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Separation of residual from Bouguer gravity data measured over a deep sedimentary basin, western Sudan

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ABSTRACT

Methods of regional–residual separation, including graphical smoothing, polynomial trend surface analysis, upward continuation and 2-D wavenumber filtering, were employed to separate the residual anomalies from gravity data measured over a deep sedimentary basin in western Sudan. The objective was to obtain a residual gravity map suitable for quantitative modelling of the geometry of the basin, in connection with a search for deep groundwater sources.

Upward continuation and 2-D wavenumber filtering results show some common features, but they do not satisfactorily define the shape of the basin. The residual gravity values obtained in this way can only be interpreted in terms of very shallow sources, which might be the superficial deposits. On the other hand, residual gravity maps obtained by graphical smoothing and polynomial trend surface define the shape of the basin clearly and show a general similarity, although there is significant offset in gravity values.

When correlated with surface geology and information from drilled holes, only the graphical smoothing residual gravity map meets the objective of quantitative modelling, with an assumption that the whole residual is produced by the sedimentary fill.

INTRODUCTION

The problem of regional–residual separation is a well-known aspect of the gravity method (Dobrin 1976). However, in spite of recent developments in techniques of gravity interpretation and data processing, regional-residual separation is difficult to achieve and the personal factor plays an important role. The bias in regional-residual separation arises because the term ‘residual’ is very subjective and has two different meanings. On one hand, the term can be used in a general sense to mean what remains of the Bouguer anomaly after subtracting a smoothed regional field. As Nettleton (1976) says ‘the regional is what you take out in order to make what’s left look like the geology’. On the other hand, the term residual can be used to mean the values that result from the convolution of Bouguer gravity values with some weighting functions or spatial filters. However, in this second meaning the resulting anomalies are not equivalent to the general sense of the term ‘residual’ and some other name such as convolution gravity can be used. Also, the effectiveness of these

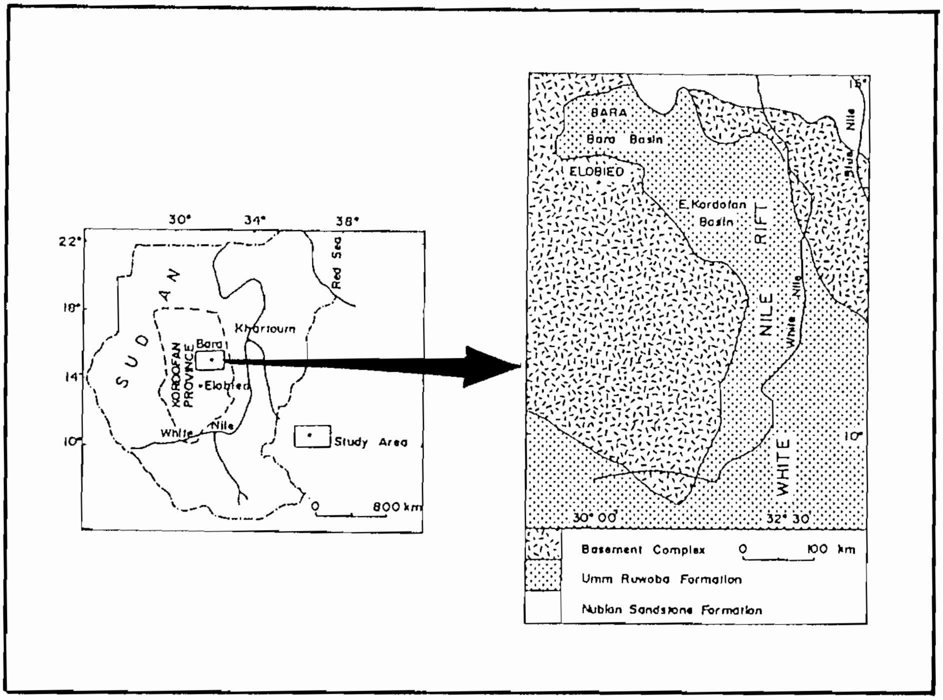

Fig. 1. Location and geological map of the Bara Basin.

residual values from the view point of quantitative interpretation (modelling) is debatable (Gupta & Ramani 1980).

Gupta & Ramani (1980) have discussed some aspects of upward continuation, spectral factorization and graphical smoothing techniques of regional–residual separation applied to Bouguer anomalies in pre-Cambrian greenstone belts in northwestern Ontario, Canada. In the present study, the subjectivity in residual separation is constrained by applying graphical smoothing, polynomial trend surface analysis, 2-D wavenumber filtering and upward continuation techniques to Bouguer gravity values over a deep sedimentary basin in western Sudan.

GRAVITY SURVEY OF THE BARA BASIN

The Bara Basin in western Sudan (Fig. 1) occupies an area of 6000 km^2 of semi-desert terrain covered by sand dunes and supporting sparse vegetation.

Geologically, the bulk of the basin is composed of fine-grained sediments known as the Umm Ruwaba Formation of Tertiary age (Whiteman 1971). The Umm Ruwaba sediments are bounded on the north, south and west by rocks of the Basement Complex and in the northeast by Jurassic–Cretaceous Nubian Sandstone Formation (Fig. 1). All these formations are overlain by a thin superficial layer of aeolian sands.

To provide El Obeid Town, the capital of Kordofan Region (Fig. 1), with a permanent source of water supply, the Rural Water Corporation (Sudan) suggested the nearby Bara Basin as a primary target for deep groundwater search. To determine

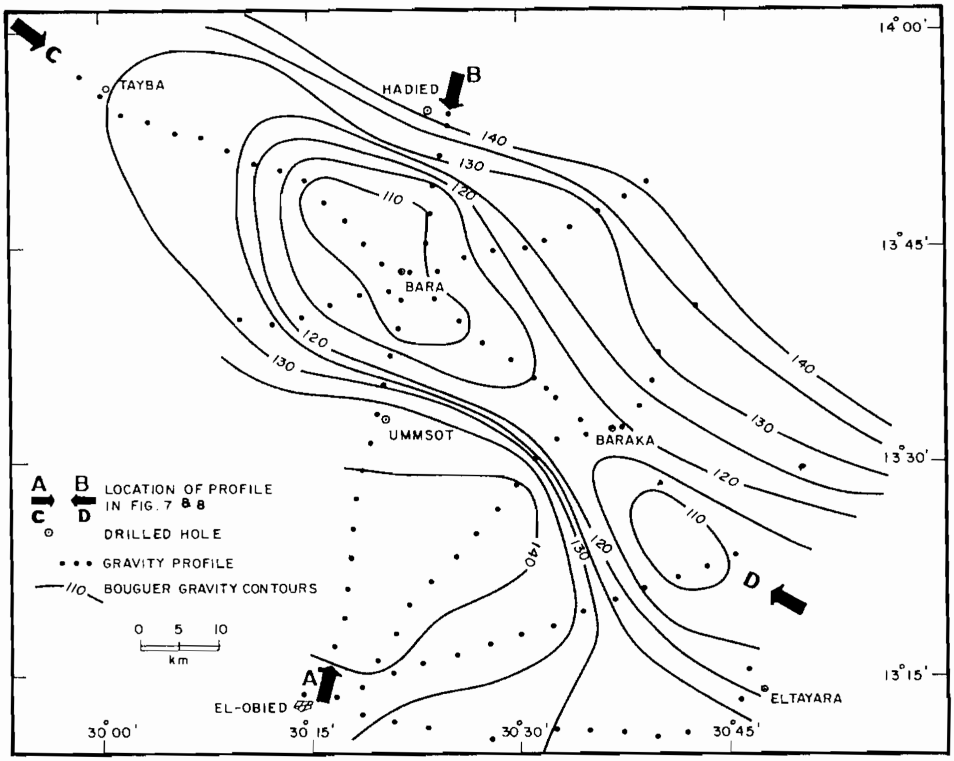

Fig. 2. Bouguer gravity map of the Bara Basin (after Ali 1979).

the lateral and the vertical extents of the Umm Ruwaba sediments a gravity survey was conducted in the Bara Basin (Ali 1979). About 800 gravity points were measured. The gravity stations were spaced at intervals of 0·5 and 1·0 km along 6 profiles, oriented approximately perpendicular to the expected strike of the basin. The measured gravity values were corrected for drift, latitude and height with respect to a base station whose absolute gravity value is 978145·91 mgal and located at El Obeid Airport (Fig. 1).

The resultant Bouguer gravity is shown in Fig. 2. The map reflects a prominent gravity high in the south where sediments are thin and the Basement rocks are exposed. The Bouguer gravity changes at a rate of −4 mgal/km northwards. The general strike of the contours suggests a narrow basin, about 45 km wide, bounded by steep sides and trending NW–SE.

REGIONAL–RESIDUAL SEPARATION

The Bouguer gravity map in Fig. 2 represents a combination of large scale (or regional) as well as local (or residual) anomalies. A major step in analysis of gravity data in the Bara Basin is to resolve the observed Bouguer anomaly in a way that highlights the local geological conditions and to obtain a residual gravity map suitable for modelling of the geometrical shape of the basin. To achieve this objective, the

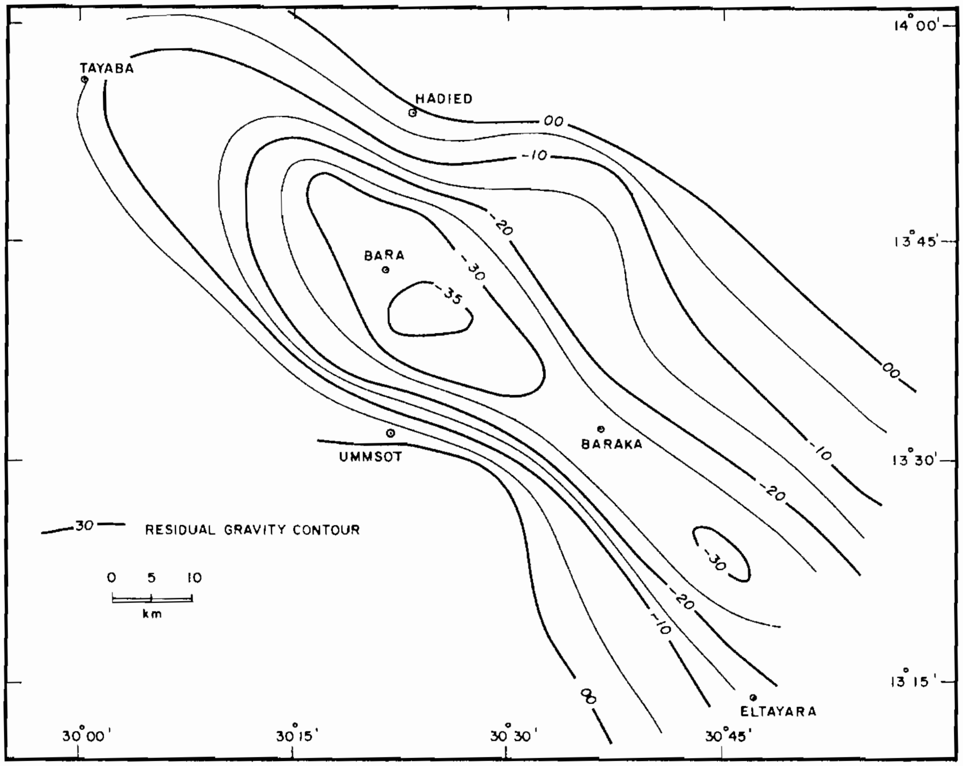

Fig. 3. Graphical smoothing residual gravity map of the Bara Basin.

graphical smoothing, polynomial trend surface, upward continuation, and 2-D wavenumber filtering techniques were used to separate the residual gravity field.

Graphical smoothing. In the graphical method the level of the regional field was visually estimated. Each profile was assessed separately and borehole information at some localities on the profiles were also used as control points by calculating the gravity effect of a slab body (Dobrin 1976). At profile intersections, the values of the residual (or regional) field were checked for consistency. The residual values from each profile were then transformed to the base map to prepare the residual gravity map as shown in Fig. 3.

Polynomial trend surface. In this method, observed Bouguer gravity data are used to compute the mathematically described 2-D surface $\{g(x, y)\}$ which gives the closest fit to the observed field $\{g_0(x, y)\}$ within a specific order. This surface $\{g(x, y)\}$ is considered as the regional field, and the residual value is the difference between it and the Bouguer gravity field $\{g_0(x, y)\}$. The decision as to what order of polynomial trend surface best approximates the regional field in a specific area is based on analysis of some statistical parameters such as the correlation coefficients and F-test function values (Davis 1973). Reviews of the method with application to gravity interpretation are given by Rankin & Lavin (1970), Coons *et al.* (1963), Simpson (1954), Dobrin (1976), Abdel-Rahman *et al.* (1985), and Ali (1987).

The main advantage of the polynomial trend surface is that irregularly spaced gravity points can be used. However, the presence of an anomalous feature of high frequency affects the residual field over the entire area.

Upward continuation. Analytical continuation of potential field vertically upward and downward has been used frequently to approximate the regional and residual potential fields (Henderson 1960; Peter 1949; Gupta & Ramani 1980). The goal of the upward continuation is to remove the high frequency local trend from the set of gravity data and leave, with minimum distortion, the low frequency variations due to broader features. Thus, the difference between the observed gravity value and the upward continued gravity value could convey information about the high frequency (shallow) sources. Henderson (1960) has derived the working upward continuation formula as follows:

$$g_k(r_1) = \sum_{i=1}^{N} g_0(r_i) F(r_i, K)$$

where $g_0(r_i)$ and $g_k(r_i)$ are respectively the observed (input) field at radial coordinate (r_i) and the upward continued field at K grid unit above the plane, $F(r_i, K)$ is a set of filters (coefficients) for computing $g_k(r_i)$. Upward continuation filters are described in Fuller (1967). Also, Gupta & Ramani (1980) have used the Fourier transform of the mapped gravity data to obtain the upward filters.

2-D wavenumber filtering. This method is based on Fourier transform (Cooley & Tukey 1965) and the concept of space and frequency domains. Its objective is to isolate near surface, high frequency components of the potential field from the low wavenumber regional trend. Therefore, successful regional–residual separation depends on anomalies being quite separate in the wavenumber domain. Nevertheless, unique separation can hardly be achieved, since a clear unambiguous break in gravity spectrum is rarely found. Filters based on space and/or frequency domains are presented in the literature by Dean (1958), Mesko (1965, 1966), Dampney (1966), Darby & Davis (1967), Zurflueh (1967), Fuller (1967), Syberg (1972), Steinger (1973, 1975), Clarke (1969), Spector & Grant (1970), Meyer (1974) and Gupta & Ramani (1980). Basically, 2-D wavenumber filtering of discrete data can be achieved by two-dimensional convolution as follows (Fuller 1967):

$$g_0(x, y) \sim \sum_{k=-x}^{x} \sum_{n=-y}^{y} W(k, n) g_i(x-k, y-n)$$

where $g_i(x, y)$ and $g_0(x, y)$ are the input gravity data and the filtered output respectively, and $W(k, n)$ are the filters (weights) which operate on the input data at coordinates (k, n) with respect to the point at which filtered output is desired.

The Wiener filter theory (Wiener 1949) provides a method of designing the filters. Optimum filter coefficients can be obtained by using inverse transformation of the frequency response and power spectrum of the gravity anomaly (Gupta & Ramani 1980).

In this study Fuller's (1967) 2-D high frequency filters have been used to derive the residual gravity values.

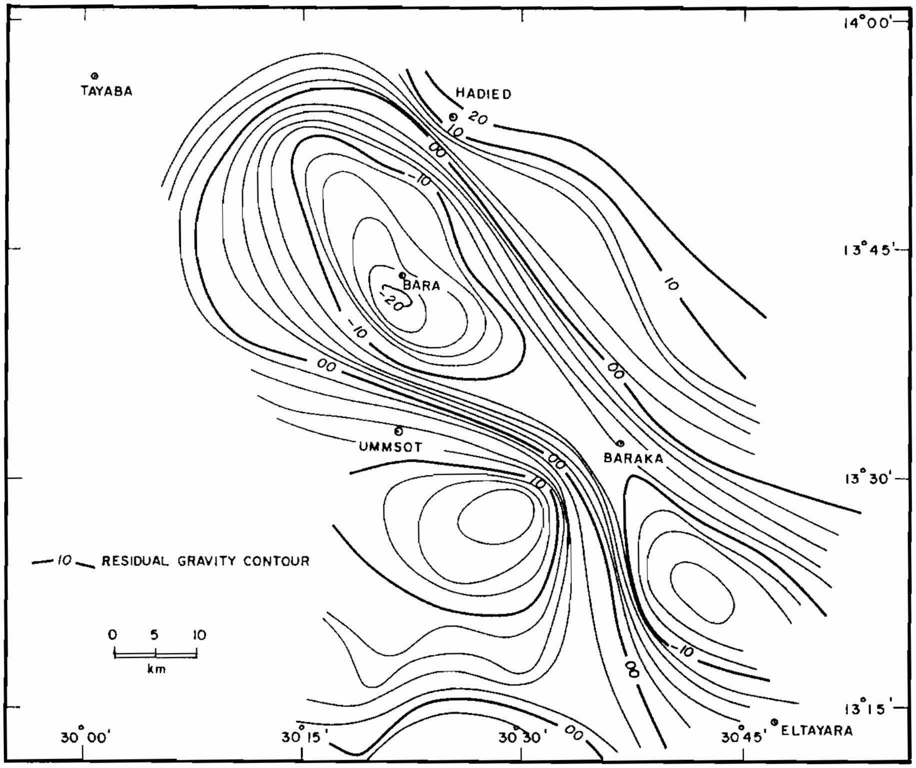

Fig. 4. First order polynomial residual gravity map of the Bara Basin.

RESULTS AND DISCUSSION

Fig. 3 shows the residual gravity map obtained by graphical smoothing. The shape of the contour lines have been controlled by automatic contouring. The map represents the effect of the sediments, which cause the whole magnitude of the residual values. Thus the residual map defines the existence of the basin clearly and provides a qualitative idea about the thickness of the sediments. The magnitudes of the residual values suggest that the maximum thickness of the sediments occurs about 2 km south of Bara Town and extends in a southeasterly direction. The southern margin of the basin lies about 45 km north of El Obeid Town (Fig. 1).

First order polynomial trend surface residual map is shown in Fig. 4. The calculation of the polynomial residual gravity values was based on a modification of the computer program by Davis (1973). In comparison with the graphical solution, the polynomial residual map defines the same features and trend of the basin as the graphical method does. However, the zero contour line, which is presumably the boundary of the basin, is shifted more towards the north in the polynomial interpretations. The low correlation coefficients of 0·41 in the first and 0·62 in the second order polynomial trend surfaces may indicate that the regional trend is weak. Analysis of the first order solution has revealed that the components of the regional gradient are 0·386 mgal/km and 0·466 mgal/km in the western and the southern directions respectively. This gives a resultant of 0·6 mgal/km of regional gradient

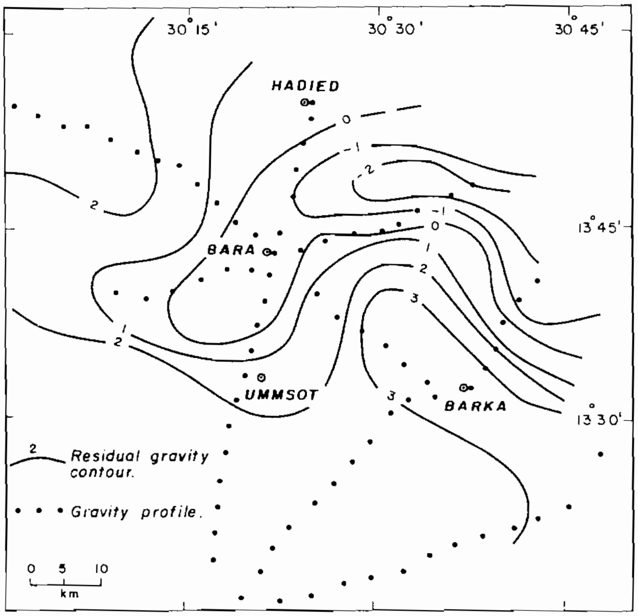

Fig. 5. Upward continuation residual gravity map of the Bara Basin.

towards S40W. This calculated regional gradient is probably inaccurate because of the limited length of the gravity profiles relative to the dimension of the Bara Basin gravity anomaly. However, the relatively low value of regional gradient may suggest that little overall density contrast exists between the Basement rocks on either side of the basin and the Nubian Sandstone Formation, which crops out on the northeastern side of the Bara Basin, and is probably relatively thin (Ali & Whiteley 1981).

To assist application of the filtering methods (upward continuation and 2-D high wavenumber filtering), the automatically contoured Bouguer gravity map (Fig. 2) was digitized into a 30 × 30 grid mosaic of 2·5 km interval, although such grid interpolation might affect the quality of the input data by adding noise components. Fuller's (1967) filters for upward continuation and high pass filters have been used to derive the residual maps. These filters have the advantage of being relatively accurate because their design is controlled by their frequency responses. Also, they work on seven data points so that the maximum loss of record is six data points on each edge of the map and they are amenable to computer programming. Gupta & Ramani (1978) suggest that the loss of data on edges of the map could be reduced by filling the surrounding of the input map (Bouguer gravity map) with data of similar kind.

Figure 5 maps residual gravity values representing the difference between observed Bouguer gravity and the upward continued gravity values to a height of 2·5 km, equivalent to one grid interval. It is clear that Fig. 5 bears little relation to the basin as it appears in the original Bouguer gravity map in Fig. 2. However, the residuals of Fig. 5 could reflect shallow inhomogeneities, possibly in the superficial deposits, or may be due to facies variation within the upper zone of the Umm Ruwaba

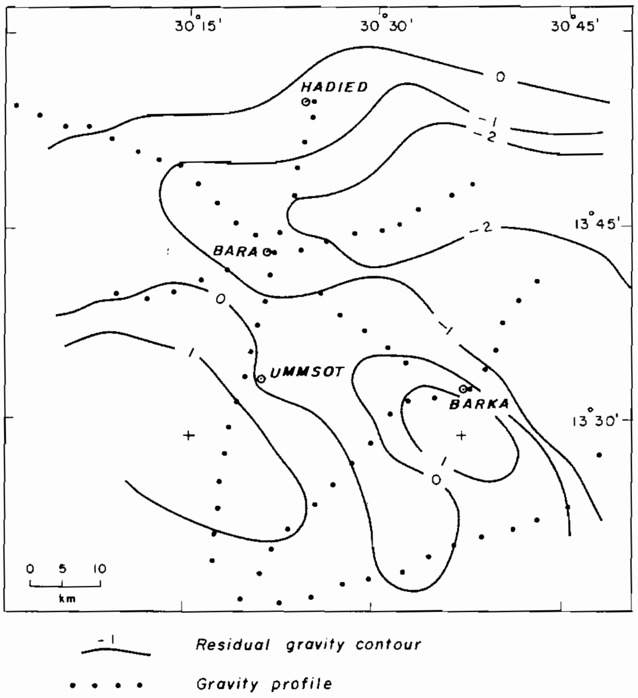

Fig. 6. High pass wavenumber filtering residual gravity map of the Bara Basin.

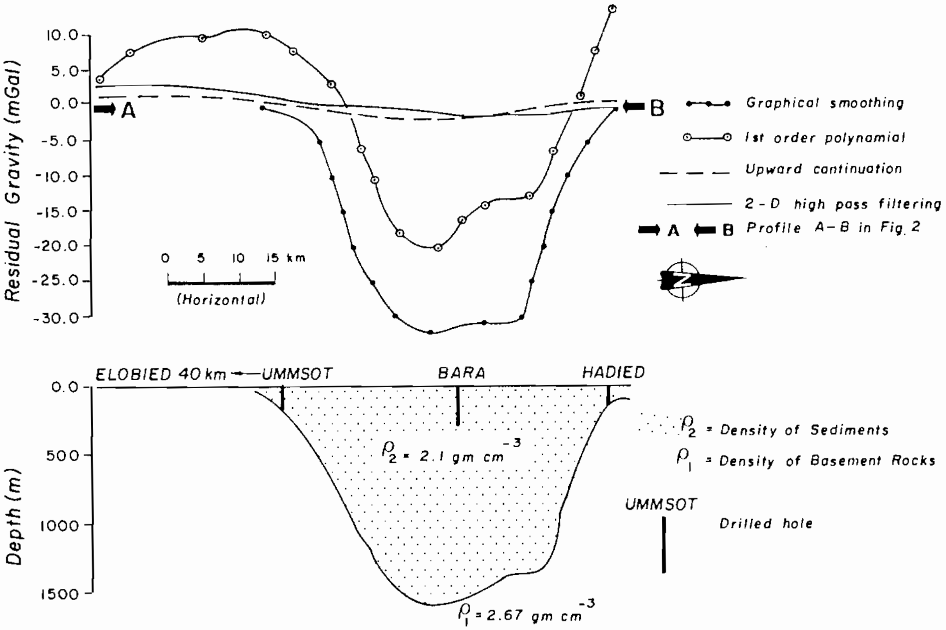

Fig. 7. Comparison of residual gravity values and interpreted 2-D geological model across the Bara Basin, based on graphical smoothing residual gravity (see Fig. 2 for location).

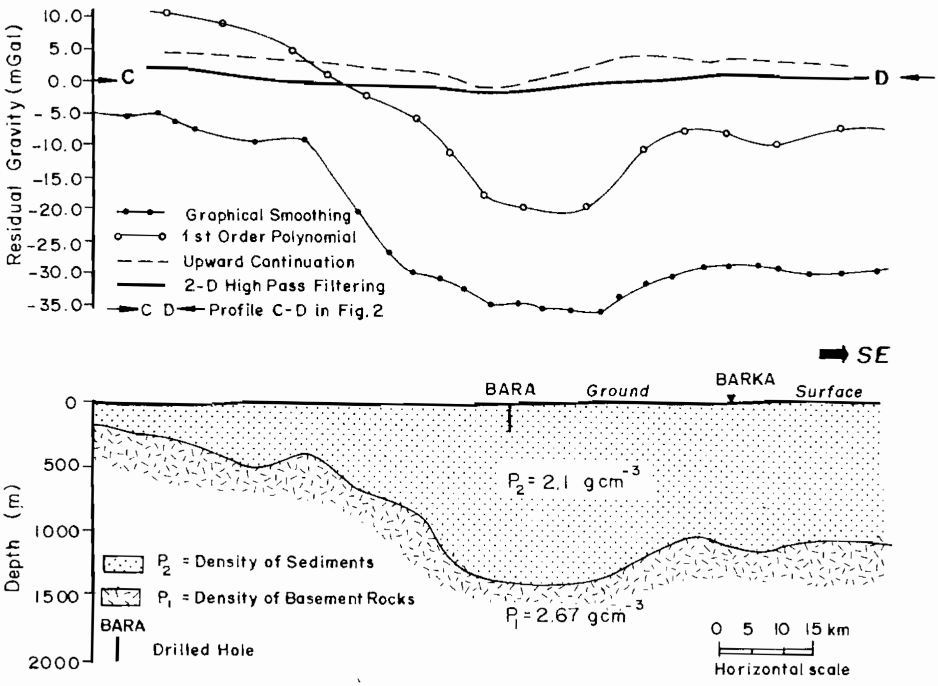

Fig. 8. Comparison of residual gravity values and interpreted 2-D geological model along the Bara Basin, based on graphical smoothing residual gravity (see Fig. 2 for location).

sediments. It is more likely that the result is less defined because the grid interval is not small enough to reveal these shallow features, added to which, extraneous noise has been created by digitization and interpolation procedures.

Fig. 6 shows the high pass filter residual map. The high pass filter in itself is a residual operator which defines high frequency sources; therefore it is likely that high pass filter and upward continued residual gravity maps reveal similar features. As shown in Fig. 6, the high pass filter map (2-D wavenumber filtering) is less defined in comparison with the upward continuation map (Fig. 5). Fig. 6 also shows a zone of negative anomaly that approximately coincides with the general trend of the basin. Though the high pass filter residual map (Fig. 6) is dissimilar to the upward continuation residual gravity map (Fig. 5), it does show some common features, especially the negative anomaly zones in the east central part of the basin. Thus the high pass filter residual gravity map in Fig. 6 can be used to get a general idea about the relative thicknesses of the shallow sources, which might be the superficial deposits.

Results of graphical smoothing, polynomial trend surface, upward continuation and 2-D high pass wavenumber filtering are compared on two profiles (A–B and C–D in Fig. 2). These are, respectively, perpendicular to and along the strike of the basin, as shown in Figs 7 and 8. These figures also show quantitative, interpreted 2-D geologic models based on the graphical residual gravity and a density contrast of $-0{\cdot}6\,\mathrm{g\,cm^{-3}}$ between the Umm Ruwaba sediments and the Basement rocks. Interpretation of these models was achieved by employing a computer method described by Qureshi & Kumar (1976). The sediments attain a maximum thickness of 1600 m around the central part of the basin. To the north, south and west the basin is bounded by steep sides (Figs 7 & 8) which may suggest that the basin has resulted from normal faulting. Tectonically, the Bara Basin represents the northwestern limit of a series of fault-bounded sedimentary basins forming the White Nile Rift (see Fig. 1), which extends from southern to central Sudan along a northwest trend (Browne *et al.* 1984).

CONCLUSIONS

Application of graphical smoothing, polynomial trend surface, upward continuation and 2-D high wavenumber filtering techniques to separate residual gravity in a deep sedimentary basin in western Sudan has highlighted the subjectivity in regional–residual separation.

Analytical techniques such as upward continuation and 2-D high wavenumber filtering tend to reduce the human bias, but the resultant residual maps do not define the geometry of the basin. Such residual maps may reflect the shallow sources such as superficial deposits in the basin.

Although graphical smoothing is a rather simple approach, the residual gravity map obtained defines the limit of the basin clearly and the residual map can be used for modelling the geometry of the basin. There is a general similarity between graphical smoothing and polynomial residual maps over much of the area despite the offset in the residual gravity values.

From this comparative study it can be concluded that, in such deep sedimentary basins with relatively high magnitude of Bouguer gravity values, graphical smoothing can be successfully used to separate the regional and residual components with little inaccuracy.

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فصل المركب الموضعي من قيم الجاذبية (بوجير) المقاسة على أحد الأحواض الرسوبية العميقة بغرب السودان

حامد عمر علي
قسم الجيولوجيا بجامعة الكويت ، ص . ب . ٥٩٦٩ ،
الصفاة ١٣٠٦٠ ، الكويت

خلاصة

طرق فصل المركب الموضعي والتي تشمل التشذيب البياني والتحليل السطحي متعدد الحدود والتتابع الارتقائي والتنقية ذات البعدين عالية التردد ، قد استخدمت لفصل قيم الجاذبية الموضعية من قياسات أخذت على أحد الأحواض الرسوبية العميقة بغرب السودان . وكان الغرض من الدراسة هو الحصول على خريطة لقيم الجاذبية الموضعية لابراز الأبعاد الحقيقية للحوض بغية التعرف على المصادر العميقة للمياه الجوفية .

أبرزت نتائج التتابع الارتقائي والتنقية عالية التردد ذات البعدين بعض السمات العامة الا انها لا تظهر شكل الحوض بوضوح .

إن معطيات الجاذبية الموضعية المتحصلة بهاتين الطريقتين ربما يمكن تقييمها فقط على مستوى مصادر ضحلة للغاية مثل الرسوبيات السطحية . ومن ناحية أخرى فإن خرائط قيم الجاذبية الموضعية الناتجة عن استخدام التشذيب البياني والتحليل السطحي متعدد الحدود توضح شكل الحوض بصورة جلية وتشير الى تماثل عام على الرغم من ان هناك تباينا ملحوظا في قيم الجاذبية .

وعند مقارنة كل هذه النتائج بالجيولوجية البائنة ومعطيات الآبار فأن خريطة الجاذبية الموضعية المستقاة من التشذيب البياني ، وحدها ، هي التي تعكس الابعاد الحقيقية للحوض وتفي بغرض التخطيط الكمي على فرض ان قيم الجاذبية الموضعية كلها ناتجة من تأثير الرسوبيات .