given in Table 1 by the logistic functions, we assume that the speed of growth of the number of students is proportional to the actual value of y_t and to the factor $(k - y_t)$. Symbol y_t denotes the number of students in the moment t, where $t = 0, 1, \ldots, 15$. The magnitude of the difference $(k - y_t)$ decreases with time and that is why it plays the role of the retarding factor. When the University approaches saturation in regard to student numbers, the retarding factor $(k - y_t)$ tends towards zero and the speed of growth $\frac{dy_t}{dt}$ also approaches zero. This means that the growth in the number of students begins to stop, i.e. their number commences to stabilize near the level $y_t = k$.

In the light of the above considerations it seems logical and justifiable to make an attempt to represent the trend curves of the development of the size of student population, both in the whole University and in its particular Faculties, by logic functions. To estimate the values of the parameters k, b and a of the logistic function we apply the so-called three-point method (Lange 1978).

The technique of finding the equation for the logistic trend by the three-point method assumes that the trend curve must be drawn through three points, the ordinates of which are given by the formulae

$$y_0 = \frac{1}{k}(1 + b), \tag{3}$$

$$y_0 = \frac{1}{k}(1 + b\, e^{-na}), \tag{4}$$

$$y_0 = \frac{1}{k}(1 + b\, e^{-2na}). \tag{5}$$

We define differences d_1 and d_2 as follows:

$$d_1 = \frac{1}{y_0} - \frac{1}{y_1} = \frac{b}{k}(1 - e^{-na}), \tag{6}$$

$$d_1 = \frac{1}{y_1} - \frac{1}{y_2} = \frac{b}{k}(1 - e^{-na}). \tag{7}$$

From (6) and (7) we obtain the ratio

$$\frac{d_2}{d_1} = e^{-na}. \tag{8}$$

Therefore

$$a = \frac{1}{n}(\ln d_1 - \ln d_2). \tag{9}$$

To estimate the value of the parameter k, let us consider the expression $\frac{d_1^2}{d_1 - d_2}$.

After some transformations we obtain the following relation:

$$\frac{d_1^2}{d_1 - d_2} = \frac{b}{k} = \frac{1}{y_0} - \frac{1}{k}. \tag{10}$$

Hence,

$$\frac{1}{k} = \frac{1}{y_0} - \frac{d_1^2}{d_1 - d_2} \tag{11}$$

and thus

$$k = \left(\frac{1}{y_0} - \frac{d_1^2}{d_1 - d_2}\right). \tag{12}$$

Finally, from Equation (3) we derive the formula

$$b = \frac{k - y_0}{y_0}. \tag{13}$$

The three-point method has been used to fit logistic curves to the dynamic series presented in the third, fourth, fifth, sixth and ninth columns of Table 1. The obtained values of the parameters k, b and a are summarized in Table 2. Furthermore, Table 2 contains information on the mean-square error, s, of individual approximations. In general, the mean-square deviation of the given trend curves from time series is equal to

$$s = \frac{\sum_{t=1}^{n} (y_t - \hat{y}_t)^2}{f} \tag{14}$$

where y_t and $\hat{y}_t$ are respectively the empirical and theoretical values of the elements of time series respectively, $f = n - m$, where n is the number of observations in time series and m is the number of unknown parameters (Cox & Hinkley 1974; Cuddy 1947; Kendall & Stuart 1968). In our case the number of estimated parameters of the logistic curve is equal to $m = 3$.

The results of calculations are presented graphically in Figs 1–4 and 7. It appears that the logistic curve reflects sufficiently well the trend of empirical series. Thus there is a possibility of application of analytical logistic models in forecasting the future development of the University. The forecasts for the whole University and its individual faculties are given in appropriate figures.

Table 2. Numerical values of the unknown parameters of the logistic function

Faculty	k	a	b	s
Science	1772	0·4323	20·3547	295·09
Arts and Education	8569	0·2394	24·5800	311·54
Commerce, Economics, and Political Science	2131	0·4504	14·2209	162·73
Law and Shari'a	690	0·3174	11·3300	45·70
Kuwait University	10080	0·4024	23·1153	601·53

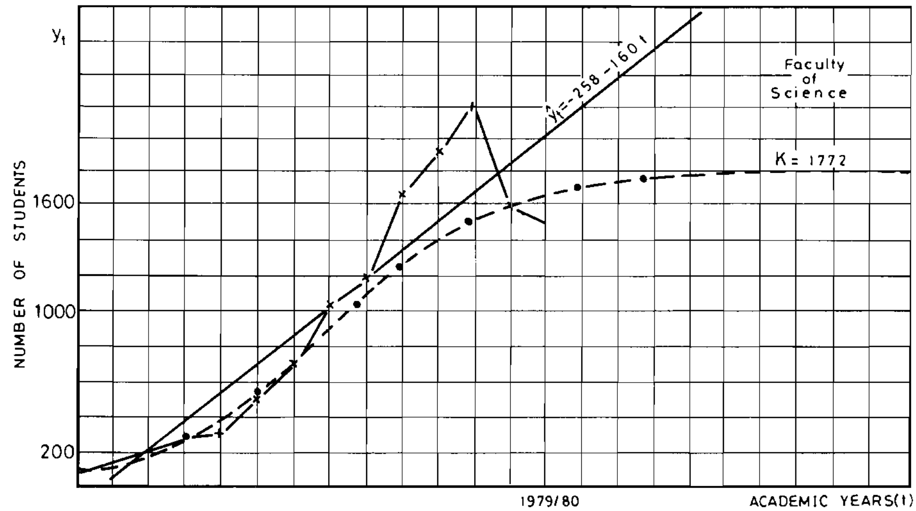

Fig. 1. Logistic curve and straight-line trend of the development of the number of students in the Faculty of Science.

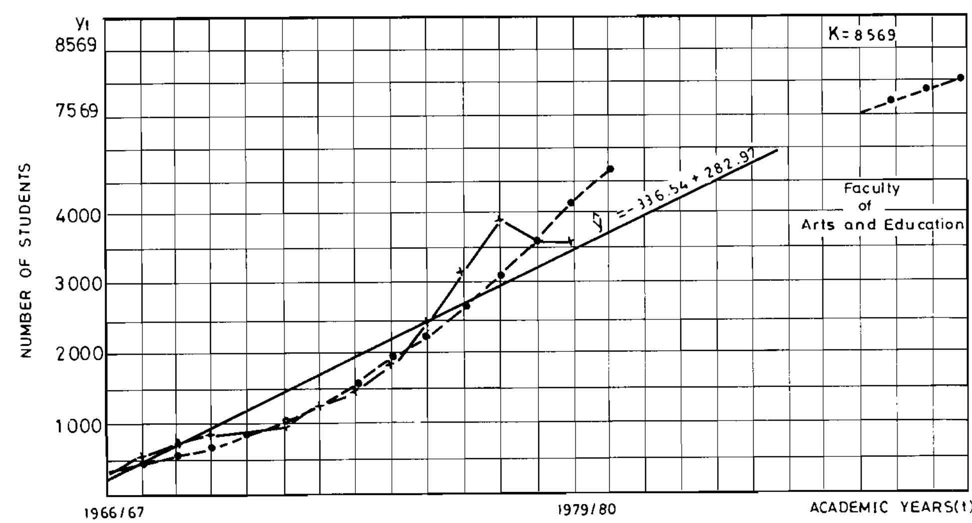

Fig. 2. Logistic curve and straight-line trend of the development of the number of students in the Faculty of Arts and Education.

IDENTIFICATION OF THE LINE TRENDS OF THE DEVELOPMENT OF KUWAIT UNIVERSITY

Let us now consider the question of analytically smoothing the time series given in Table 1 by means of a straight line

$$y_t = a + bt. \tag{15}$$

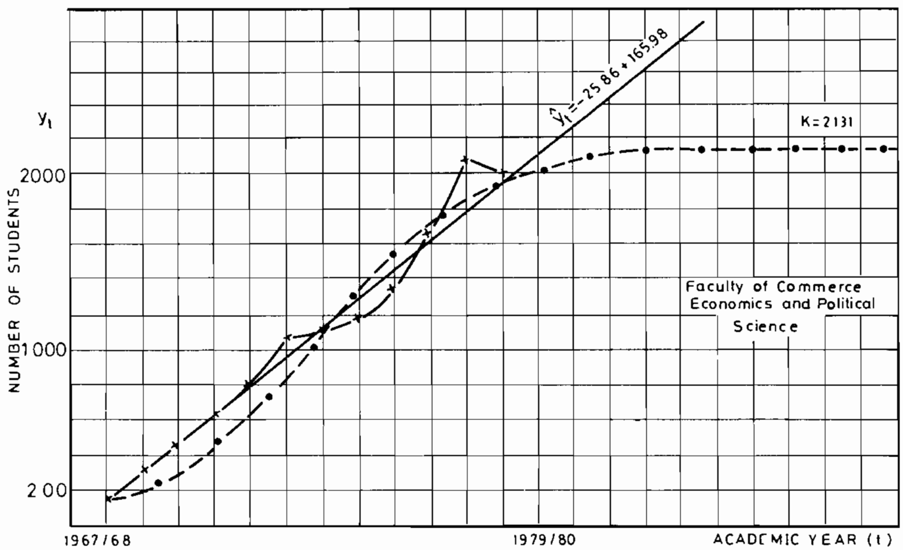

Fig. 3. Logistic curve and straight-line trend of the development of the number of students in the Faculty of Commerce, Economics and Political Science.

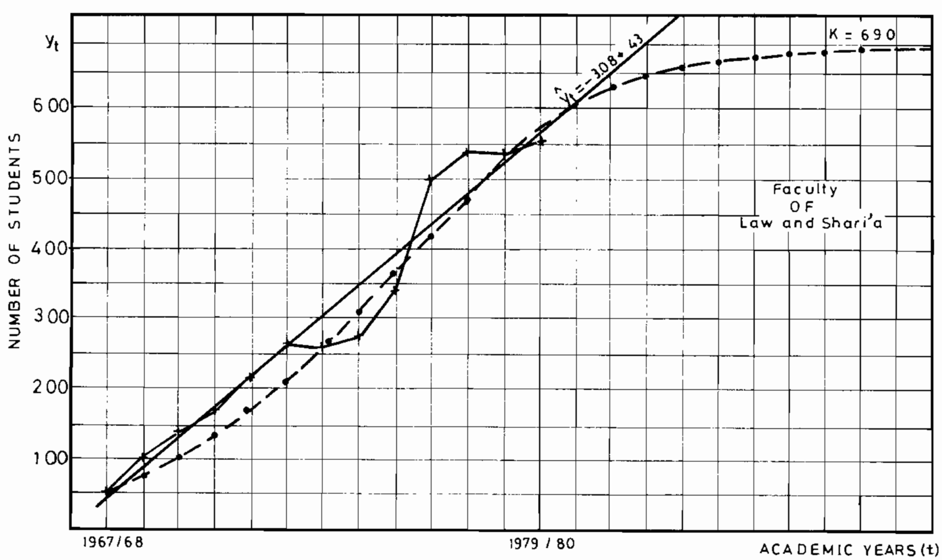

Fig. 4. Logistic curve and straight-line trend of the development of the number of students in the Faculty of Law and Shari'a.

Applying to this straight line the familiar method of least squares for smoothing the series (see for example Tintner (1952)) we obtain results which are presented in Table 3. In the case of line approximation, the number of unknown parameters is equal to $m = 2$. Hence in formula (14) for mean-square error we let $f = n - 2$. The values of mean-square error are given in Table 3 too. The line trends and forecasts obtained by the method of trend extrapolation are presented in Figs 1–7.

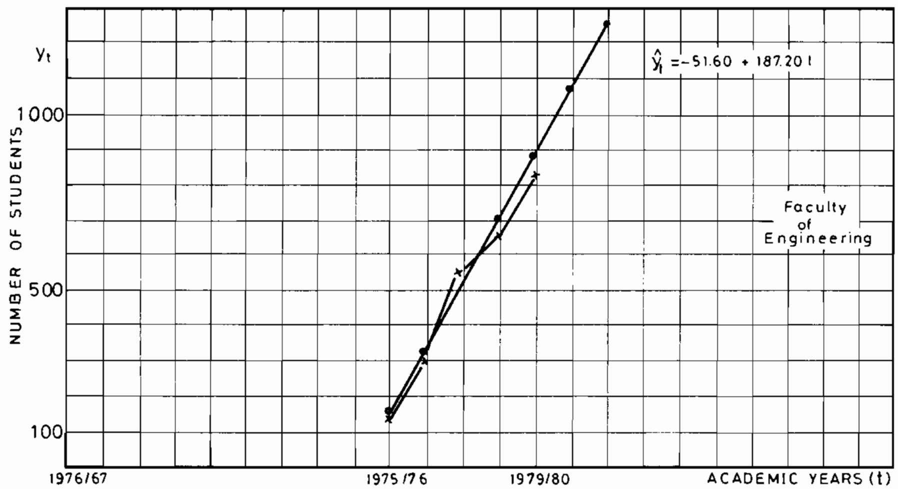

Fig. 5. Line of trend of the development of student numbers in the Faculty of Engineering and Petroleum.

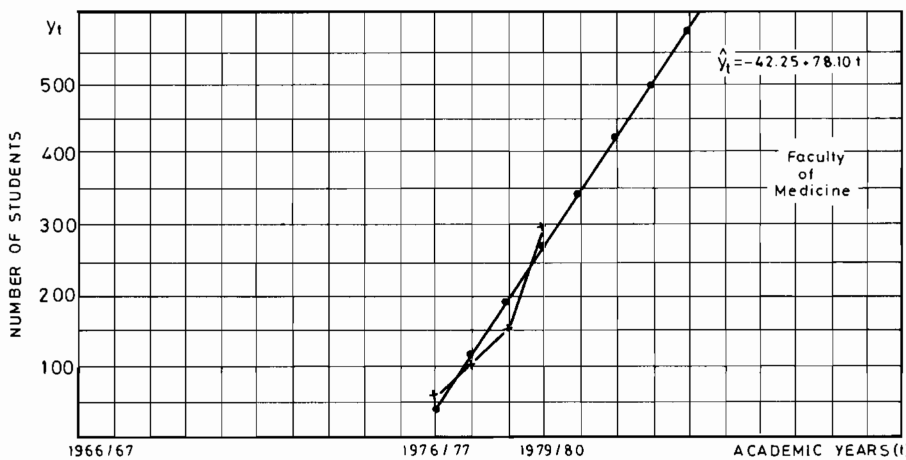

Fig. 6. Line of trend of the development of student numbers in the Faculty of Medicine.

Table 3. Values of the line trend parameters computed by the method of least squares

Faculty	*a*	*b*	*s*
Science	−258·00	160·00	293·41
Arts and Education	−336·54	282·97	446·53
Commerce, Economics, and Political Science	−25·86	165·98	116.20
Law and Shari'a	−3·08	43·64	40·85
Engineering and Petroleum	−51·60	187·20	29·85
Medicine	−42·25	78·10	37·61
Whole University	−1256·42	869·30	1510·90

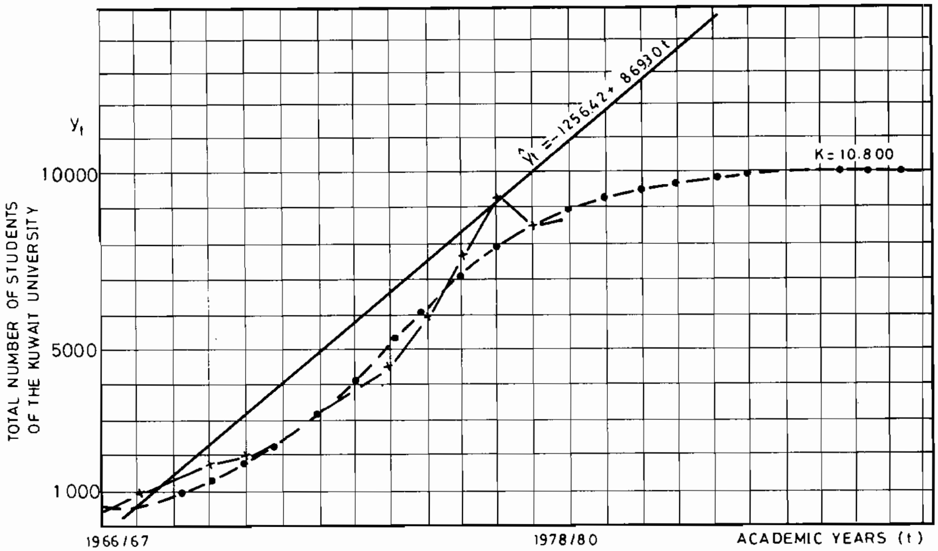

Fig. 7. Logistic curve and straight-line trend of the development of the total number of students in Kuwait University.

CONCLUDING REMARKS

This paper presents a solution of the problem of trends identification in development of Kuwait University and its individual faculties. Two analytical models have been suggested for reflecting trends which exist in time series describing, statistically, the various aspects of University activities. On one hand, there is approximation of trend by means of logistic function, and on the other we have the approximation made by the line model. It seems that both models can be used as short- and medium-term forecasting models.

From the statistical point of view, the Medical and Engineering & Petroleum Faculties are in their initial growth periods and will demonstrate a levelling of this growth in the future. Since the other faculties reached maturity in approximately 11 years, this may be true of the two newer faculties.

The logistic model is appropriate where computer facility is available. The linear model can be used to approximate Equation (15) if one does not wish to perform a statistical computer analysis in greater depth.

However, the models simplify some aspects of the factors characterizing the development of the University. Thus, impacts associated with specific faculties and activities must be handled by more detailed models. The models presented show promise, and more research regarding university systems is necessary in order to quantify other qualitative variables, which are thought to be equally important in the decision-making process. Similar analysis with larger data sets should be carried out in order to validate and generalize or revise the methodology presented in this paper.

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تحديد المؤشرات الأنمائية للتخطيط بجامعة الكويت

محمد عز الدين و نبيل قدومى
قسم الهندسة المدنية بجامعة الكويت
ص ب ٥٩٦٩ ، الصفاة ١٣٠٦٠ ، الكويت

خلاصـــــــة

يقدم هذا البحث بعض الحلول لمشكلة تحديد المؤشرات الأنمائية للتخطيط بجامعة الكويت وكلياتها ومرافقها المختلفة ، ويقترح نموذجين رياضيين من أجل محاكاة المؤشرات الراهنة بالبيئة التعليمية ، من خلال سلسلة زمنية تصف إحصائيا العوامل الأساسية للأنشطة الجامعية .

وباستخدام المعادلات المنطقية لتقدير المؤشرات ومدى توافقها مع الأنماط الفعلية بالاضافة الى النماذج الخطية ، أمكننا اختبار صلاحية النموذجين المقترحين كأدوات مناسبة للتخطيط على كل من المدى القصير والمتوسط .

ويتميز هذان النموذجان بالمرونة ، لكنهما في الواقع يبسطان بعض العوامل المؤثرة في نمو الجامعة ، لذلك نرى أن تعالج بعض الآثار المرتبطة بأنشطة أو كليات محددة بواسطة نماذج أكثر تفصيلا . فالنماذج المقدمة نماذج واعدة لكن المزيد من الأبحاث ضروري من أجل دراسة مفصلة للنظم والبرامج الجامعية بكل مكوناتها ومقوماتها الهامة واللازمة لعملية اتخاذ القرار . كما نرى ضرورة اجراء تحليل مماثل يعتمد قاعدة أكبر للبيانات من أجل قياس مدى ملاءمة وتوافق الأسلوب التحليلي المقدم في هذا البحث .