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Identification and control of flexible structures

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ABSTRACT

In this paper, the method of least square estimates is used to model actually built structures by means of measured input and output signals. The input signals can be generated from exciting the structural control mechanism itself. The identified model serves to design a discrete-time feedback control law. The derived control law is used to control the actual structure against general disturbances. An illustrative example is given to show the procedure on one-storey building frame with tendon control mechanism.

INTRODUCTION

The technique of active control as applied to flexible structure has gained much interest among researchers and structural engineers (Leipholz 1979, 1985; Leipholz & Abdel-Rohman 1986). However, the design of any structural control system is based on the knowledge of the structural characteristics required to describe the mathematical model. Previous applications with this subject have been already dealt with (Abdel-Rohman 1984, 1986, 1987). For a built structure, the mathematical model and its parameters need to be identified (Yao & Abdel-Rohman 1986). Systems identification techniques contain several methods to estimate the system parameters from a set of measurements.

In this paper, one of the system identification techniques is used to represent actually built structures by mathematical models. The models serve for the design of the feedback control law. The efficiency of the designed control law is tested by investigating the controlled response of the model. If satisfactory, the control law is then applied to the actual structure. Application on the identification and control of one-storey building frame is shown by an example.

THE LEAST SQUARE METHOD

The least square estimate is a widespread, simple identification technique (Sinha & Kuszta 1983). Considering a single-input single-output system, and assuming a noise-free situation, the measured output is related to the input by the pulse transfer

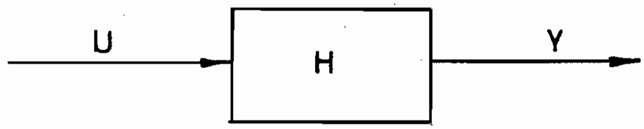

Fig. 1. Block diagram of open-loop system.

function as shown in Fig. 1 by

$$\frac{\hat{Y}(z)}{\hat{U}(z)} = H(z) = \frac{a_0 + a_1 z^{-1} + \ldots + a_m z^{-m}}{1 + b_1 z^{-1} + \ldots + b_n z^{-n}} \tag{1}$$

in which z is the complex variable of Z-transform.

Equation (1) can be written in the form of difference equation as

$$Y_k = a_0 U_k + a_1 U_{k-1} + \ldots + a_m U_{k-m} - b_1 Y_{k-1} - b_2 Y_{k-2} \ldots - b_n Y_{k-n} \tag{2}$$

in which $Y_k = Y(kT)$, and $U_k = U(kT)$, $k = 1, 2, \ldots$, and T is the time step for collecting data.

In order to determine the parameter $a_0, a_1, \ldots, a_m, b_1, b_2, \ldots, b_n$, one has to collect input-output data at some intervals. Eqn. (2) can be applied for the available data to give

$$\begin{vmatrix} U_k & U_{k-1} & \ldots & U_{k-m} & -Y_{k-1} & \ldots & -Y_{k-n} \\ U_{k+1} & U_k & \ldots & U_{k-m+1} & -Y_k & \ldots & -Y_{k-n+1} \\ \vdots & \vdots & & \vdots & & & \vdots \\ U_{k+p} & U_{k+p-1} & & U_{k-m+p} & -Y_{k+p-1} & & -Y_{k+p-n+1} \end{vmatrix} \begin{vmatrix} a_0 \\ a_1 \\ \vdots \\ a_m \\ b_1 \\ b_2 \\ \vdots \\ b_n \end{vmatrix} = \begin{vmatrix} Y_k \\ Y_{k+1} \\ \vdots \\ Y_{k+p} \end{vmatrix} \tag{3}$$

which can be expressed as

$$\mathbf{A}\boldsymbol{\theta} = \mathbf{Y} \tag{4}$$

in which $\mathbf{A}$ is of dimension $(p + 1) \times (m + n + 1)$; $\boldsymbol{\theta}$ is of order $(m + n + 1)$; and $\mathbf{Y}$ is of order $(p + 1)$.

It is obvious that if $\mathbf{A}$ is a square nonsingular matrix, one can directly determine $\boldsymbol{\theta}$. However, if the number of the available data is greater than $(m + n + 1)$, one can determine the system parameters from

$$\boldsymbol{\theta} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{Y} \tag{5}$$

which results in minimizing the sum of squares of the components of the error vector $(\mathbf{A}\boldsymbol{\theta} - \mathbf{Y})$.

For the matrix $(\mathbf{A}^T\mathbf{A})$ to be nonsingular, the input signal has to excite the system persistently (Sinha & Kuszta 1983). A sufficient condition to guarantee this is that the sequence U_k has to be a set of periodic functions, containing at least a number of sinusoids equal to the order of the system, in order to excite all modes of the system. Moreover, the sequence of these functions is not integrally related and the period of

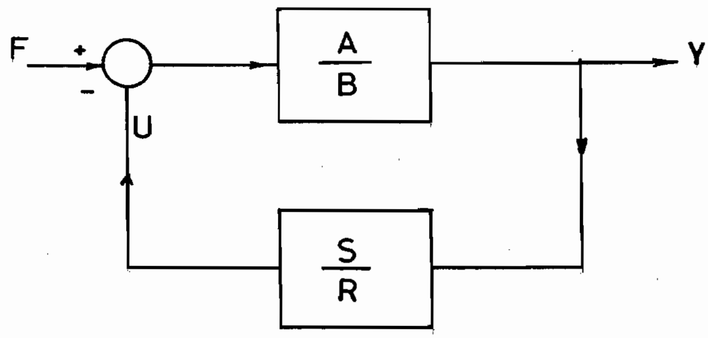

Fig. 2. Block diagram of closed-loop system.

the function is greater than the total time over which the data have been collected. This simple method is adopted in this paper in order to show how one can directly treat the identification and control of structures.

DESIGN OF CONTROL LAW

By determining the parameters θ from Eqn (4) or Eqn (5), the system model is now identified as in Eqn (1). The poles and zeros of the system can be determined by equating, respectively, the denominator and the numerator of Eqn (1) to zero. New poles are then assigned for the controlled structure. Considering Fig. 2, the transfer function of the closed-loop system is given by

$$\frac{\hat{Y}}{\hat{F}} = \frac{A/B}{1 + AS/BR} = \frac{AR}{BR + AS} \tag{6}$$

The poles of the controlled model are related to the corresponding poles in the s-plane by the relation $z = e^{sT}$, where T is the sampling period. By equating the denominator of Eqn (6) to the polynomial containing the assigned poles one obtains

$$(z - p_1)(z - p_2) \ldots (z - p_{2n-1}) = BR + AS \tag{7}$$

in which $p_1, p_1, \ldots p_{2n-1}$ are the assigned poles in z-plane.

One can then determine the parameters of the feedback control law by solving the equations resulting from equating terms of equal powers in z. When the performance of the controlled model is satisfactory, one may use the designed control law for the actual structure. The performance is said to be satisfactory if the controlled response in the presence of an unexpected disturbance is smaller and better behaved than the uncontrolled response.

EXAMPLE

We take as example the identification and control of a single-storey shear building. A tendon control mechanism such as shown in Fig. 3 is used for both identification and control. The pretensioned tendon is connected with a servohydraulic actuator. By operating the actuator to follow a certain input-forcing function and recording the sway of the frame using transducers, one can determine, off-line, the model

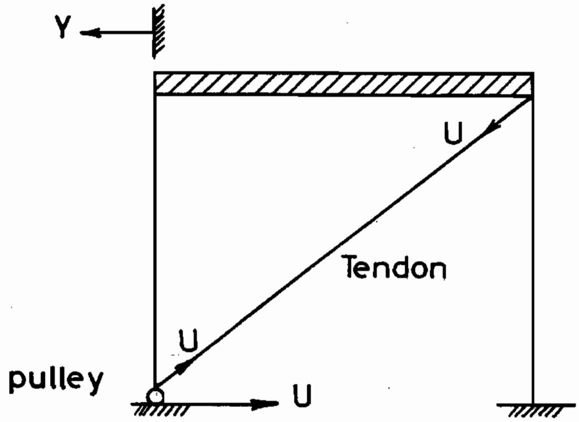

Fig. 3. One-storey building with tendon control mechanism.

parameters for the frame. In this example, the model can be assumed as a second order model expressed by

$$\frac{\hat{Y}}{\hat{U}} = \frac{a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}} = \frac{a_1 z + a_2}{z^2 + b_1 z + b_2} \tag{8}$$

which can be written as

$$Y_k = a_1 U_{k-1} + a_2 U_{k-2} - b_1 Y_{k-1} - b_2 Y_{k-2} \tag{9}$$

From the input and the measured output data one can write Eqn (9) as

$$\begin{vmatrix} Y_2 \\ Y_3 \\ Y_4 \\ \vdots \\ Y_p \end{vmatrix} = \begin{vmatrix} U_1 & U_0 & -Y_1 & -Y_0 & a_1 \\ U_2 & U_1 & -Y_2 & -Y_1 & a_2 \\ U_3 & U_2 & -Y_3 & -Y_2 & b_1 \\ \vdots & \vdots & \vdots & \vdots & b_2 \\ U_{p-1} & U_{p-2} & -Y_{p-1} & -Y_{p-2} & \end{vmatrix} \tag{10}$$

The model parameters can then be determined from Eqn (5). The measured displacement in this example was simulated by the response of a single degree of freedom undamped system with natural frequency $\omega = 1{\cdot}414$ rps subjected to a forcing function given by

$$F(t) = 3 \sin 1{\cdot}7t + \sin 1{\cdot}1t \tag{11}$$

The response of this excitation, $U(K)$, is shown in Fig. 4 and the measured displacement, $Y(k)$, is shown in Fig. 5. By substituting the values of input and output into Eqn (10), considering 1000 samples with intervals of $T = 0{\cdot}2$ s, the model parameters were determined from Eqn (5) as $a_1 = 0{\cdot}016179$, $a_2 = 0{\cdot}0161464$, $b_1 = -1{\cdot}92049$; and $b_2 = 1{\cdot}000024$. The response of the identified model using the same input function of Eqn (11) is shown in Fig. 6 which shows very close agreement with the response of the measured data.

One can now design a feedback control law for the tendon operation using the identified model. Since the poles of the uncontrolled model are approximately at $(\pm j\ 1{\cdot}414)$ in s-plane, the poles of the closed-loop controlled system are assumed at $(-0{\cdot}5 \pm j\ 1{\cdot}5)$ with the objective of introducing more damping to the structure. A

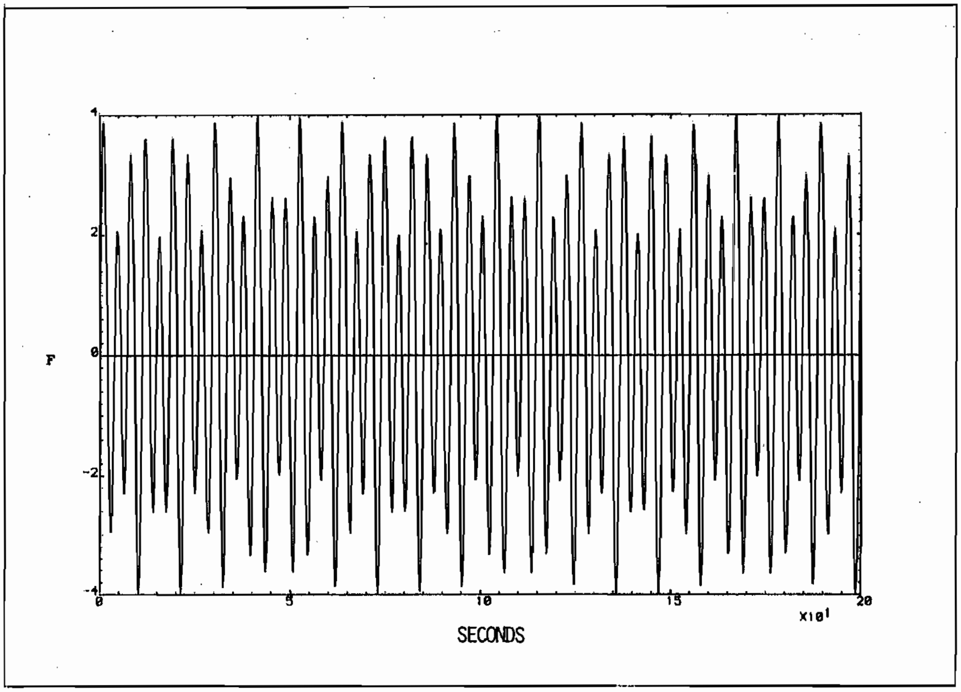

Fig. 4. Response of input-forcing function.

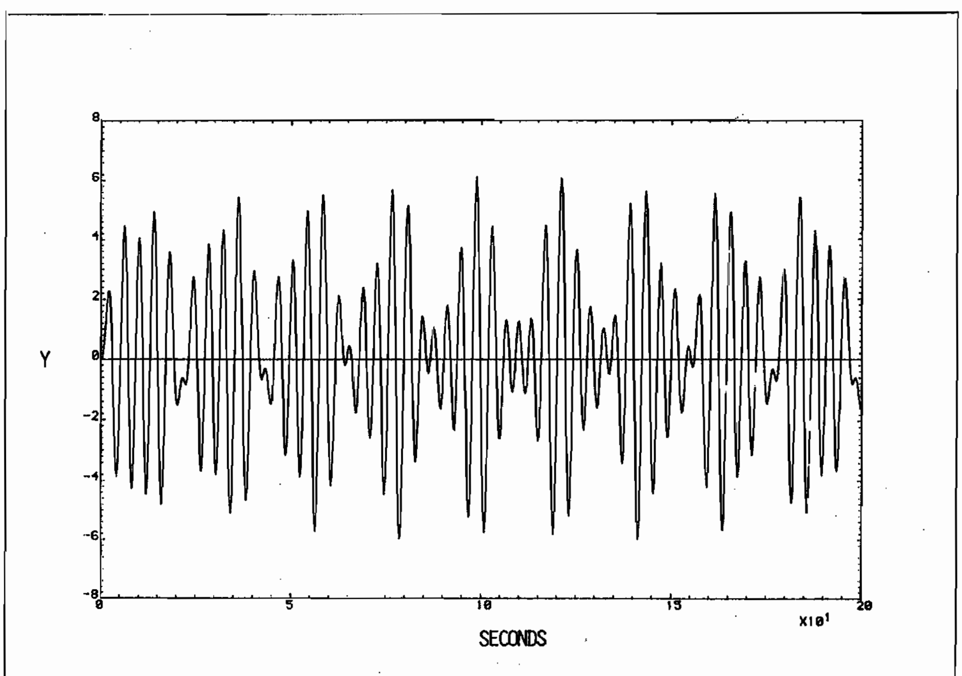

Fig. 5. Response of measured data $Y(t)$.

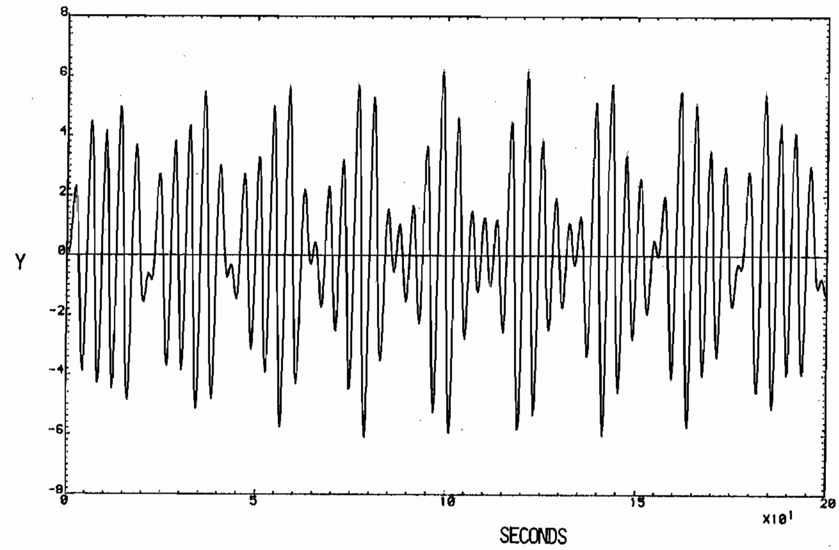

Fig. 6. Response of the identified model using the input-forcing function.

further damping can be introduced by shifting the poles towards the greater negative real parts. Using Eqn (7), one obtains

$$BR + AS = z(z - p_1)(z - p_2) \tag{12}$$

in which $p_1 = e^{(-0{\cdot}5+j1{\cdot}5)T}$, $p_2 = e^{(-0{\cdot}5-j1{\cdot}5)T}$, $B = z^2 + b_1 z + b_2$, $A = a_1 z + a_2$, $R = r_0 z + r_1$, and $S = s_0 z + s_1$.

By equating terms of equal powers of z one obtains the following equations:

$$\begin{vmatrix} 1 & 0 & a_1 \\ b_1 & a_1 & a_2 \\ b_2 & a_2 & 0 \end{vmatrix} \begin{vmatrix} r_1 \\ s_1 \\ s_0 \end{vmatrix} = \begin{vmatrix} -b_1 - p_1 - p_2 \\ -b_2 + p_1 p_2 \\ 0 \end{vmatrix} \tag{13}$$

and r_0 is unity.

The feedback transfer function is then

$$\frac{\hat{S}}{\hat{R}} = \frac{s_0 + s_1 z^{-1}}{r_0 + r_1 z^{-1}} = \frac{5{\cdot}9717 - 5{\cdot}88538\ z^{-1}}{1 + 0{\cdot}095025\ z^{-1}} \tag{14}$$

The controlled response is calculated from the closed-loop relation obtained from Eqn (6) as

$$\frac{\hat{Y}}{\hat{U}} = \frac{AR}{BR + AS} = \frac{(r_0 + r_1 z^{-1})(a_1 z^{-1} + a_2 z^{-2})}{(1 - p_1 z^{-1})(1 - p_2 z^{-1})} \tag{15}$$

To test the designed control law one simulates any disturbance on the model. The response of the model to an assumed disturbance expressed as

$$F(t) = 5 \sin 0{\cdot}7t + 2 \sin 1{\cdot}2t$$

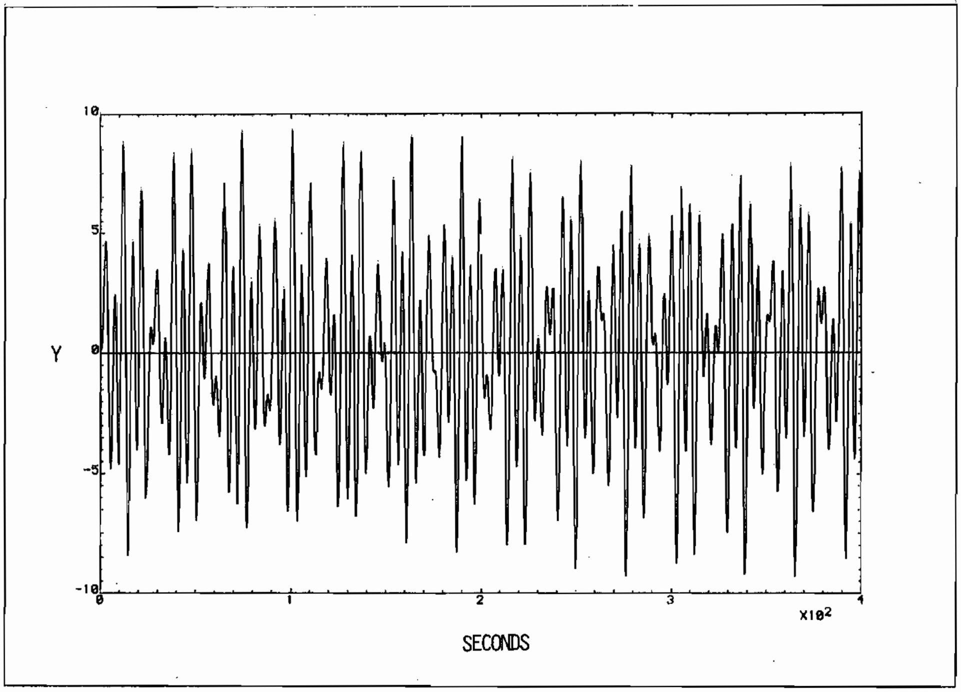

Fig. 7. Response of the model to $F = 5 \sin 0{\cdot}7t + 2 \sin 1{\cdot}2t$.

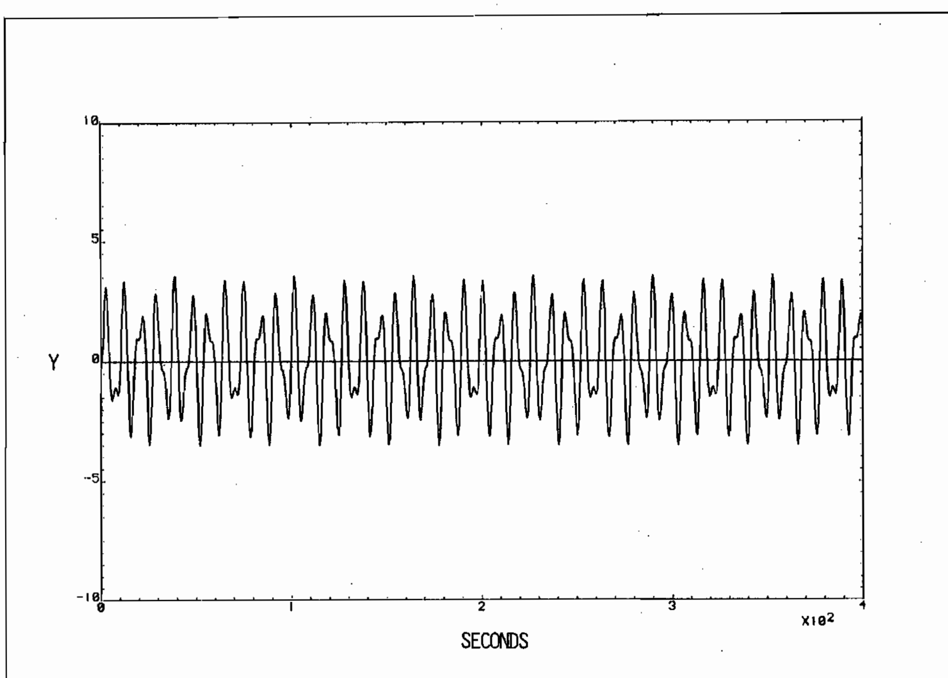

Fig. 8. Controlled response of the model.

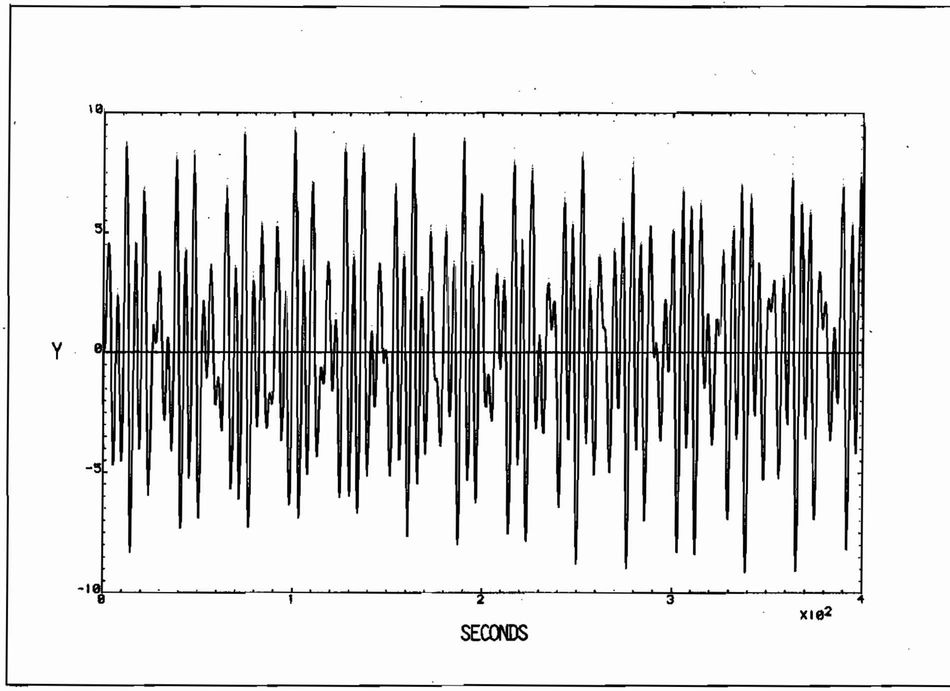

Fig. 9. Deflection response of the uncontrolled frame.

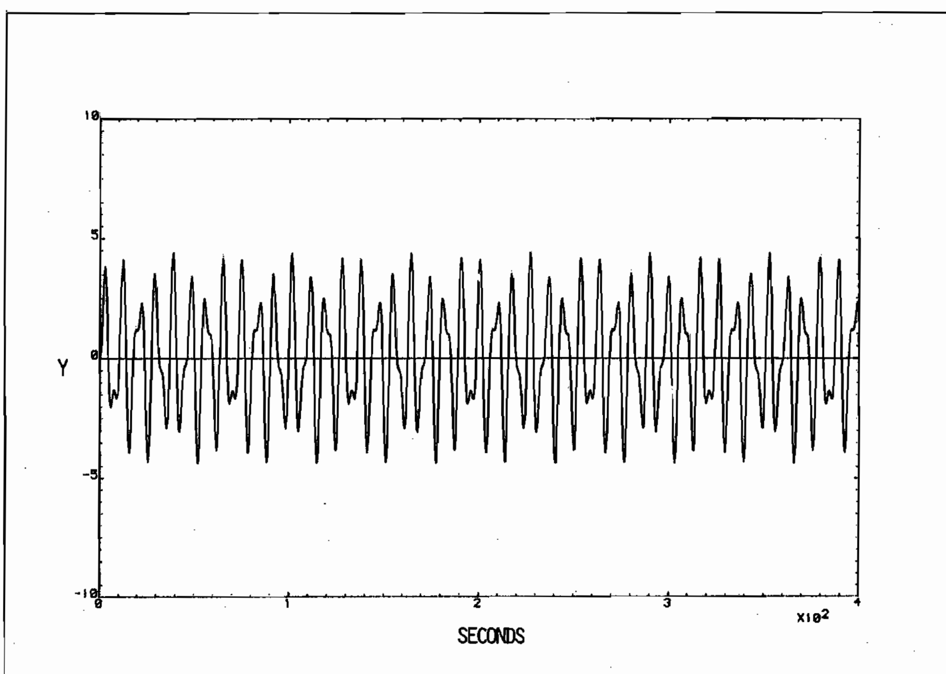

Fig. 10. Deflection response of the controlled frame.

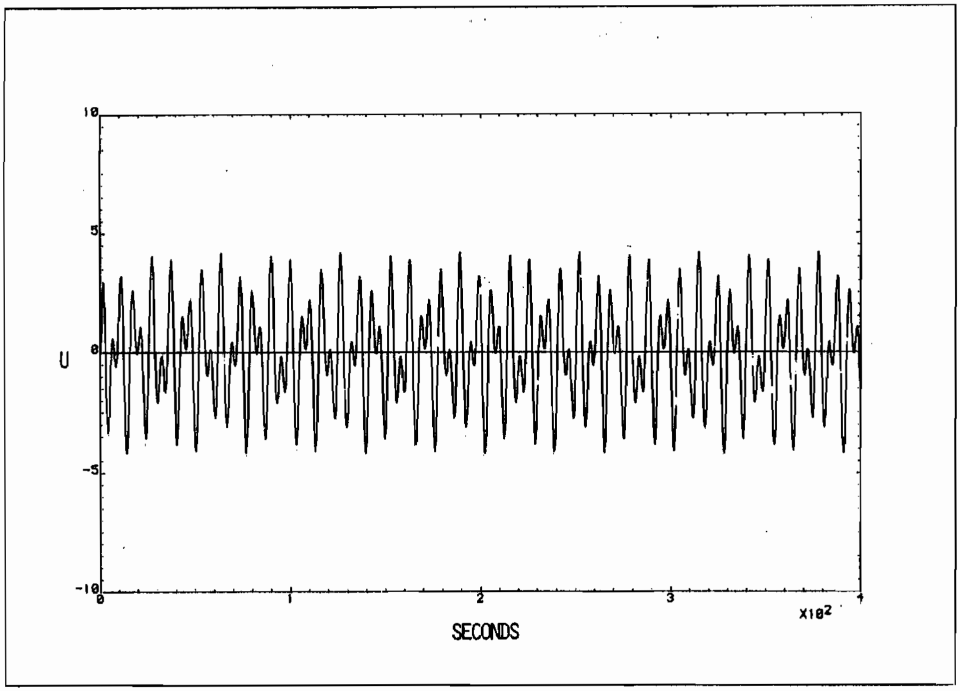

Fig. 11. Response of the control force.

is shown in Fig. 7 for the uncontrolled response and in Fig. 8 for the controlled response. Since, the controlled response of the model was satisfactory, one can now use this control law for the actual structure. The uncontrolled response of the actual frame, which was represented by a single degree of freedom system, to the assumed disturbance is shown in Fig. 9 and the anticipated controlled response is displayed in Fig. 10. The response of the applied control force is shown in Fig. 11. One observes that the identified model could represent successfully the actual structure without the need to identify specifically the parameters of the actual structure.

SUMMARY AND CONCLUSIONS

The paper approaches the identification and control of built structures by means of the least squares method. The control mechanism can be used to excite the structure with predetermined input excitation. The measured output signals and the input excitation serve to determine the model parameters. On the basis of the identified model, measured parameters and the control inputs, a discrete time feedback control law has been derived. This law can then be used to control the actual structure since it depends on the measured output and the feedback control signals at previous steps. An example has illustrated the control of building vibrations without the need to know their natural frequencies or mode shapes. However, the choice of the input signal for the purpose of identification and the time intervals of the measured data need some trials until one is convinced that the identified model is close to the real structure.

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التعرف والتحكم في المنشآت المرنة

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خلاصة

يستخدم هذا البحث طريقة التقديرات بأقل خطأ تربيعي لتمثيـل المنشآت المبنيـة الفعلية بنموذج رياضي باستعمال بيانات داخلة وخارجة من المنشأ. يمكن توليد البيانات الداخلة بتشغيل هيكل التحكم المقترح للتحكم في سلوك المنشأ نفسـه. بعد ذلك يستخدم النموذج الذي تم التعرف عليه في تصميم قانون للتحكم ذي تغذية مرتجعة للأوقات المتقطعة. يستخدم هذا القانون للتحكم في سلوك المنشأ الفعلي ضد القوى العامة التي يمكن أن يتعرض لها المنشأ. ويقدم البحث مثالا توضيحيا يبين الخطوات الواجب إتباعها في حالة مبنى هيكلي ذي طابق واحد، باستعمال هيكل التحكم بالكابلات.