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Direct flexural design of fully prestressed concrete members

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ABSTRACT

This paper presents a procedure which achieves, with fair accuracy, a quick preliminary design of prestressed concrete flexural members. The method utilises a combined analytical–graphical solution through the application of formulas and charts for the dimensioning of the most commonly used beam shapes. The derivation of these design expedients is detailed.

Recommendations for selecting the dimensions and other related properties of typical concrete sections are given. Numerical examples intended to demonstrate how the formulas, tables and charts can be used are worked out. All symbols used in the text are defined in Appendix C.

INTRODUCTION

In designing prestressed concrete structures many aspects, which are not conventional in reinforced concrete design, have to be taken into account. This, together with the various constraints imposed by practical considerations, would make preliminary design methods an attractive aid to prestressed concrete designers. Such methods can conveniently serve as an expedient step towards the final design and any warranted refinements.

This paper presents a simplified, yet fairly accurate, procedure for designing prestressed concrete beams at service-load conditions. In this study, only fully—and not partially—prestressed concrete beams are treated, i.e. no cracks are assumed to develop under working loads. In other words, extreme fibre tensile stresses are kept within the permissible limit (generally below the tensile strength of the concrete).

Once the cross-sectional dimensions, the magnitude of the prestressing force, and the number and location of the prestressing tendons are determined at service load, a check for the strength adequacy under ultimate load can easily be performed as for ordinary reinforced concrete beams; additional reinforcing steel bars are provided, if necessary.

The approach sequence outlined above is also adopted by many prominent authors (Guyon 1960; Leonhardt 1964; Bate & Bennett 1976; Libby 1977; Nilson 1978); it is even implicitly recommended by the British Code CP115 (Bennett 1973). Especially convincing in this respect is also what Abeles, a pioneer of advocating the idea of partial prestressing, has stated: 'nevertheless, most text books have been based on the concepts of full prestressing and elastic design' (Abeles *et al.* 1976).

SCOPE OF THE PROPOSED PROCEDURE

The possible limits on service loads in combination with the limiting values of prestress are:

(a) minimum load (normally the own weight) and initial prestressing force, P_0 (before shrinkage and creep take place);
(b) maximum load (including live load) and initial prestressing force;
(c) minimum load (or full dead load) and final prestressing force, P_∞ (after shrinkage and creep have taken place);
(d) maximum load and final prestressing force.

The design approach will, however, be based essentially on satisfying the last two conditions, since:

(i) case b seldom has any practical significance;
(ii) in reality it is allowed by internationally recognised codes (CP 110, 1972; AS 1481, 1974; IS 1343, 1960; DIN 4227, 1953) to take the permissible compressive stress in the concrete at transfer (case a) as large as one-third higher than that adopted in service, and the tensile stress may reach $0{\cdot}36\sqrt{f_{cu}}$ N/mm^2 in the CP 110, $0{\cdot}25\sqrt{f'_{ci}}$ or $0{\cdot}5\sqrt{f'_{ci}}$ N/mm^2 in both the ACI 77 (ACI 318, 1978) and AS 1481, and varies from 3 to 4·5 N/mm^2 in the DIN 4227;
(iii) beams are usually prestressed at ages and/or in modes sufficient to ensure the safety of both the concrete and steel at time of prestressing;
(iv) stresses at transfer can be controlled by the fabricator (Libby 1977).

GENERAL CONSIDERATIONS

A complete preliminary flexural design of a prestressed concrete element involves the determination of six parameters, viz. y_1, y_2, I, A, P and e (Fig. 1). In order to counterbalance the largest possible proportion of the dead load bending moment (or min. M), the prestressing tendons have to be placed as close as possible to the extreme fibres at the critical sections, i.e. near the bottom of the beam in the mid-span regions, and near the top over the intermediate supports. Hence, the eccentricity, e, is fixed in advance as a function of the depth, h. Note that there must be a sufficient concrete cover in order to resist the high compressive stresses.

For flanged beams, the depth is usually chosen based on practical rules; the cross-sectional area, A, is kept as small as possible to bring forth an economical beam. For a minimum A, the value of the square of the radius of gyration, r, is a maximum.

As long as the centre of compression remains within the kern of the cross section, the value of r^2/y_1 is always less than the value of y_2. Therefore, the most economical beams are those with the highest ratio $(r^2/y_1)/y_2$. This ratio will here be termed the efficiency factor, ρ, which geometrically constitutes the ratio of the depth of kern to the depth of section. Well-designed symmetrical *I*- and box-sections may have ρ-values as much as 0·5; for *T*-sections, ρ is up to 0·41; for *I*-sections with unequal flanges, the ρ-values are intermediate between the two formerly cited values (Guyon 1960;

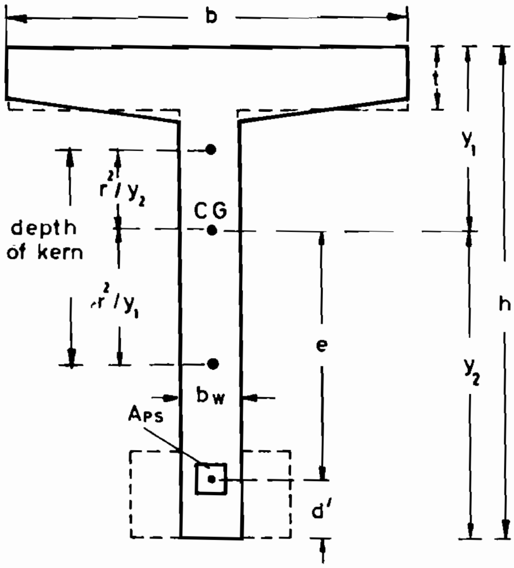

Fig. 1. Dimensional designation of a flanged section of prestressed concrete beam.

Leonhardt 1964). Therefore, by choosing h, fixing e and assuming ρ, the design is reduced to finding out three unknowns, viz. $Z_1 = I/y_i$, $Z_2 = I/y_2$ and P_∞. It follows that three conditions need only be satisfied.

Thus, for cases c and d, three of the four permissible stresses ($f_1, \bar{f}_1, f_2$ and $\bar{f}_2$) may be considered to be reached at the extreme fibres. Alternatively, two out of the four permissible stresses in the concrete may be assumed, and the third condition could be, for instance, a symmetrical section, that is $y_1 = y_2$, or a given y_1/y_2 ratio as for units of certain types of construction.

PROPOSED DESIGN FORMULAS

1. ASYMMETRICAL *I*-BEAMS

In this case, the three optimal conditions to be satisfied for a positive bending moment can be expressed as follows (Fig. 2):

$$f_p + f_D + f_L \leqslant f_1 \quad \text{(i)}$$

$$f'_P + f'_D + f'_L \geqslant \bar{f}_1 \quad \text{(ii)}$$

$$f'_p + f'_D \leqslant f_2 \quad \text{(iii)}$$

where

$$f_p = \frac{P_\infty}{A}\left(1 - \frac{ey_1}{r^2}\right) = \left(\frac{f_1 y_2 + \bar{f}_1 y_1}{h}\right)\left(1 - \frac{y_2 - d'}{\rho y_2}\right),$$

$$f_D = \frac{\text{min. } M}{Z_1}$$

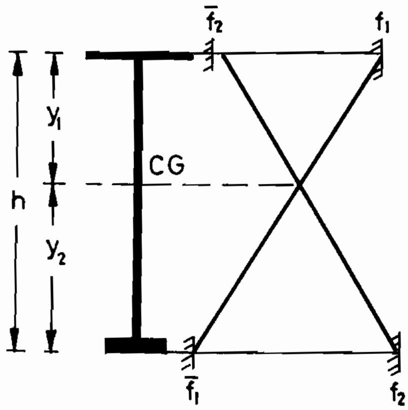

Fig. 2. Stress conditions for *I*-section.

and

$$f_L = \left(\frac{\text{max. } M - \text{min. } M}{Z_1}\right) = \frac{M_L}{Z_1}\,.$$

Note that max. $M = M_D +$ max. M_L, and min. $M = M_D$ or min. $M = M_D +$ min. M_L, if min. M_L is negative. It is best to employ the concrete section fully, and the above inequalities become, therefore, equations.

Subtracting Equation (iii) from Equation (ii), and using absolute values for the stress-variation, yield $f'_L = (f_2 - \bar{f}_1)$ and hence,

$$Z_2 = \frac{M_L}{(f_2 - \bar{f}_1)}\,.$$

As

$$\frac{f_L}{f'_L} = \frac{y_1}{y_2}\,,$$

it follows that

$$f_L = (f_2 - \bar{f}_1)\frac{y_1}{y_2}$$

and

$$f_D = \frac{\text{min. } M}{M_L}(f_2 - \bar{f}_1)\frac{y_1}{y_2}\,.$$

Substituting the above values of f_p, f_D and f_L in Equation (i) leads to

$$\left(\frac{f_1 y_2 + \bar{f}_1 y_1}{h}\right)\left(1 - \frac{y_2 - d'}{\rho y_2}\right) + (f_2 - \bar{f}_1)\frac{y_1}{y_2}\left(\frac{\text{min. } M}{M_L} + 1\right) = f_1 \qquad (1)$$

in which y_1 and y_2 are the only unknowns. But

$$y_1 + y_2 = h\,. \qquad (2)$$

Solving Equations 1 and 2, y_1 and y_2 are determined. The values of I, A and P_∞ are given by

$$\left.\begin{aligned} I &= Z_2 \cdot y_2 = \frac{M_L}{(f-\bar{f}_1)}\, y_2 \\ A &= \frac{I}{r^2} = \frac{I}{y_1 y_2} \\ P_\infty &= \left(\frac{f_1 y_2 + \bar{f}_1 y_1}{h}\right) A \end{aligned}\right| . \qquad (3)$$

Usually the permissible stresses f_1 and f_2 are identical and will, henceforth, be denoted by f; the value of $\bar{f}_1$ is normally specified as zero.

Thus Equation 1 becomes

$$\frac{y_2}{h}\left(1-\frac{y_2-d'}{y_2}\right)+\frac{y_1}{y_2}\left(\frac{\text{min. } M}{M_L}+1\right)=1\,. \qquad (1')$$

2. SYMMETRICAL FLANGED BEAMS (*I*- AND BOX-SECTIONS)

Making use of condition (i), the relevant design formula may be written as

$$\left(\frac{f_1+\bar{f}_1}{2}\right)\left(1-\frac{y-d'}{\rho y}\right)+\frac{\text{max. } M}{Z}=f_1 \qquad (4)$$

or

$$\frac{\text{max. } M}{Z} = (0{\cdot}5-2\lambda)\,(f_1+\bar{f}_1)+f_1 \qquad (5)$$

in which ρ has been taken equal to 0·5, and

$$\lambda = \frac{d'}{2y}\cdot$$

Either Equation 4 or 5 is solved for Z; I, A and P are subsequently determined from Equation 3.

When $\bar{f}_1$ equals zero, as is often the case, and f_1 is replaced by f, Equation 5 becomes

$$\frac{\text{max. } M}{Z} = (1{\cdot}5-2\lambda)f$$

or

$$\boxed{Z = \frac{\text{max. } M}{R_1}} \qquad (6)$$

where

$$R_1 = (1{\cdot}5\text{–}2\lambda)f\,. \qquad (7)$$

3. *T*-BEAMS AND CHANNEL-SECTIONS

The stress considerations accounted for in the *I*-beams are not suited for the design of *T*-beams.

As the centroidal axis for a T-beam is relatively high beyond the soffit, the most economical beams are usually achieved by making the section fully employed in the unloaded state and reaching the minimum allowable stress in the loaded state (Fig. 3).

These conditions, for a positive bending moment, can be formulated as

$$f_p + f_D \geq \bar{f}_2 \tag{iv}$$

$$f'_p + f'_D \leq f_2 \tag{v}$$

$$f'_p + f'_D + f'_L \geq \bar{f}_1 \,. \tag{vi}$$

Making the above inequalities equations and subtracting (v) from (vi) give (in an absolute concept) $f'_L = (f_2 - \bar{f}_1)$, hence

$$Z_2 = \frac{M_L}{(f_2 - \bar{f}_1)} \,.$$

Following the previous computational steps leads to

$$f_D = \frac{\text{min. } M}{M_L}(f_2 - \bar{f}_1)\frac{y_1}{y_2}$$

and

$$f_p = \left(\frac{f_2 y_1 + \bar{f}_2 y_2}{h}\right)\left(1 - \frac{y_2 - d'}{\rho y_2}\right) .$$

Upon substitution in relation (iv),

$$\left(\frac{f_2 y_1 + \bar{f}_2 y_2}{h}\right)\left(1 - \frac{y_2 - d'}{\rho y_2}\right) + \frac{\text{min. } M}{M_L}(f_2 - \bar{f}_1)\frac{y_1}{y_2} = \bar{f}_2 \,. \tag{8}$$

If $\bar{f}_1 = \bar{f}_2 = 0$, f substituted for f_2 and λ for d'/h, Equation 8 becomes

$$\boxed{y_2 = \left(\frac{\text{min. } M}{M_L} + \frac{\lambda}{\rho}\right)\frac{\rho h}{(1-\rho)} \,.} \tag{9}$$

Clearly, the rest of the solution is identical to that indicated for symmetrical I-beams.

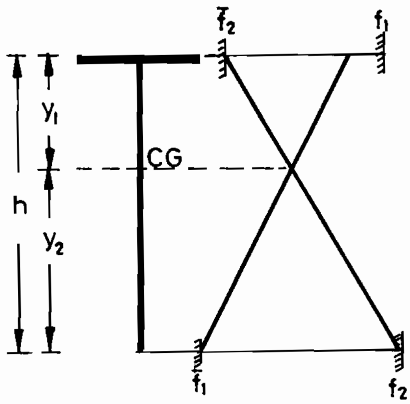

Fig. 3. Stress conditions for a T-section.

4. RECTANGULAR BEAMS

For a rectangular section $\rho = \frac{1}{3}$, and if b is given, the respective design formula (derived from Equation 4) may be written in the form

$$\frac{\text{max. } M}{bh} = \left(\frac{f_1 + \bar{f}_1}{2}\right)(\tfrac{2}{3}h - d') - \frac{f_1 h}{6} \qquad (10)$$

which is solved for the depth h. For $\bar{f}_1 =$ zero and f_1 changed to f, Equation 10 reduces to

$$\frac{\text{max. } M}{bh^2} = \left(\frac{1}{3} - \frac{\lambda}{2}\right)f$$

or

$$\boxed{h = \sqrt{\left(\frac{\text{max. } M}{R_2 b}\right)}} \qquad (11)$$

where

$$R_2 = \left(\frac{1}{3} - \frac{\lambda}{2}\right)f \qquad (12)$$

It is worth noting that the form of Equation 11 resembles that of the well-known design formula for reinforced concrete beams.

DESIGN TABLES FOR *I*- AND RECTANGULAR SECTIONS

To facilitate the application of Formulas 6 and 11, values of the coefficients R_1 and R_2 for various practical limits of the ratio λ and of the permissible compressive stress, f, have been computed and are listed in Table 1.

PROPOSED DESIGN CHARTS

1. RECTANGULAR BEAMS

The total moment (max. M) appearing in Equation 10 can be set out as

$$\text{max. } M = (C_1\omega + C_2 L)\ell^2 \qquad (13)$$

where C_1 and C_2 are moment-coefficients, ω is the own-weight of the beam and L is the superimposed load per unit length. Let the dimensions b, h and ℓ be in m, and ω and L in MN/m. Thereby, $\omega = 0{\cdot}024 C_1 bh\ell^2$.

Assuming $\bar{f}_1 = 0$, and substituting f for f_1 and λ for d'/h, Equation 10 reduces to

$$0{\cdot}024 C_1 \ell^2 + \frac{C_2 L \ell^2}{bh} = \left(\frac{1}{3} - \frac{\lambda}{2}\right)fh$$

or

$$\boxed{\left(\frac{1}{3} - \frac{\lambda}{2}\right)h^2 - 0{\cdot}024\left(\frac{C_1\ell^2}{f}\right)h - \left(\frac{C_1\ell^2}{f}\right)\left(\frac{C_2 L}{C_1 b}\right) = 0\,.} \qquad (14)$$

Table 1. Coefficients R_1 and R_2 in terms of the ratio λ and permissible concrete stress f in MPa

$\lambda=d'/h$	f	14·0	15·5	17·0	18·5	20·0	21·5	23·0	24·5
0·05	R_1	19·60	21·70	23·80	25·90	28·00	30·10	32·20	34·30
0·10	R_2	4·317	4·779	5·242	5·704	6·167	6·629	7·092	7·554
	R_1	18·20	20·15	22·10	24·05	26·00	27·95	29·90	31·85
0·15	R_2	3·967	4·392	4·817	5·242	5·667	6·092	6·517	6·942
	R_1	16·80	18·60	20·40	22·20	24·00	25·80	27·60	29·40
0·20	R_2	3·617	4·004	4·392	4·779	5·167	5·554	5·942	6·329
	R_1	15·40	17·65	18·70	20·35	22·00	23·65	25·30	26·95
0·25	R_2	3·267	3·617	3·967	4·317	4·667	5·017	5·367	5·717
	R_1	14·00	15·50	17·00	18·50	20·00	21·50	23·00	24·50
0·30	R_2	2·917	3·229	3·542	3·854	4·167	4·479	4·792	5·104
	R_1	12·60	13·95	15·30	16·65	18·00	19·35	20·70	22·05
	R_2	2·567	2·842	3·117	3·392	3·667	3·942	4·217	4·492

I- and box-sections: $Z = \dfrac{\text{max. } M}{R_1}$, $R_1 = (1{\cdot}5 - 2\lambda)f$

Rectangular sections: $h = \sqrt{\left(\dfrac{\text{max. } M}{R_2 b}\right)}$, $R_2 = \left(\frac{1}{3} - \dfrac{\lambda}{2}\right)f$.

Equation 14 is a second-degree equation of the beam depth, h. Its graphical solution is readily available by means of the charts of Fig. 4, which cover different practical values of λ as well as other factors appearing in the equation.

2. SYMMETRICAL FLANGED BEAMS (I- AND BOX-SECTIONS)

In this case the design involves the application of Equations 4 (or alternatively 6) and 3. The problem is then to shape a cross-section, i.e. finding b_ω, b and t, whose I, A and h are known.

To comply with certain practical requirements, the web width, b_ω, is usually selected according to the rules discussed later. Thus, two dimensions remain, viz. the flange width, b, and thickness, t. Therefore, for known values of I and A, the values of b and t are unique.

A solution can be achieved by cut-and-try procedure, a rather time-consuming task. A more convenient approach is to utilise the curves in Fig. 5 to effect a quick solution. These curves were developed as follows:

The moment of inertia I may be expressed as

$$I = k_1\left(\frac{b_\omega h^3}{12}\right) \tag{15}$$

in which

$$k_1 = \frac{b}{b_\omega} - \left(\frac{b}{b_\omega} - 1\right)\left(1 - \frac{2t}{h}\right)^3 \tag{16}$$

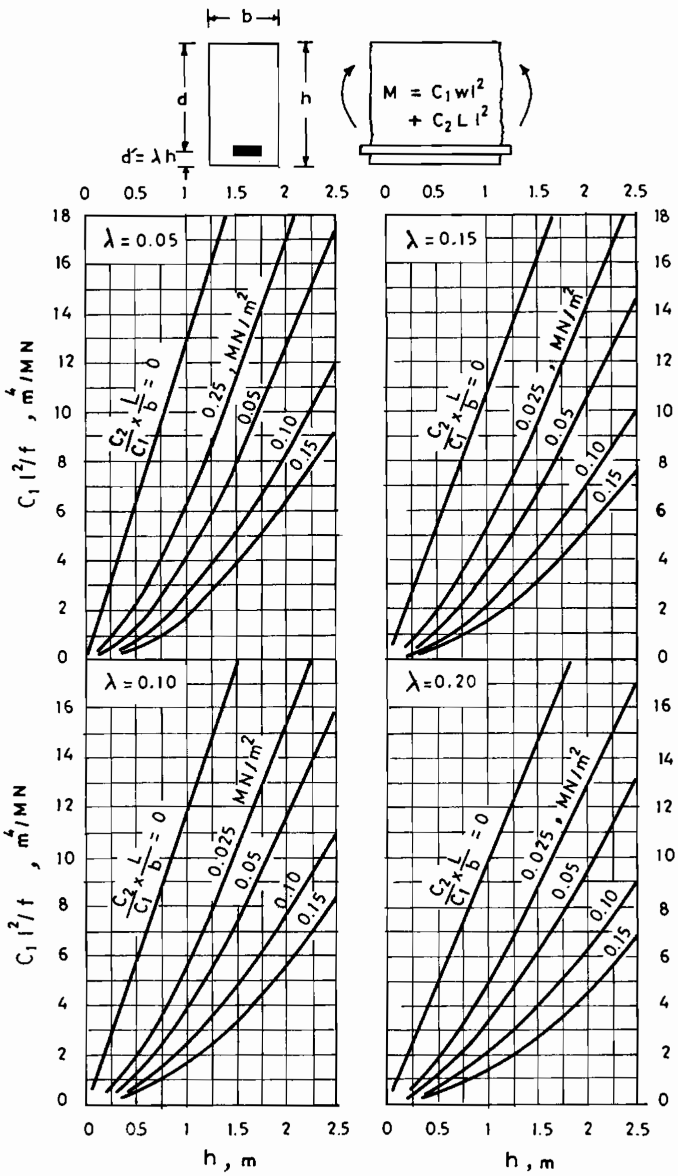

Fig. 4. Depth of rectangular beam as function of span, beam width, live load and permissible concrete stress.

Similarly

$$A = k_2(b_\omega h) \tag{17}$$

where

$$k_2 = 1+2\left(\frac{b}{b_\omega}-1\right)\frac{t}{h}\cdot \tag{18}$$

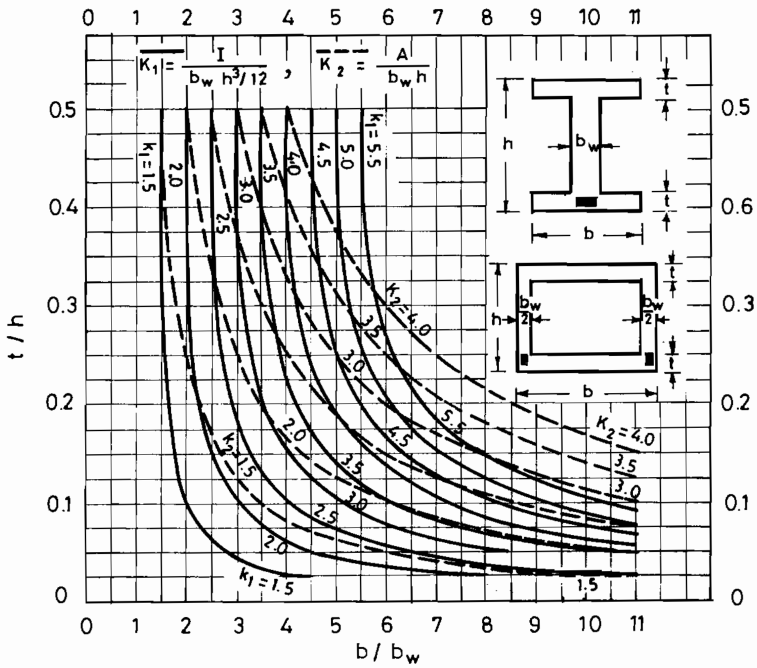

Fig. 5. Values of t/h and b/b_ω in terms of k_1 and k_2.

The two families of curves, plotted in Fig. 5, pertain to various values of k_1 and k_2; solid lines refer to k_1 and the dotted lines to k_2. Fig. 5 is entered with known values of k_1 and k_2 to locate the appropriate point of intersection whose co-ordinates will read the values of b/b_ω and t/h on the abscissa- and ordinate-axis, respectively.

3. *T*- AND CHANNEL-SECTION BEAMS

The above argument also holds in this case; it can be shown that the coefficients k_1 and k_2 relating to a *T*-section have the following expressions:

$$k_1 = 1+\left(\frac{b}{b_\omega}-1\right)\left(\frac{t}{h}\right)^3+\frac{3\left(\frac{b}{b_\omega}-1\right)\left(1-\frac{t}{h}\right)^2\frac{t}{h}}{1+\left(\frac{b}{b_\omega}-1\right)\frac{t}{h}} \tag{19}$$

$$k_2 = 1+\left(\frac{b}{b_\omega}-1\right)\frac{t}{h}\cdot \tag{20}$$

The two families of curves describing b/b_ω versus t/h, at different values of k_1 and k_2, are drawn in Fig. 6. Again, a point on the graph representing two given values of k_1 and k_2 will furnish the proportions of the required cross-section.

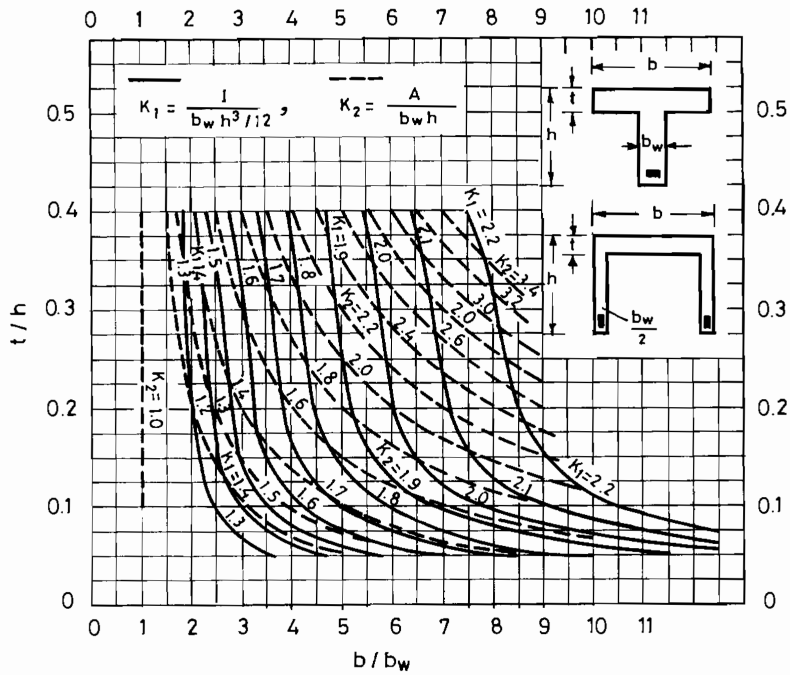

Fig. 6. Values of t/h and b/b_ω in terms of k_1 and k_2.

SUMMARY AND CONCLUSIONS

An expedient approach for dimensioning prestressed concrete flexural members at service load has been presented. The approach has the following features and advantages:

(1) the design formulas and charts proposed are simple to apply and lead to a fairly accurate preliminary design;
(2) the depth of a rectangular beam can readily be determined without making any assumption except for the width, b;
(3) if the cross-sectional dimensions were slightly increased, which would also suit practical dimensions, the stress limitations at transfer would most probably be met;
(4) the method furnishes a quick computation of the final prestressing force that would in turn enable the designer to assess the prestress losses on a systematic rather than a lump sum basis.

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APPENDIX A—GUIDE RULES FOR SECTION DIMENSIONING

When selecting and/or controlling the dimensions of a prestressed concrete section, the following practical rules are recommended:

1. BEAM DEPTH

It is economically advantageous to choose fairly large depths, unless restrictions on clearance and levels are imposed (Guyon 1960). As a general rule, depth-span ratios of from $\frac{1}{20}$ to $\frac{1}{24}$ for simple beams and $\frac{1}{22}$ to $\frac{1}{26}$ for continuous beams provide economical section (Magnel 1954; Guyon 1960; Lin & Zia 1974). Leonhardt (1964), Libby (1977) and Nilson (1978) suggest higher values varying between $\frac{1}{16}$ and $\frac{1}{22}$ depending on loading conditions, though they admit that smaller depth-span ratios have proved satisfactory under certain conditions. However, exceedingly small depths require large prestressing forces and very good concrete.

2. WEB WIDTH

Theoretically, the value of the web width, b_ω, is to be kept as low as possible, since the efficiency factor, ρ, decreases as b increases. In practice, however, b_ω cannot be reduced indefinitely because of the minimum clearances required for placing and compacting the concrete in the web and lower flange (Guyon 1960; Libby 1977). Under normal conditions of placing and compaction, it would be recommended to take

$$\text{min. } b_\omega = 125 \text{ mm when } h \leq 750 \text{ mm}$$

and

$$\text{min. } b_\omega = \frac{h}{6} \text{ to } \frac{h}{8} \text{ when } h > 750 \text{ mm} .$$

In the case of providing web reinforcement, the minimum values should further be increased by 15–20 mm.

3. FLANGE THICKNESS

For most practical cases, the minimum possible flange thickness provides a design of optimum material cost (Guyon 1960; Balaguru 1981). As useful rules for normal situations, the flange thickness, t, may be assumed ranging between 0·1 h and 0·17 h, with the higher values to be adopted for the T-beams.

4. FLANGE WIDTH

In theory, it is preferable to use wide-thin flanges (Guyon 1960). However, the maximum flange width is limited by fragility considerations and level of stresses at the junction of the flange with the web. Field experience has also demonstrated the desirability of a reasonable flange width in order to reduce the transverse flexibility of the girder during handling (Guyon 1960; Libby 1977).

Generally, the flange width should not normally exceed 0·8 h for individual beams.

5. SHAPE OF CROSS-SECTION

As for the optimum cross-sectional shape, Jacobsohn (1952) has shown that for high dead load to live load ratios and ample depths, a T-section (or a section with small tension flange) is the most appropriate choice. On the other hand, if the dead load to live load ratio is low, or if the depth is small, an I-section or a box-section would be the best selection (Leonhardt 1964; Libby 1977). The box-sections are particularly useful for slender beams where lateral stability becomes a consideration, and for those subjected to torsional moments (Warner & Faulkes 1979). For short-span beams in which the dead load is likely to be only a small fraction of the total load, rectangular members may provide the most economical solution (Nilson 1978).

APPENDIX B—DESIGN EXAMPLES

A simply supported beam spanning 25 m is to carry a useful load of 15 kN/m. Maximum and minimum permissible working stresses in the concrete are 17 MPa and zero, respectively. If λ is assumed 0·1 for cases 1 and 2, and 0·125 for case 3, it is required to dimension the critical section: (i) as a rectangular section; (ii) as an I-section, and (iii) as a T-section. In the T-section case, provide for a useful load of 10 kN/m. Determine the final prestressing force in each case.

SOLUTION

Case 1. Rectangular section

In this case $C_1 = C_2 = 0{\cdot}125$

$C_1\ell^2/f = 0{\cdot}125 \times 25^2/17 = 4{\cdot}5956\ \mathrm{m^4/MN}$

Assume $b = 0{\cdot}45$ m, hence $\dfrac{C_1L}{C_2b} = \dfrac{0{\cdot}015}{0{\cdot}45} = 0{\cdot}0333\ \mathrm{MN/m^2}$

Enter the chart designated by $\lambda = 0{\cdot}1$ in Fig. 4, proceed to the right from the ordinate 4·5956, interpolate for 0·0333 between the $\frac{C_2}{C_1}\frac{L}{b}$ curves and finally read on the abscissa scale $h = 0{\cdot}95$ m

Using Equation 3

$$P_\infty = \frac{17}{2} \times 0{\cdot}95 \times 0{\cdot}45 \times 1000 = 3633{\cdot}75 \text{ kN}.$$

Case 2. I-*section*

Choose $h = \frac{\ell}{22} = 1{\cdot}136$ m, say, 1·15 m

let min. $b_\omega = \frac{h}{8} \simeq 144$ mm

$b_\omega = 144 + 20 = 164$ mm; use $b_\omega = 165$ mm

Estimate: $\omega = 0{\cdot}024\{0{\cdot}165 \times 1{\cdot}15 + 2(0{\cdot}14 \times 1{\cdot}15)\ (0{\cdot}4 \times 1{\cdot}15)\}$
$= 0{\cdot}00811$ MN/m

max. $M = 0{\cdot}125(0{\cdot}00811 + 0{\cdot}015)\frac{25^2}{8} = 1{\cdot}8054$ MN/m

Let $\rho = 0{\cdot}5$, and from Table 1 read $R_1 = 22{\cdot}1$

Applying Equation 6 gives

$$Z = \frac{1{\cdot}8054}{22{\cdot}1} = 0{\cdot}08169 \text{ m}^3$$

Then from Equation 3 it follows that

$$I = 0{\cdot}04697 \text{ m}^4,$$

and

$$A = 0{\cdot}2841 \text{ m}^2$$

Using Equations 15 and 17, hence

$$k_1 = 2{\cdot}2462 \text{ and } k_2 = 1{\cdot}5$$

Enter for these two values Fig. 5, locate the appropriate point, and read its coordinates

Thus, $b/b_\omega = 3{\cdot}54$ and $t/h = 0{\cdot}1$

Therefore, $b = 3{\cdot}54 \times 165 = 585$ mm,
$t = 0{\cdot}1 \times 1150 = 115$ mm

Check for I and A:

$$I = \frac{0{\cdot}585 \times 1{\cdot}15^3}{12} - \frac{0{\cdot}420 \times 0{\cdot}92^3}{12}$$

$$= 0{\cdot}0469 \text{ m}^4 \simeq I_{req}, \text{ O.K.}$$

$$A = 0{\cdot}585 \times 1{\cdot}15 - 0{\cdot}420 \times 0{\cdot}92$$
$$= 0{\cdot}2864 \text{ m}^2 = 1{\cdot}008\ A_{req}, \text{ O.K.}$$

$$P_\infty \simeq \frac{17}{2} \times 0{\cdot}2852 \times 1000 = 2424{\cdot}2 \text{ kN}$$

Case 3. T-*section*

Adopt as before $h = 1{\cdot}15$ m, but $b_\omega = 155$ mm and $\lambda = 0{\cdot}125$

Let $\rho = 0{\cdot}4$

Estimate: $\omega = 0{\cdot}024\{0{\cdot}155 \times 1{\cdot}15 + (0{\cdot}16 \times 1{\cdot}15)\ (0{\cdot}4 \times 1{\cdot}15)\}$
$= 0{\cdot}00631$ MN/m

If all the superimposed load of 10 kN/m is assumed a live load, then

$$\text{min. } M = 0{\cdot}125 \times 0{\cdot}00631 \times 25^2 = 0{\cdot}493 \text{ MN/m}$$

and $M_L = 0{\cdot}125 \times 0{\cdot}01 \times 25^2 = 0{\cdot}78125$ MN/m

Thus,

$$\frac{\text{min. } M}{M_L} = 0{\cdot}631$$

Substituting in Equation 9 gives

$$y_2 = \left(0{\cdot}631 + \frac{0{\cdot}125}{0{\cdot}4}\right)\left(\frac{(0{\cdot}4 \times 1{\cdot}15)}{(1 - 0{\cdot}4)}\right)$$
$$= 0{\cdot}723 \text{ m}$$

Consequently

$$y_1 = 1{\cdot}15 - 0{\cdot}723 = 0{\cdot}427 \text{ m}$$

Based on Equation 3

$$I = 0{\cdot}78125 \times 0{\cdot}723 = 0{\cdot}033226 \text{ m}^4$$
$$A = (0{\cdot}033226/0{\cdot}4 \times 0{\cdot}723 \times 0{\cdot}427)$$
$$= 0{\cdot}2691 \text{ m}^2$$

Applying Equations 15 and 17, then

$$k_1 = 1{\cdot}69 \text{ and } k_2 = 1{\cdot}51$$

Locate on Fig. 6 a point corresponding to the above two values and read

$$b/b_\omega = 3{\cdot}94 \text{ and } t/h = 0{\cdot}174$$

Therefore, $b = 3{\cdot}94 \times 155 = 610$ mm

and $t = 0{\cdot}174 \times 1150 = 200$ mm

Checking on A and I will give

$$A = 0{\cdot}26925 \text{ m}^2 = 1{\cdot}0006\ A_{req}, \text{ O.K.}$$

and

$$I = 0{\cdot}03354 \text{ m}^4 = 1{\cdot}001\ I_{req}, \text{ O.K.}$$

Finally, $P_\infty = \left(17 \times \frac{0{\cdot}427}{1{\cdot}15}\right) 0{\cdot}2691 \times 1000 = 1699 \text{ kN}$

APPENDIX C—NOTATION

A	= gross area of section
A_{ps}	= area of prestressing tendons
b	= width of compression face of member
b_ω	= width of web
C_1	= coefficient of moment due to own weight
C_2	= coefficient of moment due to superimposed loads
d'	= distance from extreme tension fibre to centroid of tendons
e	= eccentricity of prestressing force with respect to centroid of section
f	= allowable compressive stress in concrete
f_1	= maximum allowable stress in concrete with live load acting (always compressive)
f_2	= maximum allowable stress in concrete in absence of live load (always compressive)
$\bar{f}_1$	= minimum allowable stress in concrete with live load acting (negative if tensile)
$\bar{f}_2$	= minimum allowable stress in concrete in absence of live load (negative if tensile)
f_{cu}	= specified cube strength of concrete
f'_c	= specified cylinder strength of concrete
f'_{ci}	= cylinder strength of concrete at time of prestressing (at transfer)
f_D	= extreme fibre stress at ordinate y_1 due to dead loads
f'_D	= extreme fibre stress at ordinate y_2 due to dead loads
f_L	= extreme fibre stress at ordinate y_1 due to live load
f'_L	= extreme fibre stress at ordinate y_2 due to live load
f_p	= extreme fibre prestress at ordinate y_1
f'_p	= extreme fibre prestress at ordinate y_2
h	= overall depth of section
I	= moment of inertia
k_1	= coefficient related to moment of inertia
k_2	= coefficient related to area of section
ℓ	= span length
L	= super-imposed load (including live load) per unit length
max. M	= largest design moment
min. M	= smallest design moment
M_D	= bending moment due to dead loads
M_L	= variation of bending moment = max. M − min. M
P_0	= initial prestressing force
P_∞	= final prestressing force
r	= radius of gyration
R_1	= coefficient for computing modulus of I-section (Z)

R_2 = coefficient for computing depth of rectangular section
t = thickness of compression flange
ω = own weight of member per unit length
y = half-depth of symmetrical flanged member
y_1 = distance from centroid to extreme compression fibre
y_2 = distance from centroid to extreme tension fibre
Z_1 = section modulus with respect to ordinate y_1
Z_2 = section modulus with respect to ordinate y_2
λ = ratio of d' to h
ρ = efficiency factor $= \dfrac{\text{depth of kern}}{\text{depth of section}} = \left(\dfrac{r^2}{y_2} + \dfrac{r^2}{y_1}\right)/h$

تصميم مباشر للأطراف الخرسانية
كاملة الاجهاد المسبق

محفوظ سعيد الريس
قسم الهندسة المدنية بجامعة الكويت

خلاصة

يقدم هذا البحث طريقة تحقق بدقة جيدة تصميما أوليا سريعا للاعتاب الخرسانية المسبقة الاجهاد . والطريقة تعتمد حلا تحليليا ـ بيانيا لتصميم أكثر أشكال الاعتاب شيوعا واستعمالا وذلك عبر تطبيق المعادلات والرسومات البيانية المستنبطة .

ويشتمل البحث على ارشادات تسهل اختيار أنسب الأبعاد للمقاطع وما يتصل بها من خواص هندسية وميكانيكية . كما أعطيت مسائل محلولة بغرض تبيان كيفية استخدام المعادلات والجداول والرسومات البيانية . هذا وان جميع الرموز والاصطلاحات الواردة في النص قد جرى تعريفها في آخر البحث .