J. Univ. Kuwait (Sci.) **16**, 1989

Steady-state input characteristics of induction motors fed from variable frequency sources

M. H. SALAMA

Department of Electrical and Computer Engineering, University of Kuwait, P.O. Box 5969, Safat 13060, Kuwait

ABSTRACT

This paper presents a mathematical model for the magnetizing reactance in terms of voltage, frequency, load torque and motor rating. This model is derived from no-load test data. Using this model the steady-state performance of three different motors excited from three different variable frequencies has been calculated and correlated with the results predicted by the conventional value of the magnetizing reactance. This study clearly demonstrates an improvement in predicting machine steady-state performance. Because the mathematical model simplifies the steady-state analysis, it should prove equally useful in minimizing the computational time.

All motors, when excited from sources represented by the minimum power relationship, show their best steady-state performance at frequencies less than rated frequency. When excited from sources represented by direct relationship, these motors will exhibit the best steady-state performance at frequencies close to rated value.

INTRODUCTION

Recently great interest has been shown in the application of induction motors for variable speed drive systems by varying the frequency of the source. To achieve this objective, several investigations are directed either to improving the static inverter in order to produce sinusoidal voltages free from harmonics at variable frequencies, or to designing a new motor capable of operating with inverters having an output rich in harmonics.

Al-Kababjie & Shepherd (1984) suggested a form of controller/compensator for the speed control of small induction motors. Smooth speed control was realized over a wide speed range, and the motor current remains substantially sinusoidal. Oldenkamp & Peak (1985) designed a static inverter which was used to supply an induction motor for a traction drive system. Takahashi & Noguchi (1986) studied the efficiency optimization of an induction motor in the steady state operation by presenting a new quick response control scheme which depends on the concept of the instantaneous slip frequency ccntrol. Alexa (1987) has presented a static frequency converter with inverter based on the amplitude modulation of the output voltage pulses for supplying two-phase induction motors. Pavithran *et al.* (1987) presented an optimum design of a current source inverter to feed a normal induction motor which satisfies specified performance requirements over a wide frequency range. Salama (1986b) presented three variable frequency sources which are capable of producing three different voltage/frequency relationships. These are determined according to specific drive requirements, and are classified as:

(i) a direct relationship represented by

$$V_x = kf_x$$

where $k = \dfrac{V_n \text{ (rated voltage)}}{f_n \text{ (rated frequency)}}$

(ii) a constant torque relationship which determines the voltage and frequency necessary to produce the maximum torque of the motor throughout the build-up time till the rated speed is reached.

(iii) a minimum input power relationship which, when applied, will allow the motor to develop the required torque with the minimum power input (Salama 1986a).

The objective of this paper is to calculate the steady-state input current and power factor when an induction motor is fed from a source of a specified voltage/frequency relationship in the manner discussed above. Three induction motors of ratings $\frac{1}{2}$ hp, 1 hp and 10 hp are used in the present study and their data are presented in Table 1. In the analysis, and for each voltage/frequency relationship, the variation of the magnetizing reactance X_m with the load torque is carefully examined and is represented in mathematical terms. These expressions will tend to simplify the analysis, minimize the computational time, and will fill the gap between the actual value of variable X_m and the constant value used in most texts. Comparison between the values of X_m obtained from the use of these expressions and the actual values of X_m shows good agreement. The results obtained show the same motor performance when supplied either according to the direct relationship or the constant torque relationship. A lightly loaded induction motor exhibits good steady-state performance when excited from a source of the minimum input power relationship. For all motors the minimum input power relationship will result in good steady-state performance at frequencies less than rated values. However, the direct relationship will result in less stator current and higher power factor at fre-

Table 1. Data of the 4-pole, 50 Hz induction motors used

			Parameters (ohms per phase)					
Rating, hp	Line voltage, v	Winding connections	r_1	r_2	x_1	x_2	X_m	R_m
1/2	415	star	27·00	21·10	25·30	25·30	305·50	6000
1	415	star	14·36	5·81	21·38	21·38	188·50	3707
10	380	star	0·74	0·25	1·80	1·80	27·13	428·9

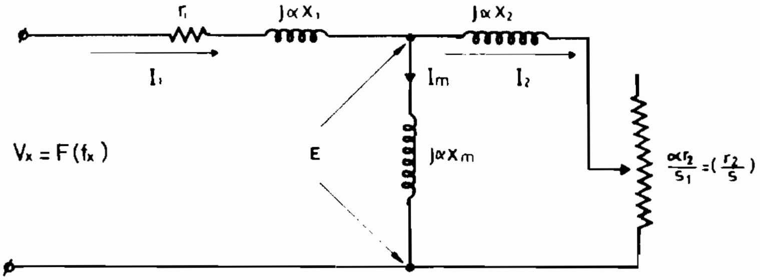

Fig. 1. Steady-state equivalent circuit for operation with variable frequency source.

quencies close to rated values. The experiments show that the source of constant torque relationship has failed to start a motor when the frequency is less than 40 Hz. This is explained in the light of the low magnetizing reactances at frequencies less than 40 Hz.

MATHEMATICAL ANALYSIS

From the motor equivalent circuit shown in Fig. 1, the rotor current I_2 is given by

$$I_2 = \frac{V_x}{\left(A + \dfrac{B}{S}\right) + j\left(\alpha F - \dfrac{H}{\alpha s}\right)} \tag{1}$$

where $A = r_1\left(1 + \dfrac{x_2}{X_m}\right)$,

$$B = r_2\left(1 + \frac{x_1}{X_m}\right),$$

$$F = x_1 + x_2 + \frac{x_1 x_2}{X_m}$$

$$H = \frac{r_1 r_2}{X_m}$$

$$\alpha = \frac{f_x \text{ (source frequency)}}{f_n \text{ (rated frequency)}}$$

s = per unit slip.

The motor input current is given by

$$I_1 = I_2\left(1 + \frac{x_2}{X_m} - j\,\frac{r_2}{s\alpha X_m}\right) \tag{2}$$

Thus

$$|I_1| = V_x\sqrt{\frac{(1 + x_2/x_m)^2 + (r_2/s\alpha X_m)^2}{\left(A + \dfrac{B}{s}\right)^2 + \left(\alpha F - \dfrac{H}{\alpha s}\right)^2}} \tag{3}$$

Assuming V_x as a reference phasor, the phase angle ϕ of the motor input current I_1, will be

$$\begin{aligned}\phi = {} & \tan^{-1}\left[\left(\frac{r_2}{s\alpha X_m}\right)\bigg/\left(1 + \frac{x_2}{X_m}\right)\right] \\ & -\tan^{-1}\left[\left(\alpha F - \frac{H}{\alpha s}\right)\bigg/\left(A + \frac{B}{s}\right)\right]\end{aligned} \tag{4}$$

Eqns (3) and (4) indicate that both the motor input current and power factor (cos ϕ) are functions of both the slip s and the magnetizing reactance X_m. Since in practice an induction motor is used to deliver a specified torque T, both the slip s and the magnetizing reactance X_m will be represented in terms of T.

The slip s as a function of T

Assuming the motor is fed at a frequency f_x, the value of V_x can be determined depending on the type of source used (Salama 1986a). On using the motor equivalent circuit, the developed electro-magnetic torque T will be

$$T = \frac{3V_x^2(r_2/s)\left(\dfrac{1}{\alpha\omega_s}\right)}{\left(r_1 + \dfrac{r_2}{s} + \dfrac{r_2 x_1}{sX_m} + \dfrac{r_1 x_2}{X_m}\right)^2 + \left[\alpha\left(x_1 + x_2 + \dfrac{x_1 x_2}{X_m}\right) - \dfrac{r_1 r_2}{\alpha s X_m}\right]^2} \tag{5}$$

where $\omega_s = 2\pi \times$ rated synchronous speed.

Rearranging Eqn (5) we get

$$Ks^2 + Ls + M = 0 \tag{6}$$

where $K = A^2 + \alpha^2 F^2$,

$$L = 2(AB - FH) - \left(\frac{3V_x^2 r_2}{\alpha\omega_s T}\right),$$

$$M = B^2 + \left(\frac{H}{\alpha}\right)^2$$

Solving Eqn (6), the per unit slip s which satisfies the type of drive required will be

$$s = \frac{1}{2K}\left(-L + \sqrt{L^2 - 4KM}\right) \tag{7}$$

X_m as function of T

For a specified voltage/frequency relationship and for a frequency f_x, a no-load test is carried out for each motor under consideration when the source voltage is varied

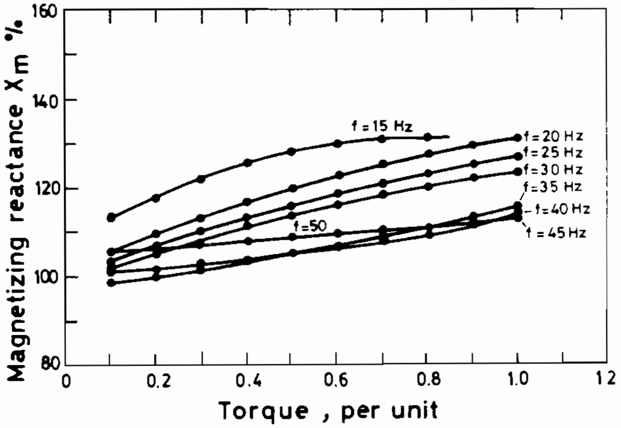

Fig. 2a. Variation of magnetizing reactance X_m with torque for 1 hp motor excited according to the direct relationship.

between 0 and V_x. The no-load magnetization curve can generally be represented by the following mathematical form:

$$I_m = a_1 + a_2 E + a_3 E^2 + a_4 E^3 + a_5 E^4 \tag{8}$$

where a_1, a_2, a_3, a_4 and a_5 are constants for each selected frequency.

E is shown in Fig. 1

Comparison between the results obtained from Eqn (8) and the experimental no-load curve shows good agreement for the part corresponding to a load torque variation between 0·1 T_{FL} and T_{FL}. Also, the X_m–I_m curve is represented by the following mathematical expression:

$$X_m = [A_1 e^{-\gamma I_m} - B_1 \sin(\delta I_m - C_1)] X_{m\,\text{rated}} \tag{9}$$

where A_1, γ, B_1, δ and C_1 are constants which are obtained from the open circuit characteristics.

$X_{m\,\text{rated}}$ is the magnetizing reactance at rated voltage and frequency.

For a given torque T, the rotor current I_2 can be determined from the following:

$$|I_2| = \sqrt{\frac{Ts\alpha\omega_s}{3r_2}} \tag{10}$$

Hence, from the motor equivalent circuit of Fig. 1

$$|E| = \sqrt{\frac{Ts\alpha\omega_s}{3r_2}\,[(r_2/s)^2 + (\alpha x_2)^2]} \tag{11}$$

Using Eqn (11), the value of E corresponding to the given torque T will be used in Eqn (8) to determine the magnetizing current I_m. These calculated values of I_m are then used in Eqn (9) to determine the value of X_m for the motor under test and according to the type of source used.

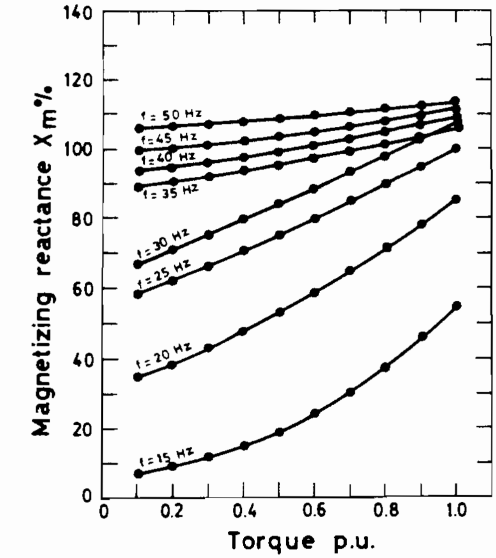

Fig. 2b. Variation of magnetizing reactance X_m with torque for 1 hp motor excited according to the constant torque relationship.

By applying the previous procedure with the help of the experimental no-load magnetization curve, Figs 2a–c are obtained to present the variations of X_m with load torque when 1 hp motor was fed from the three variable frequency sources.

For sources represented by direct relationship, a motor will perform a higher magnetizing reactance at low frequencies although the reactance is gradually

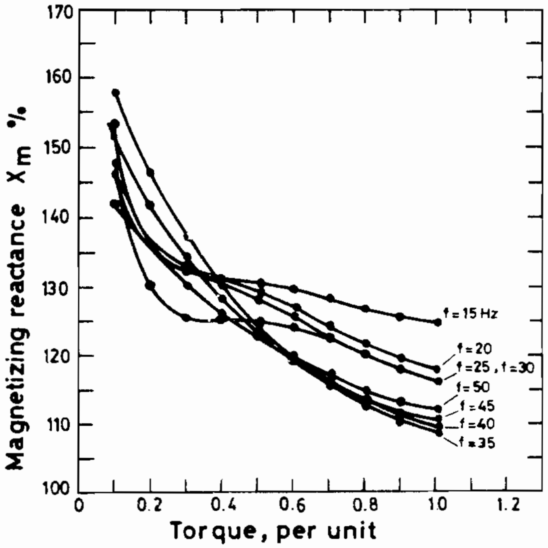

Fig. 2c. Variation of magnetizing reactance X_m with torque for 1 hp motor excited according to the minimum input power relationship.

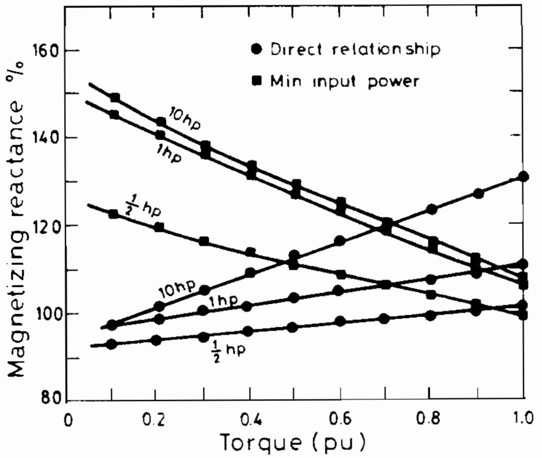

Fig. 3. Typical curves for the variations of X_m at frequency of 35 Hz.

increased with load torque. But for sources represented by the minimum input power relationship, and for the frequency range from 15 Hz to 50 Hz, a large change in motor's magnetizing reactance is recognized when load torque varies from no-load to full-load.

The X_m curves obtained for a source represented by the constant torque relationship, show that X_m exhibits very low values for frequencies less than 35 Hz which denotes a highly saturated motor. Consequently, and for the purpose of comparison between the three variable frequency sources, the operating frequency range used in the present paper is limited between 35 and 50 Hz.

The effect of motor rating on the variations of X_m with load torque is presented in Fig. 3. It was realized that both the direct and the constant torque relationships show almost the same variations of X_m with respect to load torque for frequencies above 30 Hz. By using the least-squares of fitting principle, the curves are represented by:

(i) $X_m\% = a + (b + cS)T,$ (12)

for sources represented by the direct or the constant torque relationship;

(ii) $X_m\% = (d + gS)e^{-(h+kS)T},$ (13)

for sources represented by the minimum input power;

where $a = 100$, $b = 10$, $c = 2{\cdot}5$
$d = 125$, $g = 3$, $h = 0{\cdot}24$
$g = 0{\cdot}013$
S is the motor rating in hp.

STEADY-STATE INPUT PERFORMANCE

The input current I_1 and its corresponding power factor $\cos\phi$ computed from Eqns (3) and (4) are presented in Figs 4 and 5. The results show that the source of the minimum input power relationship gives, at all frequencies, the least values of

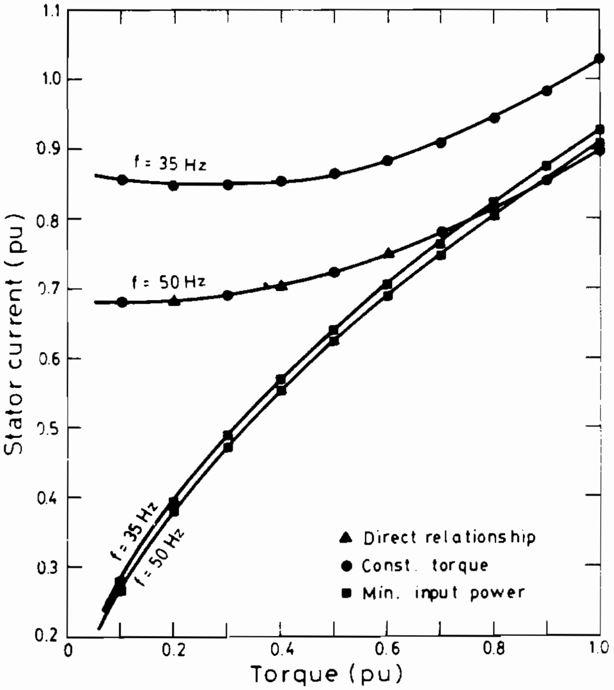

Fig. 4a. The input current-torque characteristics for $\frac{1}{2}$ hp motor at frequencies above 30 Hz.

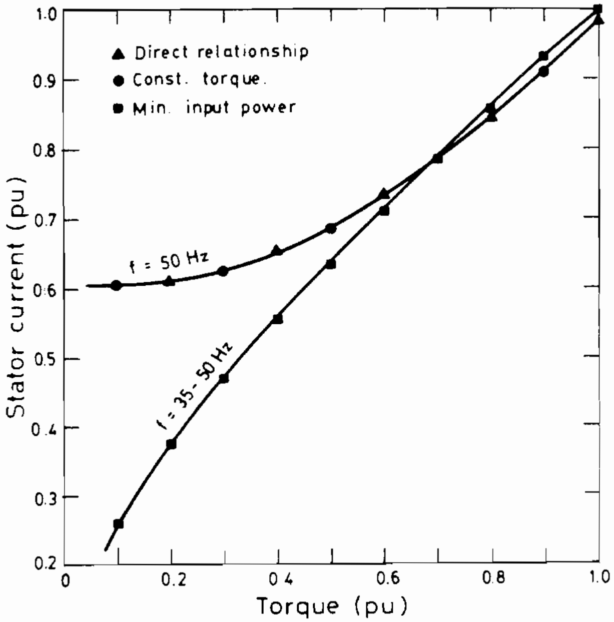

Fig. 4b. The input current-torque characteristics for 1 hp motor at frequencies above 30 Hz.

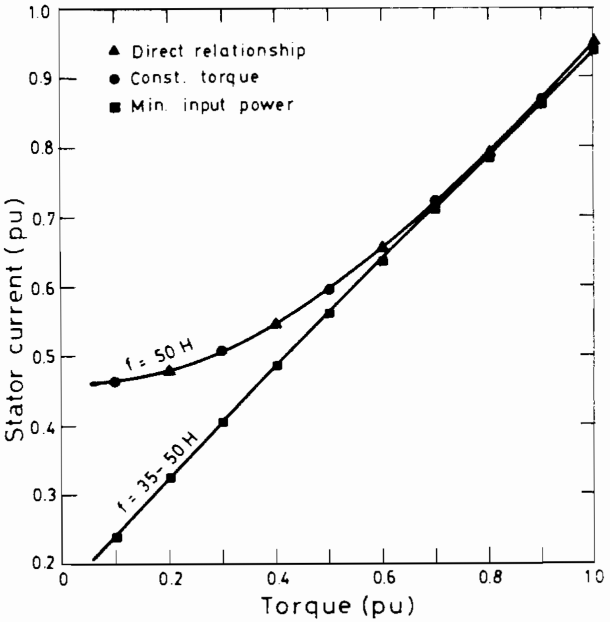

Fig. 4c. The input current-torque characteristics for 10 hp motor at frequencies above 30 Hz.

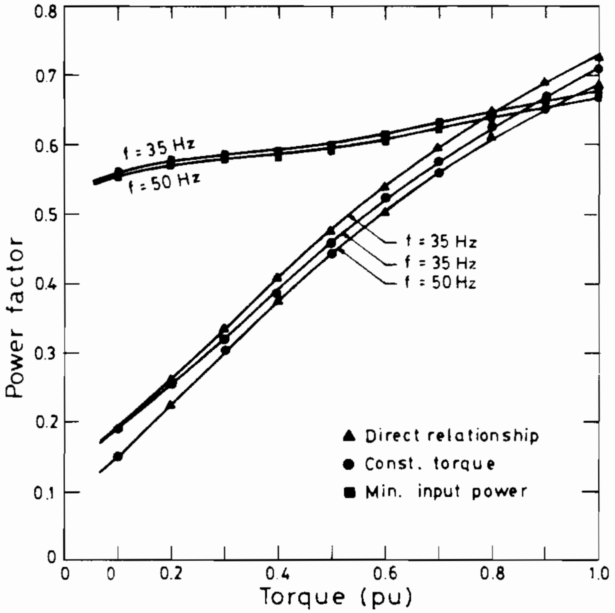

Fig. 5a. The power factor-torque characteristics for $\frac{1}{2}$ hp motor at frequencies above 30 Hz.

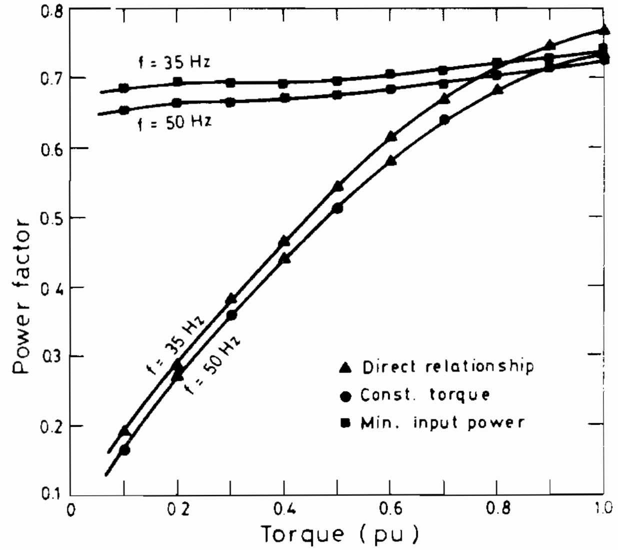

Fig. 5b. The power factor-torque characteristics for 1 hp motor at frequencies above 30 Hz.

current with the highest values of power factor. The results also indicate that at the same load torque the input current will increase as the size of the motor is reduced for all sources. To verify the accuracy of the mathematical expressions used for the computations (Eqns (12) and (13)), Tables 2 and 3 are presented. They show a comparison between the values of both the input current and power factor when X_m used in the motor's equivalent circuit is obtained from curves similar to those shown

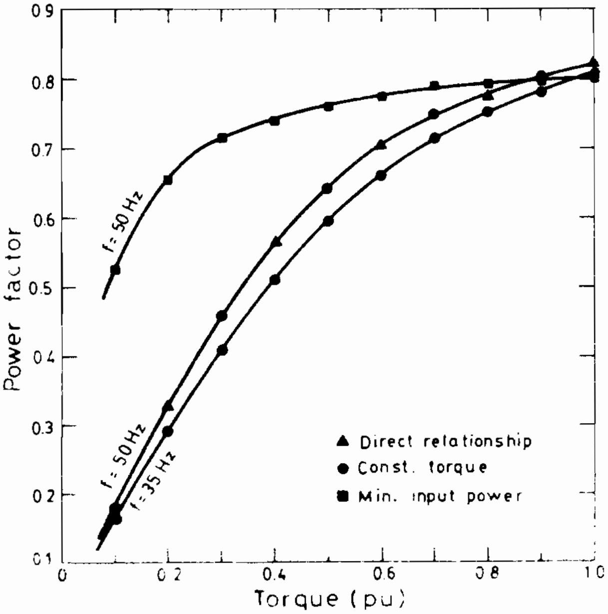

Fig. 5c. The power factor-torque characteristics for 10 hp motor at frequencies above 30 Hz.

Table 2. Per unit input currents corresponding to actual X_m and mathematical X_m at 35 Hz.

	For direct relationship source						For minimum input power relationship source					
	$\frac{1}{2}$ hp		1 hp		10 hp		$\frac{1}{2}$ hp		1 hp		10 hp	
Torque (pu)	Act. X_m	Math. X_m	Act. X_m	Math. X_m	Act. X_m	Math. X_m	Act. X_m	Math. X_m	Act. X_m	Math. X_m	Act. X_m	Math. X_m
0·1	0·702	0·729	0·639	0·648	0·460	0·439	0·272	0·276	0·253	0·262	0·237	0·237
0·5	0·729	0·748	0·700	0·708	0·591	0·569	0·638	0·629	0·633	0·630	0·568	0·563
0·8	0·818	0·831	0·856	0·864	0·794	0·778	0·819	0·811	0·854	0·851	0·795	0·790
1·0	0·909	0·919	1·010	1·020	0·962	0·949	0·925	0·923	0·995	1·000	0·946	0·947

in Figs. 2a–c (named actual X_m) and when X_m is obtained from the mathematical expressions presented by Eqns (12) and (13) (named mathematical X_m).

The results shown in Tables 2 and 3 prove that Eqns (12) and (13) satisfactorily present the nonlinear mutual reactance of induction motors of ratings less than 10 hp when fed from variable frequency sources. Since frequencies higher than 35 Hz were selected for the computation of the steady-state performance, the results show that the constant torque relationship produces almost the same performance as that obtained with the direct relationship.

CONCLUSION

A method is developed which utilizes the results obtained from the no-load experimental results for the development of a mathematical expression for X_m as function of load torque and motor rating. The effect of X_m variations on the steady-state performance of induction motors has been examined. This effect is recognized when motors failed to run when excited from sources of the constant torque relationship at frequencies less than 35 Hz. The variable frequency source of the constant torque relationship proves to be unsuccessful because of its limited range of frequency variations and since little differences in the steady-state performance are obtained in comparison to that of the direct relationship. Variable frequency sources of the minimum input power relationship offer better steady-state performance of any induction motor in comparison to the other two sources. The mathematical expres-

Table 3. Calculated power factors corresponding to actual X_m and mathematical X_m at 35 Hz.

	For direct relationship source						For minimum input power relationship source					
	$\frac{1}{2}$ hp		1 hp		10 hp		$\frac{1}{2}$ hp		1 hp		10 hp	
Torque (pu)	Act X_m	Math. X_m	Act. X_m	Math. X_m	Act. X_m	Math. X_m	Act X_m	Math. X_m	Act. X_m	Math X_m	Act X_m	Math. X_m
0·1	0·184	0·185	0·187	0·187	0·190	0·197	0·563	0·556	0·692	0·663	0·516	0·516
0·5	0·475	0·466	0·545	0·540	0·656	0·684	0·600	0·607	0·690	0·695	0·750	0·758
0·8	0·644	0·636	0·710	0·702	0·792	0·813	0·642	0·647	0·712	0·715	0·785	0·791
1·0	0·724	0·717	0·768	0·761	0·830	0·846	0·673	0·674	0·732	0·727	0·802	0·800

sions representing the mutual reactance of an induction motor will be useful in future work concerning the dynamic analysis of motors when fed from variable frequency sources.

ACKNOWLEDGEMENT

The author gratefully acknowledges the help of Mr Mohamed Rayan. The financial support from Kuwait University Research Council under Project No. EE 012 is also acknowledged.

REFERENCES

Alexa, D. 1987. Static frequency converter for supplying an asynchronous two-phase motor. Institute of Electrical Engineers Proceedings **134B**: 57–60.

Al-Kababjie, M.F. & Shepherd, W. 1984. A speed and power factor controller for small three-phase induction motor. Institute of Electrical and Electronic Engineering Transactions on Industry Applications **20**: 1260–66.

Oldenkamp, J.L. & Peak, S.C. 1985. Selection and design of an inverter-driven induction motor for a traction drive system. Institute of Electrical and Electronic Engineering Transactions on Industry Applications **21**: 259–63.

Pavithran, K.N., Parimelalagan, K.N., Sridhara Rao, G. & Holtz, J. 1987. Optimum design of an induction motor for operation with current source inverters. Institute of Electrical Engineers Proceedings **134B**: 1–8.

Salama, M.H. 1986a. A simplified approach for determining the minimum input power conditions for induction motors fed from variable frequency sources. Electric Machines and Power Systems Journal **11**: 41–52.

Salama, M.H. 1986b. Characterization of variable frequency sources for induction motors drives. Electric Machines and Power Systems Journal **11**: 523–34.

Takahashi, I. & Noguchi, T. 1986. A new quick-response and high efficiency control strategy of an induction motor. Institute of Electrical and Electronic Engineering Transactions on Industry Applications **22**: 820–27.

(Received 5 May 1987, revised 13 February 1989)

أداء حالة الاستقرار لمحرك تأثيري ثلاثي الوجه يتغذى من مصدر للجهد ذي تردد متغير

محمد حسن سلامه
قسم الهندسة الكهربائية والكمبيوتر
بجامعة الكويت ، ص . ب . ٥٩٦٩ ، الصفاة ١٣٠٦٠ ، الكويت

خلاصة

يقدم هذا البحث نموذجا رياضيا يمثل علاقة المانعة الحثية للمجال المغناطيسي للمحركات التأثيرية بدلالة كل من جهد المنبع والتردد وعزم دوران الحمل وقدرة المحرك . ولقد تم استخلاص هذا النموذج من البيانات التي توفرت عن إختبار اللاحمل للمحرك .

وباستخدام هذا النموذج في تمثيل المانعة الحثية للمجال المغناطيسي أمكن حساب أداء المحرك أثناء حالة الإستقرار عند تغذيته من ثلاثة مصادر مختلفة ذات تردد متغير . وعند مقارنة هذه النتائج بنظيرتها في حالة استخدام القيم الفعلية للمانعة الحثية للمجال المغناطيسي اتضح توافقهما الجيد .

وتمثل هذه الدراسة تطورا هاما نحو حساب أداء حالة الإستقرار للمحركات التأثيرية وذلك من خلال تبسيط العمليات الحسابية وخفض زمن إجراء هذه العمليات . وقد أوضحت الدراسة أن المحركات التي تتغذى من مصدر يخضع لعلاقة الحد الأدنى سوف تعطي أفضل أداء لها عند تردد أقل من التردد المقنن ، بينما تقوم نفس المحركات بأدائها الأفضل عند تردد مساو تقريبا للتردد المقنن وذلك عند تغذيتها من مصدر يخضع للعلاقة المباشرة بين الجهد والتردد .