J. Univ. Kuwait (Sci.) **22**, 1995

Linear programming algorithm applied to power system measurement for frequency relaying

AHMAD M. AL-KANDARI

Faculty of Technological Studies, P.O. Box 42325, 70654 Shuwaikh, Kuwait

ABSTRACT

A new application for linear programming (*LP*) algorithm to power system measurements for frequency relaying is presented. The technique is based on the minimization of the sum of the absolute value of the error of the digitized bus voltage at the relay location. The proposed technique is superior to the well known least error squares (*LS*) technique, when the digitized voltage samples are contaminated with gross errors. It has the ability, in most cases, to reject these bad measurements in contrast to the *LS* which is adversely affected by the presence of bad data. Two models are used to estimate the frequency of a power system from the digitized bus voltage at the relay location. The first, is the constant-frequency model (*CFM*), which is used to estimate the steady state frequency deviation, while the second is the variable frequency model (*VFM*) which is used to estimate the transient frequency and the rate of change of frequency.

Effects of sampling rate, data window size, and time-reference-location on the behaviors of the algorithm are examined. Numerical results are reported in this paper, where we compare the proposed *LP* algorithm, and the *LS* algorithm. The proposed technique can easily be applied to a microprocessor based algorithm for digital frequency relaying.

1. INTRODUCTION

The frequency of a power system remains constant if generated power is equal to the sum of all loads plus system losses. However, the frequency starts to change as soon as system generation and load become unequal. The frequency starts to decay if the system load exceeds the available generated power. On the other hand, the frequency starts to increase as soon as the total generated power exceeds the loads.

Recently, the use of digital computers for power system protection, control and monitoring has received considerable attention, and many techniques have been developed in the last decade. These include the use of Discrete Fourier Transform (*DFT*), and Fast Fourier Transform (*FFT*) (Sachdev & Giray 1985 and Phadke *et al.* 1983), Kalman Filtering Algorithm *KFA* (Giray & Sachdev 1989, Sachdev *et al.* 1985) and the least error squaries technique (Giray & Sachdev 1985).

This paper discusses a new application for linear programming (*LP*) algorithm to power system measurements for frequency relaying. The proposed technique is based on the minimization of the sum of the absolute value of the error of the digitized bus voltage at the relay location. Two models are used to estimate the frequency of a power system. The first model is the constant frequency model (*CFM*), which is used to estimate the steady state frequency deviation, while the second model is the variable frequency model (*VFM*) which is used to estimate the transient frequency and the rate of change of frequency. The proposed technique is superior to the techniques currently used when the measurements set is contaminated with errors, noise or gross errors. It has the ability, in most cases, to reject the bad data, in contrast to other techniques which are adversely affected by the presence of bad data.

Effects of the sampling rate, data window size and location of reference time on the behaviour of the algorithm are examined. Numerical results are reported, where we compare the proposed *LP* algorithm and the *LS* algorithm.

2. MATHEMATICAL MODELS FOR THE ALGORITHM

2.1. *Constant Frequency Model* (Giray & Sachdev 1985)

In this model, we assume that the voltage does not change during a data window size, the frequency of the power system at the relay location does not change rapidly and the voltage waveforms are sinusoids of a fundamental frequency. The system signal can be written as

$$v(t) = V_m \sin(\omega t + \phi) \tag{1}$$

where:

$v(t)$ is the instantaneous value of the voltage at time t.
V_m is the peak value of the voltage.
ω is the system angular frequency $= 2\pi f$.
f is the frequency of the waveform.
ϕ is the arbitrary phase angle.

Using the well known trigonometric identities, Eqn. 1 can be written as

$$v(t) = (V_m \cos\phi)(\sin\omega t) + (V_m \sin\phi)(\cos\omega t) \tag{2}$$

Without loss of generality, replacing $\sin(2\pi ft)$ and $\cos(2\pi ft)$ by their first four terms of the Taylor series expansion (Effects of number of terms taken from Taylor's Series expansion are discussed in section 4.3.) in the neighbourhood of the nominal frequency f_0, Eqn. 3 is obtained

$$\begin{aligned} v(t) = {} & a_{11}(t)x_1 + a_{12}(t)x_2 + a_{13}(t)x_3 + a_{14}(t)x_4 + a_{15}(t)x_5 \\ & + a_{16}(t)x_6 + a_{17}(t)x_7 + a_{18}(t)x_8 \end{aligned} \tag{3}$$

Where

$$\begin{aligned} x_1 &= V_m \cos\phi, & x_2 &= \Delta f\, V_m \cos\phi \\ x_3 &= V_m \sin\phi, & x_4 &= \Delta f\, V_m \sin\phi \\ x_5 &= (\Delta f)^2 V_m \cos\phi, & x_6 &= (\Delta f)^2 V_m \sin\phi \\ x_7 &= (\Delta f)^3 V_m \cos\phi, & x_8 &= (\Delta f)^3 V_m \sin\phi \end{aligned}$$

and

$$a_{11}(t) = \sin(2\pi f_0 t), \qquad a_{12}(t) = 2\pi t \cos(2\pi f_0 t)$$
$$a_{13}(t) = \cos(2\pi f_0 t), \qquad a_{14}(t) = -2\pi t \sin(2\pi f_0 t)$$
$$a_{15}(t) = -2\pi^2 t^2 \sin(2\pi f_0 t)$$
$$a_{16}(t) = 2\pi^2 t^2 \cos(2\pi f_0 t)$$
$$a_{17}(t) = \tfrac{4}{3}\pi^3 t^3 \cos(2\pi f_0 t)$$
$$a_{18}(t) = -\tfrac{4}{3}\pi^3 t^3 \sin(2\pi f_0 t)$$
$$\Delta f = f - f_0$$

If the voltage is sampled at a pre-selected rate, we would obtain its samples at equal time intervals, say Δt seconds. A set of m samples designated at $v(t_1), \dots, \dots, v(t_m)$ is obtained. These are the digitized samples of the voltage where: $t_2 = t_1 + \Delta t$, $t_3 = t_1 + 2\Delta t, \dots, \dots, t_m = t_1 + (m-1)\Delta t$, and t_1 is an arbitrary time reference. Eqn. 3 can be written as

$$\begin{bmatrix} v(t_1) \\ v(t_2) \\ | \\ | \\ | \\ v(t_m) \end{bmatrix} = \begin{bmatrix} a_{11}(t_1) \cdots\cdots a_{18}(t_1) \\ a_{21}(t_2) \cdots\cdots a_{28}(t_2) \\ | \qquad\qquad | \\ | \qquad\qquad | \\ | \qquad\qquad | \\ a_{m1}(t_m) \cdots\cdots a_{m8}(t_m) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ | \\ | \\ | \\ x_8 \end{bmatrix} + \omega(t) \tag{4}$$

In compact form, Eqn. 4 can be written as

$$z(t) = A(t)x + \omega(t) \tag{5}$$

where

$z(t)$ is an m by 1 vector of voltage samples
$A(t)$ is an m by 8 measurements matrix
x is an 8 by 1 parameters vector to be estimated
$\omega(t)$ is an m by 1 errors vector associated with the measurements to be minimized. The sources of the error in this equation come from the noises contaminated the samples during the A/D conversion as well as the terms truncated from Taylor's series expansion.

The matrix $A(t)$ can be pre-determined in an off-line mode. Its elements depends on the time reference t_1 and sampling rate Δt. The x's are unknown and functions of V_m, ϕ, Δf. To determine the x vector Eqn. 5, at least eight samples of the voltage would be required for a 4th order Taylor series expansion. To obtain a good estimate, we assume that $m > 8$. Two techniques are used to solve Eqn. 5 the first one is based on the minimum least error squares (LS), while the second technique is the linear programming algorithm (LP). Both techniques are available in the IMSL/STAT routine library. Having estimated the elements of the vector x, the signal magnitude and frequency can be estimated using the following equation

$$V_m = \sqrt{x_1^2 + x_3^2} \tag{6}$$

The frequency deviation Δf can be calculated using the following equation

$$\Delta f = \frac{x_2}{x_1} = \frac{(\Delta f)(V_m \cos \phi)}{V_m \cos \phi} \tag{7}$$

Also, Δf can be calculated using the variables x_3 and x_4

$$\Delta f = \frac{x_4}{x_3} = \frac{(\Delta f)(V_m \sin \phi)}{V_m \sin \phi} \tag{8}$$

Another possible approach is to estimate frequency deviation using the variables x_1, x_2, x_3 and x_4 as

$$(\Delta f)^2 = \frac{x_2^2 + x_4^2}{x_1^2 + x_3^2} \tag{9}$$

In this paper, we calculate Δf using Eqn. 9. Furthermore, the phase angle ϕ can be obtained using one of the following equations

$$\tan \phi = \frac{x_3}{x_1} = \frac{x_4}{x_2} = \frac{x_6}{x_5} = \frac{x_8}{x_7} \tag{10}$$

2.2. *Variable Frequency Model (VFM)*

If we assume that the data window size is small, then the frequency may be assumed to change linearly with time during the measurement period. The voltage at a power system bus can be written as

$$v(t) = V_m \sin [\phi(t)] \tag{11}$$

where $\phi(t)$ is the voltage phase angle, and is given by

$$\omega(t) = \frac{d\phi(t)}{dt} = 2\pi f(t) \tag{12}$$

In Eqn. 11, we assume that the amplitude of the voltage remains constant during each data window. Since we assume that the frequency changes linearly with time, then

$$f(t) = a + bt \tag{13}$$

where a is the frequency in Hz at $t = 0.0$, and b is the rate of change of frequency in Hz/s. Substituting from Eqn. 13 into Eqn. 12, we get

$$\phi(t) = 2\pi^*(a + bt)\, dt + \theta \tag{14a}$$

which yields

$$\phi(t) = 2\pi(at + \tfrac{1}{2}bt^2) + \theta \tag{14b}$$

and the voltage signal of Eqn. 11 becomes

$$v(t) = V_m \sin[2\pi(at + \tfrac{1}{2}bt^2) + \theta] \tag{15}$$

Replacing sin (.) by the Taylor series expansion in the neighbourhood of $a = a_0 = f_0$ and $b = 0$ to obtain

$$v(t) = b_{11}(t)y_1 + b_{12}(t)y_2 + \cdots + \cdots + b_{19}(t)y_9 + b_{110}(t)y_{10} + \xi(t) \tag{16}$$

where

$$y_1 = V_m \cos\theta, \qquad y_2 = (a - a_0)V_m \cos\theta$$

$$y_3 = V_m \sin\theta, \qquad y_4 = (a - a_0)V_m \sin\theta$$

$$y_5 = -bV_m \sin\theta - 2\pi(a - a_0)^2 V_m \cos\theta$$

$$y_6 = bV_m \cos\theta - 2\pi(a - a_0)^2 V_m \sin\theta$$

$$y_7 = b(a - a_0)V_m \cos\theta$$

$$y_8 = b(a - a_0)V_m \sin\theta$$

$$y_9 = b^2 V_m \cos\theta$$

$$y_{10} = b^2 V_m \sin\theta.$$

$$b_{11}(t) = \sin(2\pi a_0 t)$$

$$b_{12}(t) = 2\pi t \cos(2\pi a_0 t)$$

$$b_{13}(t) = \cos(2\pi a_0 t)$$

$$b_{14}(t) = -2\pi t \sin(2\pi a_0 t)$$

$$b_{15}(t) = (\pi t^2)\sin(2\pi a_0 t)$$

$$b_{16}(t) = (\pi t^2)\cos(2\pi a_0 t)$$

$$b_{17}(t) = -(2\pi^2 t^3)\sin(2\pi a_0 t)$$

$$b_{18}(t) = -(2\pi^2 t^3)\cos(2\pi a_0 t)$$

$$b_{19}(t) = -\tfrac{1}{2}(\pi t^2)^2 \sin(2\pi a_0 t)$$

$$b_{110}(t) = -\tfrac{1}{2}(\pi t^2)^2 \cos(2\pi a_0 t)$$

In Eqn. 16, the y's are the parameters to be estimated and they are functions of V_m, a, b and θ. If the voltage is sampled at a pre-selected rate, we would obtain its samples at equal time intervals, say Δt seconds. A set of m samples designated as $v(t_1)$, $v(t_2), \ldots, v(t_m)$ is obtained, these are the digitized samples of the voltage, where $t_2, t_3, \ldots, t_m$ are defined earlier, Eqn. 16 can be written in a way similar to Eqn. 5 as:

$$z(t) = B(t)y + \xi(t) \tag{17}$$

where

$z(t)$ is an m by *1* measurements vector.
$B(t)$ is an m by *10* measurement matrix. The elements of the matrix depend on the time reference and the sampling interval Δt. This matrix can be calculated in an off-time mode after the sampling rate and time reference have been selected.
y is a *10* by 1 parameters vectors to be estimated.
$\xi(t)$ is an m by *1* errors vector to be minimized. The sources of these errors come from the noises associated during the A/D conversion, as well as the errors due to the terms truncated from Taylor's Series expansion.

Having estimated the elements of the vector y, the amplitude, frequency and rate of change of frequency of the voltage can be calculated as

$$V_m^2 = (y_1^2 + y_3^2) \tag{18}$$

The frequency deviation can be calculated by using one of the following equations

$$(a - a_0) = \Delta f = \frac{y_2}{y_1} = \frac{(a - a_0) V_m \cos\theta}{V_m \cos\theta} \tag{19}$$

or

$$(a - a_0) = \Delta f = \frac{y_4}{y_3} = \frac{(a - a_0) V_m \sin\theta}{V_m \sin\theta} \tag{20}$$

Another alternative technique is to use the variable y_1, y_2, y_3 and y_4 to estimate the frequency deviations

$$(\Delta f)^2 = (a - a_0) = \left(\frac{y_2^2 + y_4^2}{y_1^2 + y_3^2}\right) \tag{21}$$

If $y_1 \ll 0$, then Δf calculated from Eqn. 19 will not be accurate, also if $y_3 \ll 0$, then Δf calculated from Eqn. 20 will not be accurate. In this paper, we calculate Δf by using Eqn. 21.

The rate of change of frequency can be estimated from one of the following equations

$$b = \frac{y_7}{y_2} = \frac{b(a - a_0) V_m \cos\theta}{(a - a_0) V_m \cos\theta} \tag{22}$$

or

$$b = \frac{y_8}{y_4} = \frac{b(a - a_0) V_m \sin\theta}{(a - a_0) V_m \sin\theta} \tag{23}$$

If the frequency has its nominal value or is in the neighbourhood of its nominal value, $(a - a_0) \ll 0$, hence b calculated from Eqns. 22 & 23 will not be accurate. The rate of change of frequency can be calculated from one of the following equations

$$b^2 = \left(\frac{y_9}{y_1}\right) \tag{24}$$

or

$$b^2 = \left(\frac{y_{10}}{y_3}\right) \tag{25}$$

As we explained earlier, if $V_m \cos\theta$ is very small ($y_1 \ll 0$) the rate of change of frequency estimated by using Eqn. 24 will not be accurate. This also happens using Eqn. 25, when $V_m \sin\theta$ is very small ($y_3 \ll 0$).

Another way, which may be accurate, to calculate the rate of change of frequency is to use the variables y_1, y_3, y_5 and y_6 as follows

$$b = \frac{y_1 y_6 - y_3 y_5}{y_1^2 + y_3^2} \tag{26}$$

2.3. *Measurement of the rate of change of frequency by curve fitting*

Using the *VFM* model for measuring the rate of change of frequency, as explained above, may not be accurate. We may assume that the frequency varies with the time as

$$f = a + bt \tag{27}$$

where a & b having the same meaning as explained in Eqn. 13. Note that, we can assume higher order for the function given in Eqn. 27. If the frequency of the voltage signal is measured m times at regular intervals Δt second, then Eqn. 27 can be written as

$$f_k = a + b(k\Delta t), \quad k = 1, \ldots, m \tag{28}$$

which can be written as

$$\begin{bmatrix} f_1 \\ f_2 \\ | \\ | \\ | \\ f_m \end{bmatrix} = \begin{bmatrix} 1 \cdots\cdots \Delta t \\ 1 \cdots\cdots 2\Delta t \\ | \quad\quad | \\ | \quad\quad | \\ | \quad\quad | \\ 1 \cdots\cdots m\Delta t \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} + v \tag{29}$$

Equation 29 can be written in vector form as

$$z = H\theta + v \tag{30}$$

where z is an m by *1* measurements vector and is given by

$$z = col\,(z_1, z_2, \ldots, z_m).$$

θ is n by *1*, parameters vector to be estimated.
H is m by n measurements matrix.
v is m by *1* errors vector to be minimized.

3. PARAMETER ESTIMATION PROBLEM

In the previous discussions, the models used for frequency relaying and rate of change of frequency, Eqns. 5, 17 & 30 are transferred into a state estimation problem which simply states that: given m measurements and the model of the system

$$z_i = H_i x + v_i, \quad i = 1, \ldots, m \tag{31}$$

It is required to estimate the n of the unknown parameters x such that, the error v_i, $i = 1, \ldots, m$ contaminating the measurement i is a minimum, according to a certain performance index. The performance index for the *LP* used in this paper is to minimize the sum of the absolute value of the errors vector as

$$J_1(x) = \sum_{i=1}^{m} |v_i| \tag{32a}$$

or it can be written as

$$J_1(x) = \sum_{i=1}^{m} |z_i - H_i x| \tag{32b}$$

The Solution to Eqn. 32 can be obtained by different techniques. In this paper, we use the linear programming technique to solve this problem. The subroutine RLAV available in IMSL STAT/LIBRARY is the new version for the *LP* algorithm and is used to solve the problem formulated in Eqn. 32.

4. TESTING OF THE ALGORITHM

The proposed algorithm is tested in the off-line mode to study the effects of sampling frequency; data window size, time reference location and the number of terms truncated from the Taylor series expansion, on the performance of the algorithm. For this purpose a computer program is written to generate the data, i.e. we simulate the voltage signal on software program to generate the required voltage samples. For *CFM* the voltage signal is assumed to have constant frequency during the data window size, and is sampled at a pre-selected rate Δt. For *VFM*, we assume that the voltage signal frequency varies linearly with the time and is sampled at a pre-selected rate.

The voltage samples are used to estimate the steady state voltage amplitude, the steady state frequency deviation and the rate of change of frequency. The algorithm is tested first when the voltage samples are contaminated with bad data (gross error and/or gaussian noises), where we compare the *LP* and the well known least error squares (*LS*) Technique.

4.1. *Data Window Size*

Table 1 reports the results obtained at different window sizes where the *LS* and *LP* are used to estimate the system parameters. This table reveals that with no bad data, the

Table 1. The estimated voltage amplitude, and the frequency deviation, $\Delta f = f - f_0$, $f_0 = 60$ Hz, for sampling frequency = 720 Hz, with time reference in the middle of the data window size and the number of parameters to be estimated $x = 8$, *CFM* model.

Case #			1	2	3	4	5	6
No. of samples			12	24	36	48	60	72
Window size (m sec)			16.67	33.35	50	66.67	83.33	100
No bad data	*LS*	V	1.414	1.414	1.414	1.414	1.414	1.414
		Δf	0.0	0.0	0.0	0.0	0.0	0.0
	LP	V	1.414	1.414	1.414	1.414	1.414	1.414
		Δf	0.0	0.0	0.0	0.0	0.0	0.0
		# of Iter.	8	8	16	10	8	12
With bad data	Bad Data Location		10	10	1, 2	1, 11	1, 10	1, 10, 20
	LS	V	1.36	0.914	1.464	1.351	1.375	1.349
		Δf	137.0	14.18	1.59	2.11	1.15	1.103
	LP	V	1.414	1.414	1.414	1.414	1.414	1.414
		Δf	0.0	0.0	0.0	0.0	0.0	0.0
		$\|r\|$	2.73	2.73	2.73	2.73	3.47	5.46
		# of Iter.	10	8	17	11	12	13

$|r| = \sum_{i=1}^{m} |r_i|$ = is the sum of the absolute value of the residuals.

two techniques estimate exactly the voltage amplitude ($V_m = \sqrt{2}$) and the frequency deviation ($\Delta f = 0$).

However, if the measurements set is contaminated with bad data (Table 1), where we change the sign of the specified measurement), the *LS* algorithm gives a bad estimate, while the *LP* gives the optimal estimate, but it takes a great number of iterations. However, as the data window increase the *LS* estimate is improved.

Table 2 gives the results for the *VFM* under the specified conditions in the head of the table. With no bad data the *LS* and *LP* algorithm estimate correctly the voltage phasor amplitude ($V_m = \sqrt{2}$), the frequency deviation Δf and the rate of change of frequency b (=0.2) except for the first case which has a slight error in the rate of change of the frequency; but when the measurement samples are contaminated with bad errors, *LS* fails to estimate the frequency deviation and the voltage phasor amplitude. In contrast, *LP* estimates these parameters optimally better than *LS* algorithm.

4.2. *Sampling Rate*

The algorithm is tested at different sampling frequencies. Table 3 gives the results obtained for *CFM* when the sampling frequencies are 180, 360, 540, 720 and 900 Hz. It can be seen from this table that, if no bad data has contaminated the measurements set, both the *LS* and the *LP* algorithm produce the same estimate. However, if the measurements set is contaminated with bad data, the *LS* algorithm gives a poor estimate as the sampling frequency increases. In contrast, the *LP* algorithm gives a good estimate, and it does not change as the sampling frequency increases. The same

Table 2. The estimated voltage amplitude, the frequency deviation and the rate of change of frequency, with the time reference shifted to the left of the middle of the data window size by $3\Delta t$. The number of parameters to be estimated $x = 10$, sampling frequency = 720 Hz variable frequency model (*VFM*).

Case #			1	2	3	4	5
No. of samples			24	36	48	60	72
Window size (m sec)			33.33	50.00	66.67	83.33	100.0
No bad data	*LS*	V^+	1.414	1.414	1.414	1.414	1.414
		$\Delta f^\times$	0.00	0.00	0.00	0.00	0.02
		b^*	0.194	0.201	0.200	0.200	0.201
	LP	V	1.414	1.414	1.414	1.414	1.414
		Δf	0.00	0.00	0.00	0.00	0.00
		b	0.199	0.200	0.200	0.200	0.200
With bad data	*LS*	V	1.148	1.107	1.112	1.545	1.127
		Δf	6.601	2.380	3.960	0.726	1.317
		b	Fail	Fail	Fail	Fail	Fail
	LP	V	1.414	1.414	1.414	1.414	1.414
		Δf	0.00	0.00	0.00	0.00	0.00
		b	0.200	0.201	0.200	0.200	0.200
		$\lvert r \rvert$	2.732	7.465	6.927	8.926	12.931
		# of Iter.	14	18	28	22	27

+ the actual value is $\sqrt{2}$.
× the actual value is $\Delta f = 0.0$.
* the actual value is $b = 0.20$.

Table 3. The estimated voltage amplitude, and the frequency deviation, for a number of samples = 60, with time reference in the middle of the data window size and the number of parameters to be estimated $x = 8$, *CFM* model.

Case #			1	2	3	4	5
Sampling Freq.			180	360	540	720	900
Window size (m sec)			333.33	166.60	111.10	83.33	66.60
No bad data	*LS*	V	1.414	1.414	1.414	1.414	1.414
		Δf	0.00	0.00	0.00	0.00	0.00
	LP	V	1.414	1.414	1.414	1.414	1.414
		Δf	0.00	0.00	0.00	0.00	0.00
with bad data	*LS*	V	1.467	1.425	1.476	1.375	1.380
		Δf	0.069	0.080	0.281	1.545	1.028
	LP	V	1.414	1.414	1.414	1.414	1.414
		Δf	0.00	0.00	0.00	0.00	0.00
		$\|r\|$	4.900	2.829	5.571	3.464	2.900
		# of Iter.	11	13	11	15	16

test was conducted for *VFM*. Table 4 gives the results obtained, and the above discussion is held true for this table. Figures 1 & 2 give the variation of error in the estimated frequency deviation with the number of samples when the sampling frequency is 720 Hz and the signal frequency is 58 Hz, for no bad data and with bad data contaminating the measurement set studing these curves, carefully, reveals that with no bad data, Fig. 1, the error in the estimated frequency deviation is almost zero, and does not change as the number of samples change for *LS* and *LP*

Table 4. The estimated voltage amplitude, the frequency deviation, and the rate of change of frequency, with the time reference at the middle of the data window size. The number of parameters to be estimated $x = 10$, Number of samples = 60, variable frequency model (*VFM*).

Case #			1	2	3	4	5
Sampling Freq.			180	360	540	720	900
Window size (m sec)			333.33	166.60	111.10	83.33	66.60
No bad data	*LS*	V	1.414	1.414	1.414	1.414	1.414
		Δf	0.00	0.00	0.00	0.00	0.00
		b	0.200	0.200	0.200	0.200	0.200
	LP	V	1.414	1.414	1.414	1.414	1.414
		Δf	0.00	0.00	0.00	0.00	0.00
		b	0.201	0.200	0.200	0.200	0.200
With bad data	*LS*	V	1.389	1.413	1.400	1.514	1.487
		Δf	0.069	0.078	0.316	1.267	0.848
		b	Fail	4.283	35.42	85.60	Fail
	LP	V	1.414	1.414	1.414	1.414	1.414
		Δf	0.00	0.00	0.00	0.00	0.00
		b	0.200	0.200	0.200	0.200	0.200
		$\|r\|$	4.934	2.844	5.572	3.467	2.693
		# of Iter.	26	19	28	17	23

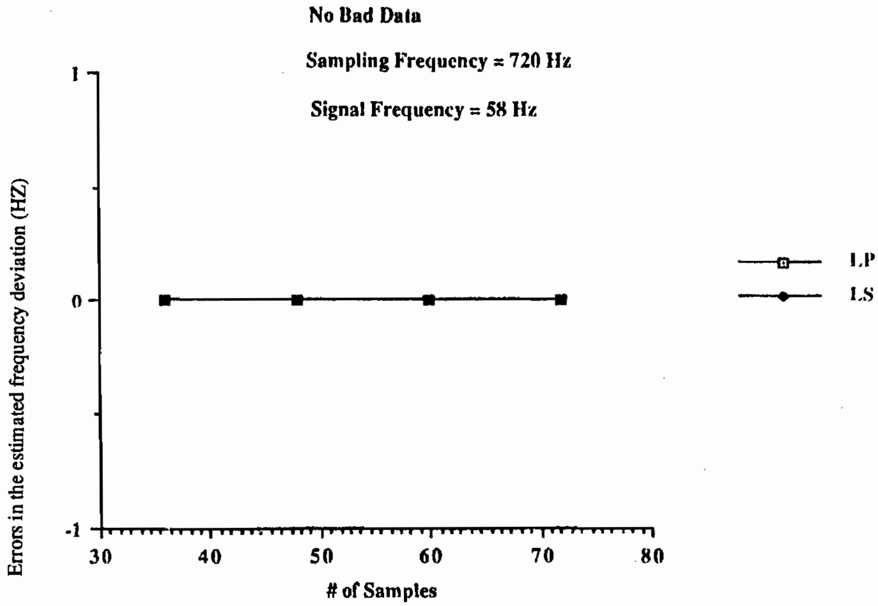

Fig. 1. Variation of errors in the estimated frequency deviation with number of samples.

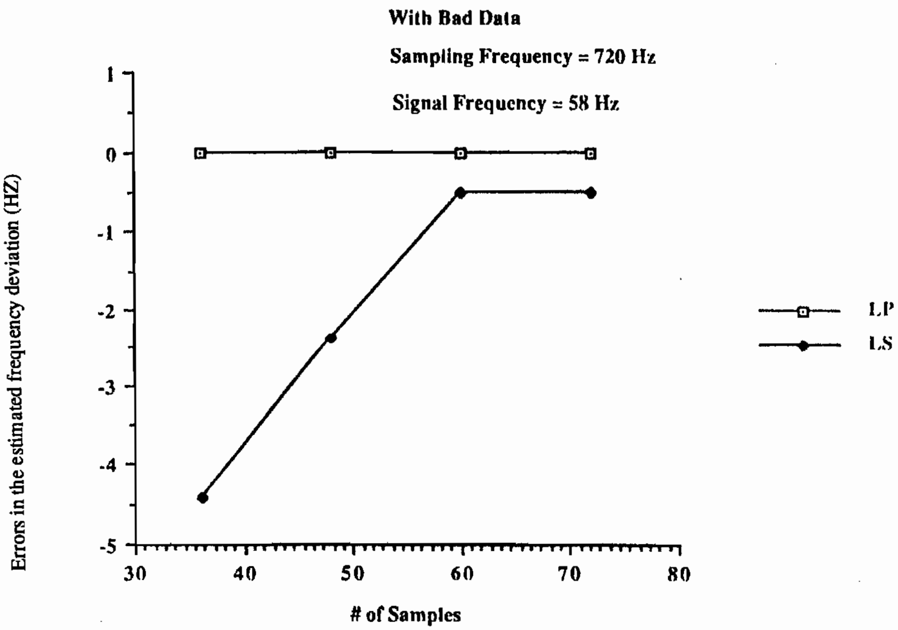

Fig. 2. Variation of errors in the estimated frequency deviation with number of samples.

algorithms. However, with bad data the error in the estimated frequency deviation using *LP* is almost zero (Fig. 2), while it decreases, for *LS* algorithm, as the number of samples increases. In conclusion, a suitable combination of data window size and sampling rate must be selected for *LS* algorithm, but it is not necessary for the *LP* algorithm.

4.3. *Taylor Series Expansion*

If the data window size is small, a few terms of Taylor series expansion of sine and cosine functions would be adequate for reasonable accuracy, but if the measurements window is large, more terms are needed. Table 5 gives the results obtained when the number of terms kept from the Taylor Series are 3, 4 and 5 of sine and cosine functions (number of parameters to be estimated are 6, 8 & 10). With no bad data, the two techniques produce the same optimal estimate for *CFM*, but when the measurements set is contaminated with gross error, the *LS* algorithm produces bad estimates, while *LP* produces good estimates. It can be seen from Table 5 that the number of terms truncated from Taylor series expansion does not affect the optimal estimate for the *LP* algorithm, but it does affect the *LS* algorithm.

4.4. *Variation of the Signal Frequency*

The *LP* algorithm together with the *LS* algorithm were tested when the signal frequency was varied from 57 to 63 Hz. Figure 3 gives the results obtained with no bad data, the number of samples equals 72 and the sampling frequency is 720 Hz. This curve shows that the error in the estimated frequency deviation around the nominal frequency (60 Hz) is almost zero, and it changes a little if the signal frequency is greater than 61 Hz and less than 58 Hz. Figure 4 shows the same results obtained when the measurement samples are contaminated with bad data. Indeed, as we see from this figure the *LP* algorithm does not produce error in the estimated frequency deviation as the signal frequency changes (unique estimate). However, the *LS* algorithm does produce errors in the estimated frequency deviations as the signal frequency changes. Note that the matrix A of Eqn. 5 is calculated once at the nominal frequency and is used for different signal frequencies.

Table 5. The estimated voltage amplitude, and the frequency deviation, for different terms truncated from Taylor series of sine and cosine functions. Number of samples = 60, sampling frequency = 720 Hz, with time reference in the middle of the data window size, *CFM* model.

Case #			1	2	3
No. of variables			6	8	10
No bad data	*LS*	V	1.414	1.414	1.414
		Δf	0.00	0.00	0.00
	LP	V	1.414	1.414	1.414
		Δf	0.00	0.00	0.00
With bad data	*LS*	V	1.362	1.351	1.511
		Δf	0.540	2.110	1.267
	LP	V	1.414	1.414	1.414
		Δf	0.00	0.00	0.00
		$\lvert r\rvert$	3.46	3.46	3.46
		# of Iter.	10	12	14

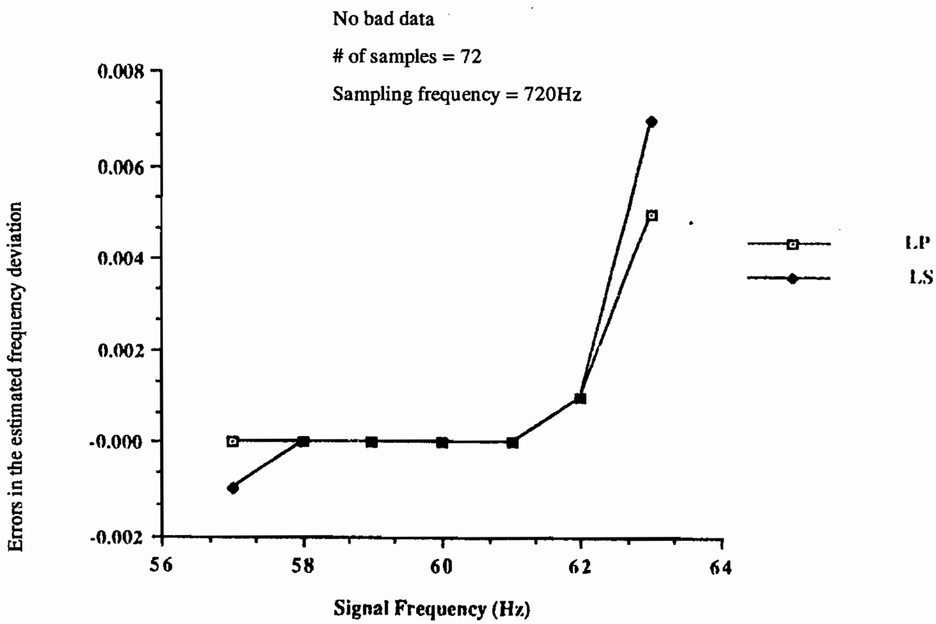

Fig. 3. Errors in the estimated frequency deviation.

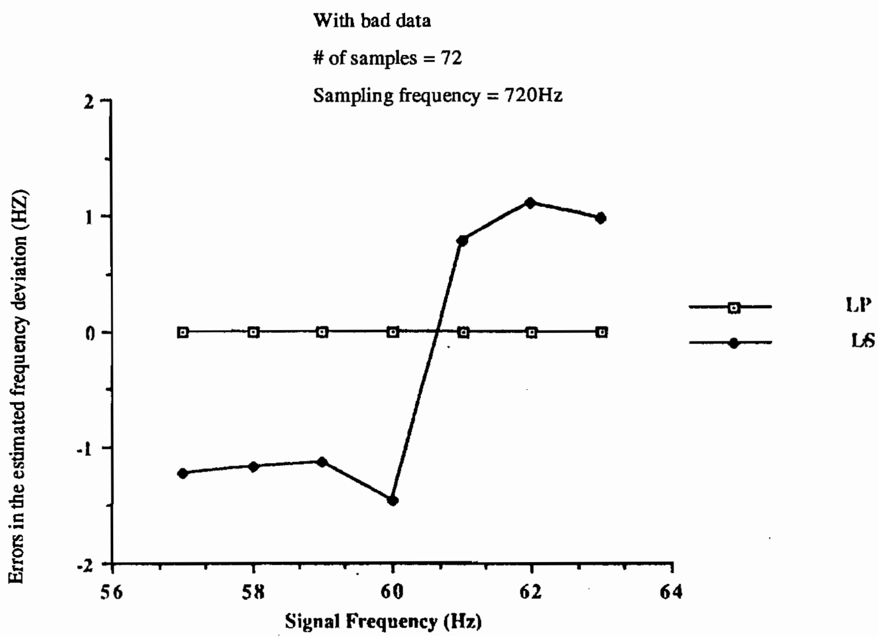

Fig. 4. Errors in the estimated frequency deviation.

4.5. *Location of Reference Time*

It has been shown that the location of the reference time does not affect the frequency deviation estimate and the rate of change of frequency as well as the amplitude of the phasor voltage, when the *LP* algorithm used. However, to decrease the computing time using the *LS* algorithm, it is recommended that the reference time should be in the middle of the data window size (Giray & Sachdev 1989).

4.6. *The Rate of Change of Frequency b*

The two algorithms, *LP* and *LS*, were used to estimate the parameters a and b of Eqn. 27 from the simulated data, when the measurements contain no bad data and they contain bad data. Table 6 reports the results obtained. For no bad data the two algorithms produce the same good estimates, but with bad data, the *LP* produces estimates better than the *LS* which fails to estimate the parameters.

Table 6. The estimated nominal frequency and the rate of change of frequency using linear regression. Sampling frequency = 720 Hz.

Case #			1	2	3	4
No. of samples			3	6	9	12
No bad data	*LS*	a	60.00	60.00	60.00	60.00
		b	0.20	0.20	0.20	0.20
	LP	a	60.00	60.00	60.00	60.00
		b	0.20	0.20	0.20	0.20
With bad data	*LS*	a	56.70	54.70	55.00	55.00
		b	0.20	411.63	240.20	126.10
	LP	a	60.00	60.00	60.00	60.00
		b	0.20	0.20	0.20	0.20
		$\|r\|$	10.00	20.00	30.00	50.00
		# of Iter.	2.00	4.00	2.00	3.00

5. CONCLUSIONS

This paper concerns frequency measurements and their estimations for frequency relaying. Two distinct cases are considered, namely a constant frequency model, *CFM* (the frequency is constant during the window size) and a variable frequency model, *VFM*, (the frequency varies linearly during the window size).

Two methods are used for estimating the frequency namely, Least Squares Error (*LS*) methods and linear programming. In all cases considered we consider the measurements first with no bad data and second with bad data. In every instant the results clearly show that the least error squares methods is adversely affected by the presence of gross error whereas the linear programming methods is not affected by the presence of bad data. Hence it can be concluded that when we expect bad data to be present it is much preferable to use linear programming over least error squares estimations.

6. REFERENCES

Giray, M.M. & Sachdev, M.S. 1989. Off-nominal frequency measurements in electric power systems. IEEE Transaction on Power Delivery **4 (3):** 1573–8.

Giray, M.M. & Sachdev, M.S. 1985. Amplitude and frequency measurements by discrete fourier transform. Transaction CEA, Power System Planning and Operation Section **24 (85-SP-150).**

Phadke, A.G., Thorp, J.S. & Adamiak, M.G. 1983. A new measurement technique for tracking voltage phasor, local system frequency, and rate of change of frequency. IEEE Transaction on Power Apparatus and Systems, **PAS-102 (5):** 1025–38.

Sachdev, M.S. & Giray, M.M. 1985. A least error squares technique for determining power system frequency. IEEE Transaction on Power Systems and Apparatus. **PAS-104 (2):** 437–44.

Sachdev, M.S., Wood, H.C. & Johnson 1985. Kalmen filtering applied to power system measurements for relaying. IEEE Transaction on Power Apparatus and Systems, **PAS-104 (12):** 3573.

(Received 5 July 1993, re-revised 21 February 1994)

تطبيق خوارزم البرنامج الخطي في قياسات نظم القوى الكهربية لمرحلات التردد الرقمية

أحمد محمد الكندري
قسم تكنولوجيا الهندسة الكهربائية بكلية الدراسات التكنولوجية،
ص.ب. 42325 الشويخ 70654 الكويت

خلاصة

يقدم هذا البحث تطبيق جديد للبرنامج الخطي في قياسات نظم القوى الكهربية المستخدمة في مراحل التردد والمبني على أساس خوارزم أقل خطأ مطلق للقراءات الرقمية للجهد، المأخوذة في موقع المرحل.

ويتميز هذا الخوارزم عن خوارزم مربع أقل خطأ المعروف في تصنيف القراءات الرديئة للجهد وحذفها من عملية الحسابات للتردد. وقد تم بناء نموذجين في هذا البحث لقياس التردد.

النموذج الأول:

ويسمى بنموذج التردد الثابت وفيه يفترض أن التردد ثابت خلال شباك القراءات.

النموذج الثاني:

ويسمى بنموذج التردد المتغير وفيه يفترض أن التردد متغير خلال شباك القراءات المستخدم ولهذا النموذج يتم قياس التردد العابر وكذلك معدل تغير هذا التردد.

ولقد تم كذلك في هذا البحث دراسة تأثير كل من معدل أخذ القراءات الرقمية وحجم شباك المعلومات وكذلك تأثير وضع بداية أخذ القراءات على الخوارزم المقترح وقد تم دراسة الخوارزم المقترح على أمثلة رقمية ومقارنته بخوارزم مربع أقل خطأ.

ويعتبر الخوارزم المقترح سهل التطبيق على المعالج الدقيق.