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Almost quaternion structure on cross-section in the cotangent bundle

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ABSTRACT

Yano & Ako (1973) obtained certain conditions under which the complete lift of tensor fields in M admits an almost quaternion structure of first and second kind in tangent bundle. Yano (1967) defined tensor fields and a connection on cross-section in the cotangent bundle. Here we obtain conditions under which the complete lift of tensor fields in M admitting almost quaternion structure defines a similar structure on cross-section in the cotangent bundle. Further, by introducing a symmetric affine connection in M we obtain equivalent conditions for the set $\{F^C, G^C, H^C\}$ to be almost quaternion.

NOTATION

Yano (1967) describes indices $a, b, c, \ldots, h, i, j, l, q, \ldots$ which have ranges in $l, \ldots, n$, and indices $A, B, C, \ldots, \lambda, \mu, \nu, \ldots$ which have range in $l, \ldots, n, n+1, \ldots, 2n$. We put $\bar{i} = i + n$. Summation over repeated indices is always implied. Entries of matrices are written as A_i^j, A_{ji} or A^{ji} and in all cases, j is the row index while i is the column index. We follow the same notation and definition.

INTRODUCTION

Let (M^n, g) be a C^∞, n-dimensional differentiable manifold and $C_T(M^n)$, its cotangent bundle. Let $\pi : C_T(M^n) \to M^n$ be the natural projection of $C_T(M^n)$ onto M^n. If $\{U, x^h\}$ is a coordinate system of M^n, then $\{U, x^h\}$ induces in a natural way a coordinate system $\{\pi^{-1}(U), (x^h, p_h)\}$ in $C_T(M^n)$, where p_h is the component of l-form at the point x^h. This new coordinate system is called an induced coordinate system in $C_T(M^n)$. Yano & Ako (1973) defined a manifold which possesses an almost quaternion structure of first kind if there exist a set of three distinct tensor fields F, G, H of type (1, 1) such that

$$\begin{aligned} &F^2 = -1, \qquad G^2 = -1, \qquad H^2 = -1 \\ &F = GH = -HG, \qquad G = HF = -FH, \qquad H = FG = -GF \end{aligned} \tag{1}$$

Similarly a manifold is called an almost quaternion manifold of second kind if for the tensor fields F, G, H of type (1, 1), the following are satisfied:

$$F^2 = -1, \qquad G^2 = 1, \qquad H^2 = 1$$
$$F = GH = -HG, \qquad G = HF = -FH, \qquad H = FG = -GF. \tag{2}$$

Let w be a global 1-form defined in M^n whose local expression is $w = w_i(x)\, dx^i$. Then w defines a cross-section in the cotangent bundle $C_T(M^n)$ whose parametric representation is

$$x^h = x^h, \qquad p_h = w_h(x). \tag{3}$$

Thus the tangent vectors $B_i^A = \partial_i x^A$ to the cross-section have components

$$B_i^A = \begin{bmatrix} \delta_i^h \\ \partial_i w_h \end{bmatrix}. \tag{4}$$

On the other hand, the fibre being represented by

$$x^h = \text{constant}, \qquad p_h = p_h \tag{5}$$

and the tangent vectors $C_{\bar{i}}^A = \partial_{\bar{i}} x^A$ to the fibre have components

$$C_{\bar{i}}^A = C^{iA} = \begin{bmatrix} 0 \\ \delta_i^h \end{bmatrix}. \tag{6}$$

The vectors B_i^A and $C_{\bar{i}}^A$ being linearly independent, form a frame along the cross-section. We call this the frame (B, C) along the cross-section (Yano 1967). The coframe $(B_A^h, C_A^{\bar{h}})$ corresponding to this frame is given by

$$B_A^h = (\delta_i^h, 0)$$
$$C_A^{\bar{h}} = C_{hA} = (-\partial_i w_h, \delta_h^i). \tag{7}$$

We call this the coframe (B, C) along the cross-section. The basic 1-form $p = p_i\, dx^i$ has the expression $p = w_i\, dx^i$ and the basic 2-form the expression $dp = 1/2(\partial_j w_i - \partial_i w_j)\, dx^j\, dx^i$ on the cross-section. The complete lift X^C of a vector field X in M to $C_T(M)$, has components

$$\begin{bmatrix} X^h \\ -\mathcal{L}_X w_h \end{bmatrix} \tag{8}$$

with respect to the frame (B, C) along the cross-section. Thus we have

$$X^C : B_i^A X^i - C^{iA}(\mathcal{L}_X w_i) \tag{9}$$

Yano (1967) proved that the complete lift X^C of a vector field X in M to $C_T(M)$ is tangent to the cross-section determined by an 1-form w in M if and only if the Lie derivative of w with respect to X vanishes in M. In this way Yano (1967, p. 38) characterized N^C as

$$N^C(X^C, Y^C) = (N(X, Y))^C - ((\mathcal{L}_X N)_Y - (\mathcal{L}_Y N) + N_{[X, Y]})^V \tag{10}$$

where V denotes the vertical lift.

ALMOST QUATERNION STRUCTURE ON CROSS SECTION IN THE COTANGENT BUNDLE

Yano (1967) shows that the complete lift $F^C = \tilde{F}$ in M^n to $C_T(M^n)$ has components

$$\tilde{F}_B^A = \begin{bmatrix} F_i^h & 0 \\ p_a(\partial_i F_h^a - \partial_h F_i^a) & F_h^i \end{bmatrix}. \tag{11}$$

If we consider the components $\tilde{F}_B^A$ on a cross-section in the cotangent bundle with respect to frame (B, C), we have

$$\tilde{F}_B^A = \begin{bmatrix} F_i^h & 0 \\ (\partial_i F_h^a - \partial_h F_i^a)w_a - F_i^t \partial_t w_h + F_h^t \partial_i w_t & F_i^h \end{bmatrix}$$

or, in short, components of $\tilde{F}$ are rewritten as

$$\tilde{F}_B^A = \begin{bmatrix} F_i^h & 0 \\ P_{ih} & F_h^i \end{bmatrix}$$

where

$$P_{ih} \overset{\text{def}}{=} (\partial_i F_h^a - \partial_h F_i^a)w_a - F_i^t \partial_t w_h + F_h^t \partial_i w_t$$

Now, we will investigate the condition for F to be an almost quaternion on $C_T(M_n^n)$. We denote

$$w_a(\partial F_h^a/\partial x^i - \partial F_i^a/\partial x^h) \quad \text{by} \quad w_{a|i} \quad F_{h|}^a$$

$$(F^C)^2 = \tilde{F}_B^A \tilde{F}_C^B = \begin{bmatrix} F_i^h & 0 \\ p_{ih} & F_h^i \end{bmatrix}\begin{bmatrix} F_j^i & 0 \\ P_{ji} & F_i^j \end{bmatrix}$$

$$= \begin{bmatrix} F_i^h F_j^i & 0 \\ P_{ih} F_j^i + P_{ji} F_h^i & F_h^i F_i^j \end{bmatrix}$$

Since $F^2 = -1$,

$$(F^C)^2 = \begin{bmatrix} -\delta_j^h & 0 \\ P_{ih} F_j^i + P_{ji} F_h^i & -\delta_h^j \end{bmatrix}.$$

Thus $(F^C)^2 = -1$ if $P_{ih} F_j^i + P_{ji} F_h^i = 0$, i.e.

$$\{(\partial_{|i} F_{h|}^a)w_a - F_i^t \partial_t w_h + F_h^t \partial_i w_t\}F_j^i + \{(\partial_{|j} F_{i|}^a)w_a - F_j^t \partial_t w_i + F_i^t \partial_j w_t\}F_h^i = 0. \tag{13}$$

Similarly, $(G^C)^2 = -1$ and $(H^C)^2 = -1$, i.e.

$$\{(\partial_{|i} G_{h|}^a)w_a - G_i^t \partial_t w_h + G_h^t \partial_i w_t\}G_j^i + \{\partial_{|j} G_{i|}^a)w_a - G_j^t \partial_t w_i + G_i^t \partial_j w_t\}G_h^i = 0 \tag{14}$$

and

$$\{(\partial_{|i} H_{h|}^a)w_a - H_i^t \partial_t w_h + H_h^t \partial_i w_t)H_j^i + \{(\partial_{|j} H_i^a)w_a - H_j^t \partial_t w_i + H_i^t \partial_j w_t\}H_h^i = 0 \tag{15}$$

Again

$$G^C = \tilde{G}_B^A = \begin{bmatrix} G_i^h & 0 \\ Q_{ih} & G_h^i \end{bmatrix} \quad \text{and} \quad H^C = \tilde{H}_C^B = \begin{bmatrix} H_j^i & 0 \\ R_{ji} & H_i^j \end{bmatrix}$$

where

$$Q_{ih} \overset{\text{def}}{=} (\partial_i G^a_h - \partial_h G^a_i)w_a - G^t_i \partial_t w_h + G^t_h \partial_i w_t$$

and

$$R_{ji} \overset{\text{def}}{=} (\partial_j H^a_i - \partial_i H^a_h)w_a - H^t_j \partial_t w_i + H^t_i \partial_j w_t$$

Thus

$$G^C H^C = \begin{bmatrix} F^h_j & 0 \\ Q_{ih} H^i_j + R_{ji} G^i_h & F^j_h \end{bmatrix}.$$

We know that $G^C H^C = F^C$; on simplifying, the equation becomes

$$w_a\{H^i_j \partial_{|i} G^a_{h|j} + G^i_h \partial_{|j} H^a_{i|}\} + G^t_h H^i_j \partial_i w_t + H^t_i G^i_h \partial_j w_t$$
$$= w_a\{G^a_i \partial_{|j} H^i_{h|} + H^i_j \partial_j G^a_i - H^i_j \partial_h G^a_i\} + G^t_i H^i_h \partial_j w_t + H^t_j G^i_h \partial_t w_i \quad (16)$$

Similarly we obtain conditions for $F^C G^C = H^C$ and $H^C F^C = G^C$ respectively, i.e.

$$w_a\{G^i_j \partial_{|i} F^a_{h|} + F^i_h \partial_{|j} G^a_{i|}\} + F^t_h G^i_j \partial_i w_t + G^t_i F^i_h \partial_j w_t$$
$$= w_a\{F^a_i \partial_{|j} G^i_{h|} + G^i_h \partial_j F^a_i - G^i_j \partial_h F^a_i\} + F^t_i G^i_h \partial_j w_t + G^t_j F^i_j \partial_t w_i \quad (17)$$

and

$$w_a\{F^i_j \partial_{|i} H^a_{h|} + H^i_h \partial_{|j} F^a_{|j}\} + H^t_h F^i_j \partial_i w_t + F^t_i H^i_h \partial_j w_t$$
$$= w_a\{H^a_i \partial_{|j} F^i_{h|} + F^i_h \partial_j H^a_i - F^i_j \partial_h H^a_i\} + H^t_i F^i_h \partial_j w_t + F^t_j H^i_j \partial_t w_t. \quad (18)$$

Further, since

$$HG = \begin{bmatrix} -F^h_j & 0 \\ R_{ih} G^i_j + Q_{ji} H^i_h & -F^j_h \end{bmatrix}$$

$$G^C H^C + H^C G^C = \begin{bmatrix} 0 & 0 \\ Q_{ih} H^i_j + R_{ji} G^i_h + R_{ih} G^i_j + Q_{ji} H^i_h & 0 \end{bmatrix}$$

From above we obtain $G^C H^C + H^C G^C = 0$ if

$$\{(\partial_{|i} G^a_{h|})w_a - G^t_i \partial_t w_h + G^t_h \partial_i w_t\}H^i_j + \{(\partial_{|j} H^a_{i|})w_a - H^t_j \partial_t w_i + H^t_i \partial_j w_t\}G^i_h$$
$$+ \{(\partial_{|i} H^a_{h|})w_a - H^t_i \partial_t w_h + H^t_h \partial_i w_t\}G^i_j + \{(\partial_{|j} G^a_{i|})w_a - G^t_j \partial_t w_i + G^t_i \partial_j w_t\}H^i_h = 0$$

or,

$$w_a\{\partial_{|i} G^a_{h|} H^i_j + \partial_{|i} H^a_{h|} G^i_j + \partial_{|j} G^a_{i|} H^i_h + \partial_{|j} H^a_{i|} G^i_h\}$$
$$+ (G^t_h H^i_j + H^t_h G^i_j)\partial_i w_t - (G^t_j H^i_h + H^t_j G^i_h)\partial_t w_i = 0.$$

As we know $G^C H^C + H^C G^C = 0$, so the above equation becomes

$$w_a\{\partial_{|i} G^a_{h|} H^i_j + \partial_{|i} H^a_{h|} G^i_j + \partial_{|j} G^a_{i|} H^i_j + \partial_{|j} H^a_{i|} G^i_h\} = 0 \quad (19)$$

Similarly, we obtain $H^C F^C + F^C H^C = 0$ and $F^C G^C + G^C F^C = 0$ as

$$w_a\{\partial_{|i} F^a_{h|} G^i_j + \partial_{|i} G^a_h F^i_j + \partial_{|j} F^a_{i|} F^i_j + \partial_{|j} F^a_{i|} F^i_h \neq 0 \quad (20)$$

$$w_a\{\partial_{|i} H^a_{h|} F^i_j + \partial_{|i} F^a_{h|} H^i_j + \partial_{|j} H^a_{i|} F^i_h + \partial_{|j} F^a_{i|} H^i_h\} = 0 \quad (21)$$

Thus we obtain

Theorem 1: If a manifold M^n has an almost quaternion structure (F, G, H) of first kind (respectively, second kind), then (F^C, G^C, H^C) also has an almost quaternion structure of first kind (respectively, second kind) on the cross-sections in the cotangent bundle if an only if Eqns (12) to (21) are true. Yano (1965) defined a connection coefficient

$$F^a_{h,i} = \frac{\partial F^a_h}{\partial x^i} + \Gamma^a_{1i} F^1_h - \Gamma^k_{hi} F^a_k$$

where Γ^k_{hi} are the components of an affine connection ∇ in M^n. Assuming Γ is symmetric

$$\Gamma^a_1 F^1_{|hi|} \overset{\text{def}}{=} \tfrac{1}{2}(\Gamma^a_{1h} F^1_i - \Gamma^a_{1i} F^a_h).$$

Eqns (13) to (21) can be equivalently expressed in terms of connection coefficients.

$$\begin{aligned}[t] &[\{(F^a_{h,i} - F^a_{i,h}) + 2\Gamma^a_q F^q_{[hi|}\}w_a + (F^t_h \partial_i w_t - F^t_i \partial_t w_h)]F^i_j \\ &\quad + [\{(F^a_{i,j}) + F^a_{j,i}) + 2\Gamma^a_q F^q_{[ij|}\}w_a + (F^t_i \partial_j w_t - F^t_j \partial_t w_i)]F^i_h = 0, \end{aligned} \tag{22}$$

$$\begin{aligned}[t] &[\{(G^a_{h,i} - G^a_{i,h}) + 2\Gamma^a_q G^q_{[hi|}\}w_a + (G^t_h \partial_i w_t - G^t_i \partial_t w_h)]G^i_j \\ &\quad + [\{(G^a_{i,j} - G^a_{j,i}) + 2\Gamma^a_q G^q_{[ij|}\}w_a + (G^t_i \partial_j w_t - G^t_j \partial_t w_i)]G^i_h = 0 \end{aligned} \tag{23}$$

and

$$\begin{aligned}[t] &[\{(H^a_{h,i} - H^a_{i,h}) + 2\Gamma^a_q H^q_{[hi|}\}w_a + (H^t_h \partial_i w_t - H^t_i \partial_t w_h)]H^i_j \\ &\quad + [\{(H^a_{i,j} - H^a_{j,i}) + 2\Gamma^a_q H^q_{[ij|}\}w_a + (H^t_i \partial_j w_t - H^t_j \partial_t w_i)]H^i_h = 0 \end{aligned} \tag{24}$$

$$\begin{aligned}[t] &\partial_{|h} F^a_{j|} w_a + 2w_a \Gamma^a_q F^q_{[hj|} - F^t_h \partial_t w_j + F^t_j \partial_t w_h \\ &\quad = w_a\{H^i_j \partial_{|i} G^a_{h|} + G^i_h \partial_{|j} H^a_{i|}\} + 2wa\Gamma^a_1 G^1_{|ih|} H^i_j + 2w_a \Gamma^a_1 H^1_{|ji|} \\ &\qquad \times G^i_h - G^t_i H^i_j \partial_t w_h - H^t_j G^i_h \partial_t w_i + G^t_h H^i_j \partial_i w_t + H^t_i G^i_h \partial_j w_t, \end{aligned} \tag{25}$$

$$\begin{aligned}[t] &\partial_{|h} H^a_{j|} w_a + 2wa\Gamma^a_1 H^1_{|hj|} - H^t_h \partial_t w_j + H^t_j \partial_t w_h \\ &\quad = w_a\{G^i_j \partial_{|i} F^a_{h|} + F^i_h \partial_{|j} G^a_{i|}\} + 2w_a\Gamma^a_1 F^1_{|ih|} G^i_j + 2w_a \Gamma^a_1 G^1_{|ji|} F^i_h - F^t_i G^i_j \partial_t w_h \\ &\qquad - G^t_j F^i_h \partial_t w_i + F^t_h G^i j \partial_i w_t + G^t_i F^i_h \partial_j w_t, \end{aligned} \tag{26}$$

$$\begin{aligned}[t] &\partial_{|h} G^a_{j|} w_a + 2w_a \Gamma^a_1 G^1_{|hj|} - G^t_h \partial_t w_j + G^t_j \partial_t w_h \\ &\quad = w_a\{F^i_j \partial_{|i} H^a_{h|} + H^i_h \partial_i F^a_i\} + 2w_a \Gamma^a_1 H^1_{|ih|} F^i_j + 2w_a \Gamma^a_1 F^1_{|ji|} H^i_h \\ &\qquad - H^t_i F^i_j \partial_t w_h - F^t_j H^i_h \partial_t w_i + H^t_h F^i_j \partial_i w_t + F^t_i H^i_h \partial_j w_t, \end{aligned} \tag{27}$$

$$\begin{aligned}[t] &w_a(G^a_{h,i} - G^a_{i,h})H^i_j + 2w_a \Gamma^a_1 G^1_{|ih|} H^i_j + w_a(H^a_{h,i} - H^a_{i,h})G^i_j \\ &\quad + 2w_a \Gamma^a_1 H^1_{|ih|} G^i_j + w_a(G^a_{i,j} - G^a_{j,i})H^i_h + 2w_a \Gamma^a_1 G^1_{|ji|} H^i_h \\ &\qquad + w_a(H^a_{i,j} - H^a_{j,i})G^i_h + 2w_a \Gamma^a_1 H^1_{|ji|} G^i_h = 0 \end{aligned} \tag{28}$$

$$\begin{aligned}[t] &w_a(F^a_{h,i} - F^a_{i,h})G^i_j + 2w_a \Gamma^a_1 F^1_{|ih|} G^i_j + w_a(G^a_{h,i} - G^a_{i,h})F^i_j \\ &\quad + 2w_a \Gamma^a_1 G^1_{|ih|} F^i_j + w_a(F^a_{i,j} - F^a_{j,i})G^i_h + 2w_a \Gamma^a_1 F^1_{|ji|} G^i_h \\ &\qquad + w_a(G^a_{i,j} - G^a_{j,i})F^i_h + 2w_a \Gamma^a_1 G^1_{|ji|} F^i_h = 0, \end{aligned} \tag{29}$$

$$w_a(H^a_{h,i} - H^a_{i,h})F^i_j + 2w_a \Gamma^a_1 H^1_{|ih|} F^i_j + w_a(F^a_{h,i} - F^a_{i,h})H^i_j$$
$$+ 2w_a \Gamma^a_1 F^1_{|ih|} H^i_j + w_a(H^a_{i,j} - H^a_{j,i})F^i_h + 2w_a \Gamma^a_1 H^1_{|ji|} F^i_h$$
$$+ w_a(F^a_{i,j} - F^a_{j,i})H^i_h + 2w_a \Gamma^a_1 F^1_{|ji|} H^i_h = 0. \quad (30)$$

Thus, we have

Theorem 2. In an almost quaternion manifold M of the first kind (respectively second kind) with almost quaternion structure (F, G, H), the cross-section in cotangent bundle $C_T(M^n)$ also possesses an almost quaternion structure (F^C, G^C, H^C) of the first kind (respectively second kind) if and only if Eqns (22) to (30) are true where ∇ is a symmetric affine connection in M. Yano (1965) showed

$$w_a(F^a_{h,i} - F^a_{i,h}) = v_{h,i} - v_{i,h} = \text{Curl } v.$$
$$w_a(G^a_{i,j} - G^a_{j,i}) = \bar{v}_{i,j} - \bar{v}_{j,i} = \text{Curl } v$$
$$w_a(H^a_{j,k} - H^a_{k,j}) = \bar{\bar{v}}_{j,k} - \bar{\bar{v}}_{k,j} = \text{Curl } \bar{\bar{v}}$$

In case, $F^a_{h,i} = G^a_{i,j} = H^a_{j,k} = 0$. Hence, in view of the above relation, the Eqns (22) to (30) become

$$w_a \Gamma^a_1 F^1_{|hi|} F^i_j + w_a \Gamma^a_1 F^1_{|ij|} F^i_h = 0, \quad (31)$$
$$w_a \Gamma^a_1 G^1_{|hi|} G^i_j + w_a \Gamma^a_1 G^1_{|ij|} G^i_h = 0, \quad (32)$$
$$w_a \Gamma^a_1 H^1_{|hi|} H^i_j + w_a \Gamma^a_1 H^1_{|ij|} H^i_h = 0, \quad (33)$$

$$2w_a \Gamma^a_1 F^1_{|hj|} - F^t_h \partial_t w_j + F^t_j \partial_t w_h$$
$$= 2w_a \Gamma^a_1 G^1_{|ih|} H^i_j + 2w_a \Gamma^a_1 H^1_{|ji|} G^i_h - G^t_i H^i_j \partial_t w_h$$
$$- H^t_j G^i_h \partial_t w_i + G^t_h H^i_j \partial_i w_t + H^t_i G^i_h \partial_j w_t, \quad (34)$$

$$2w_a \Gamma^a_1 H^1_{|hj|} - H^t_h \partial_t w_j + H^t_j \partial_t w_h$$
$$= 2w_a \Gamma^a_1 F^1_{|ih|} G^i_j + 2w_a \Gamma^a_1 G^1_{|ji|} F^i_h - F^t_i G^i_j \partial_t w_h$$
$$- G^t_j F^i_h \partial_t w_i + F^t_h G^i_j \partial_i w_t + G^t_i F^i_h \partial_j w_t, \quad (35)$$

$$2w_a \Gamma^a_1 G^1_{|hj|} - G^t_h \partial_t w_j + G^t_j \partial_t w_h$$
$$= 2w_a \Gamma^a_1 H^1_{|ih|} F^i_j + 2w_a \Gamma^a_1 F^1_{|ji|} H^i_h - H^t_i F^i_j \partial_t w_h - F^t_j H^i_h \partial_t w_i$$
$$+ H^t_h F^i_j \partial_i w_t + F^t_i H^i_h \partial_i w_t, \quad (36)$$

$$w_a \Gamma^a_1 G^1_{|ih|} H^i_j + w_a \Gamma^a_1 H^1_{|ih|} G^i_j + w_a \Gamma^a_1 G^1_{|ji|} H^i_h + w_a \Gamma^a_1 H^1_{|ji|} G^i_h = 0 \quad (37)$$
$$w_a \Gamma^a_1 F^1_{|ih|} G^i_j + w_a \Gamma^a_1 G^1_{|ih|} F^i_j + w_a \Gamma^a_1 F^1_{|ji|} G^i_h + w_a \Gamma^a_1 G^1_{|ji|} F^i_h = 0, \quad (38)$$
$$w_a \Gamma^a_1 H^1_{|ih|} F^i_j + w_a \Gamma^a_1 F^1_{|ih|} H^i_j + w_a \Gamma^a_1 H^1_{|ji|} F^i_h + w_i \Gamma^a_1 F^1_{|ji|} H^i_h = 0 \quad (39)$$

Thus we have the following corollaries:

Corollary A. In an almost quaternion manifold (F, G, H) of first kind (respectively second kind), if the covariant derivatives of F, G, H vanish, then (F^C, G^C, H^C) defines a quaternion structure of first kind (respectively second kind) on cross-section in the cotangent bundle if and only if Eqns (31) to (39) hold.

Corollary B. If in a manifold having quaternion structure (F, G, H) of first kind (respectively second kind), Curl $v =$ Curl $\bar{v} =$ Curl $\bar{\bar{v}} = 0$, then (F^C, G^C, H^C) defines an almost quaternion structure of first kind (respectively second kind) in cross-section in the cotangent bundle if and only if Eqns (31) to (39) hold. Yano & Davis (1975) proved that in an almost quaternion manifold M if any two of six Nijenhuis tensors

$$[F, F], [P, G], [G, H], [H, F], [G, G], [H, H]$$

vanish, then the others must vanish. We have

Theorem 3. Suppose that a manifold M has an almost quaternion structure (F, G, H) of first kind, (respectively second kind), then the cross-section determined by an 1-form w in $C_T(M^n)$ also defines an almost quaternion structure (F^C, G^C, H^C) of the same kind if Nijenhuis tensors

$$[F^C, F^C], [G^C, G^C], [H^C, H^C], [F^C, G^C], [G^C, H^C], [H^C, F^C]$$

vanish. The proof follows by straight forward calculations by virtue of Eqn (10).

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بنية رباعية تقريبا على مقطع عرضي في حزمة المماس المشترك

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خلاصة

لقد حصل يانو وآكو (١٩٧٣) على شروط يسمح فيها الصعود الكامل للحقول التنسورية في M ببناء رباعي تقريبا من النوعين الأول والثاني في حزمة مماسية ، كما عرّف يانو (١٩٦٧) الحقول التنسورية والإرتباط على مقطع عرضي في حزمة المماس المشترك .

وفي هذا البحث حصل المؤلف على شروط تسمح للصعود الكامل للحقول التنسورية في M ببناء رباعي تقريبا ، أن يُعرِّف بناء مماثلا على المقطع العرضي في حزمة المماس المشترك . وكذلك ، بادخال إرتباط أفيني متناظر في M ، حصل على شروط مكافئة للمجموعة (F^C, G^C, H^C) لتكون رباعية تقريبا .