J. Univ. Kuwait (Sci.) **21**, 1994

Interactive minimization scheme of copper loss in induction motors by capacitive effect

YOUSEF A. SAFAR, MOHAMED H. SALAMA AND
MAGDI S. MAHMOUD

*Electrical and Computer Engineering, University of Kuwait, P.O. Box 5969,
Safat 13060, Kuwait*

ABSTRACT

The paper has presented a method for minimizing copper loss of wound-rotor induction motor using series resonance tank in series with the rotor circuit. When an external circuit is connected in series with the rotor circuit, the values of the circuit's parameters which ensures minimum copper loss at any desired speed are computed. The computations are performed for both a fixed and a continuously-varying capacitance with respect to rotor speed. It has been found that the later case is better than the former. The optimization technique is then generalized to allow the motor to be driven from a variable frequency source.

INTRODUCTION

The high cost of energy provides the incentives to reduce the energy loss in industrial systems. This is the reason for which the problem of reducing the global losses of induction motors attract the attention of several investigators. Tsivitse & Klingshirn (1970) stated that an induction motor can be made to operate at the minimum loss for any torque and speed by an adjustment in source voltage and frequency. Mohan (1980) claimed that substantial energy savings are possible in lightly-loaded induction motors by means of controlling the applied voltage. Galler (1980) presented a feedback control system for energy efficient control of an induction motor. Kusko & Galler (1983) used an open loop controller to solve a set of loss minimization equations of induction motor by using preset drive parameters. Kirschen *et al.* (1987) presented a method based on the adaptive control of the rotor flux for optimizing the efficiency of adjustable frequency induction motor drives. Ioannides & Tegopoulos (1988) presented a technique for the optimization of efficiency of the doubly-fed induction motor in slip power recovery drives.

Baghouz & Tan (1989) presented an optimal method to control the speed of a wound-rotor induction motor by adjustable external rotor impedance. They claimed that motor efficiency is improved if compared with the conventional speed control by adjustable rotor resistance only.

This paper describes an efficient procedure to reduce the copper loss of wound-rotor induction motors by inserting a series resonant circuit in series with the rotor. A mathematical model is used to represent the motor when operating with a fixed load torque at different slip values. An optimization technique is developed based on the proposed mathematical model. This technique is used to compute the values of the external circuit to be connected in series with the rotor in order to yield minimum copper loss.

The idea of minimizing the copper loss of wound-rotor induction motors using a series-resonant circuit in series with the rotor was first presented by Baghzouz & Tan (1989). In their work they discussed a method to improve the efficiency of the rotor-resistance controlled induction motor. Their method ties the circuit parameters together to meet a constraint on the operating torque. The work described in this paper may be considered as an extension to their contribution. According to the analysis presented here, the values of the external circuit parameters are determined which, when connected in series with the rotor circuit, will ensure minimum copper loss at any desired speed. Also, the paper discusses in detail the steady-state performance obtained by inserting a capacitor of either a fixed or a continuously adjustable value with respect to the rotor speed. The optimization technique is expanded to allow the motor to be driven from an adjustable frequency source. Both the minimum copper loss and the corresponding value of capacitance are computed for a frequency range from 0.5 per unit up to the rated value.

A MATHEMATICAL MODEL

The steady-state mathematical model is obtained from the per phase equivalent circuit of wound rotor induction motor shown in Fig. 1. In this equivalent circuit, the shunt resistance accounting for the core loss is omitted since it is relatively large and the core loss is commonly assumed a fixed value.

The basic idea of the proposed mathematical technique is to search for the minimum value of the copper loss when inserting an external adjustable capacitance in series with an adjustable inductor and an adjustable resistor, in series with the rotor circuit. In the analysis, all electrical rotor variables are referred to the stator side.

The equivalent circuit reveals the following parameters

$$R_2 = \frac{R_r + R_e}{s} \tag{1}$$

$$X_2 = \omega(L_r + L_e) - \frac{1}{s^2 \omega C_e} \tag{2}$$

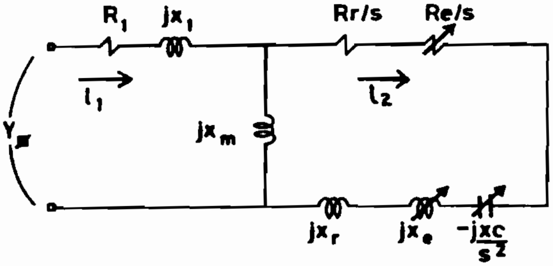

Fig. 1. Equivalent circuit of induction motor.

The stator current phasor is related to the applied voltage phasor by

$$I_1 = \frac{V_\phi}{R_1 + jX_1 + \dfrac{jX_m(R_2 + jX_2)}{R_2 + j(X_m + X_2)}} \tag{3}$$

The rotor current phasor is related to the stator applied voltage phasor by

$$I_2 = \frac{V_\phi jX_m}{R_1 + jX_1 + \dfrac{jX_m(R_2 + jX_2)}{R_2 + j(X_m + X_2)}} \times \frac{1}{jX_m(R_2 + jX_2)} \tag{4}$$

The electromagnetic torque developed by the motor is given by

$$T_e = 3|I_2|^2 R_2 \Big/ \frac{2\pi n_s}{60} \tag{5}$$

The copper loss dissipated in the motor is given by

$$P_{cu} = 3(|I_2|^2 R_1 + |I_2|^2 R_2) \tag{6}$$

$$P_{cu} = T_e \omega \left(s + \left(\frac{R_1}{X_m^2}\right) \left\{ \frac{R_r + R_e}{s} + \frac{s}{R_r + R_e} \left[X_m + \omega(L_r + L_e) - \frac{1}{s^2 \omega C_e} \right]^2 \right\} \right) \tag{7}$$

Eqn (7) indicates that the copper loss of a wound rotor induction motor is influenced by the parameters of the external circuit inserted in series with the rotor terminals.

For a constant load torque the following constraints must be considered in applying the optimization technique necessary to calculate the conditions required for minimum copper loss.

1. $s \leqslant \dfrac{R_r}{R_r + R_e}$ (8)

This inequality constraint assures a non-negative value of the external resistance R_e. Thus the optimization technique used will be valid only for slip values satisfying Eqn (8).

2. $C_e < \dfrac{1}{s^2 \omega^2 (L_r + L_e)}$ (9)

The second inequality constraint identifies the values of the capacitor necessary to start the optimization technique.

3. $R_e \geqslant 0$ (10)

The third constraint insures a positive value for the external resistance.

INTERACTIVE OPTIMIZATION PROCEDURE

Minimization of Eqn (7) with respect to L_e, R_e and C_e subject to Eqns (8) to (10) is typically a multivariable constrained optimization problem. Instead of solving a complex set of equations in terms of the gradients of Eqn (7), a variable-step search procedure in the function space has been used. The procedure is implemented in an interactive mode using equation solving software on a desk-top computer

(MATHEMATICA package). It is summarized in the following steps:

1. Select an arbitrary point (R_e^0, L_e^0 & C_e^0) satisfying constraints Eqns (8) to (10).
2. Call equation solution routines and compute P_{cu} using Eqn (7).
3. Fix R_e^0 and compute P_{cu} in the ($L_e - C_e$) space at successive points around a rectangular mesh of preselected size.
4. Plot P_{cu} surface in the three dimensional space (P_{cu}, L_e, C_e).
5. Increment R_e and repeat the procedure. Go back to step 3.
6. Examine the developed P_{cu} surfaces (R_e^*, C_e^* & L_e^*) and decide upon the desired one.
7. Pick another point (R_e, L_e & C_e) and go back to step 2. Stop when the P_{cu}^* surface does not change in successive iterations.

The computation is started by assuming that the motor is delivering rated torque at a given slip. Thus for a constant load torque and a specified slip the corresponding values of the external circuit parameters are computed during a search for the minimum value of the copper loss. The results obtained from the optimization technique are in the form of three-dimensions plots, a sample of which is shown in Fig. 2. The mathematical analysis of these extensive number of plots indicates that minimum copper loss is obtained when both values of the external resistance and inductance are very close to zero while the value of the external capacitance required to ensure minimum copper loss is found to be of decreasing value as the value of slip is increasing towards unity, as shown in Fig. 3.

To verify both the mathematical model and the optimization procedure used, three wound-rotor induction motors of different ratings, as described in Appendix 2, are used and the results obtained show the same features discussed above.

STEADY-STATE PERFORMANCE

The steady-state performance of a wound-rotor induction motor with an external capacitor connected in series with the rotor, is computed for a load of constant

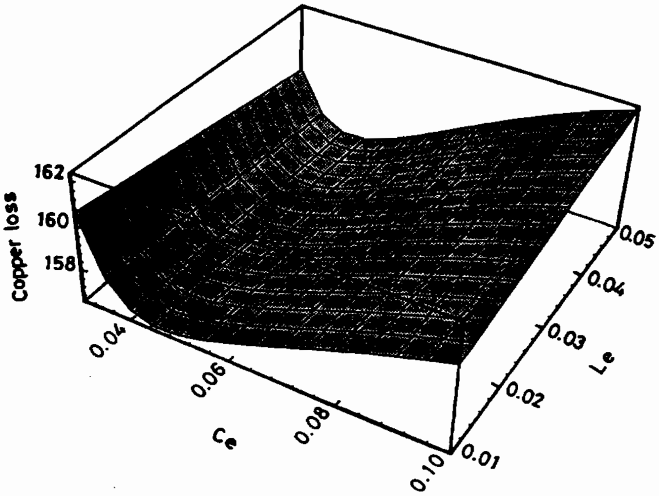

Fig. 2. Three dimension plot for copper loss variations at $R_e = 0.1R_r$ for 2 hp motor.

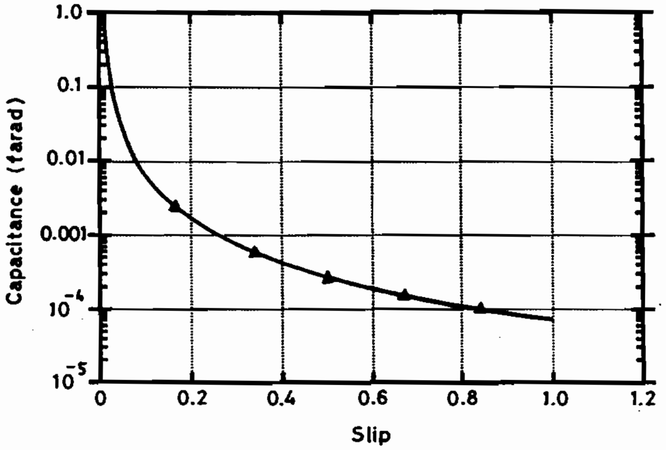

Fig. 3. Capacitance for minimum copper loss at different slips for 30 hp motor.

torque. Two study-cases are considered: First, the case when the external series capacitance is assumed continuously adjusted with slip, as in the pattern described in Fig. 3, and which ensures minimum copper loss over the full range of slip. Second, the case when the external series capacitance is assumed of a fixed value corresponding to minimum copper loss at full load slip. The steady-state performances obtained for both cases are then compared with the normal operating conditions when no external capacitor is connected to the rotor circuit. The results obtained are summarized in Table 1 and Table 2. The base quantities assumed are the copper loss and the input current at full load when no external capacitance is used.

The results presented in Table 1 & 2 show that excellent performance is obtained when the external capacitance is continuously adjusted as the speed of motor is changed. However, for a fixed value of capacitance good performance is obtained only at a slip corresponding to the chosen value of the capacitance with no practical improvement at any other slip values.

Samples of the performance characteristics are presented in Figs 4 to 6. The curves

Table 1. The steady-state performance of 30 hp wound-rotor induction motor.

The slip	The external capacitance used	The steady-state performance		
		Copper Loss pu	Input current pu	Power factor
$s_{FL} = 0.02$	$C_e = 0$	1.0	0.893	0.907
	$C_e =$ fixed value	0.5004	0.903	0.88
	$C_e =$ continuously adjusted	0.4984	0.8367	0.9499
$s_{\max} = 0.141$	$C_e = 0$	6.706	1.23	0.7358
	$C_e =$ fixed value	6.556	0.9994	0.9009
	$C_e =$ continuously adjusted	3.17	0.7944	0.9989
$s = 1.0$	$C_e = 0$	47.5117	5.6401	0.281
	$C_e =$ fixed value	47.4904	5.5872	0.283
	$C_e =$ continuously adjusted	22.4344	0.7949	0.998

Table 2. The steady-state performance of 50 hp wound-rotor induction motor.

The slip	The external capacitance used	The steady-state performance		
		Copper Loss pu	Input current pu	Power factor
	$C_e = 0$	1.0	0.8985	0.902
$s_{FL} = 0.02$	$C_e =$ fixed	0.4616	0.8229	0.9646
	$C_e =$ continuously adjusted	0.4615	0.805	0.986
	$C_e = 0$	3.2	1.209	0.702
$s_{\max} = 0.065$	$C_e =$ fixed	2.888	1.076	0.779
	$C_e =$ continuously adjusted	1.459	0.797	0.995
	$C_e = 0$	49.3	12.08	0.136
$s = 1.0$	$C_e =$ fixed	49.28	11.98	0.137
	$C_e =$ continuously adjusted	22.39	4.35	0.182

show that copper loss varies almost linearly with slip. For continuously adjusted capacitance, the input current fluctuates about its rated value while unity power factor is obtained at two values of slip; one of these being approximately the full load slip. However, the curves indicate that excellent improvement of the power factor and considerable reduction of the input current is achieved when the external capacitance is continually adjusted over the whole range of the operating speeds.

ADJUSTABLE FREQUENCY CONTROL

The motor under test is operated from an adjustable frequency source while delivering a load torque corresponding to that of its full load slip. The adjustable frequency source will ensure a constant air gap flux at all frequencies. The optimization technique is used to determine both the value of the minimum copper loss and the corresponding value of capacitance to be connected in series with the rotor circuit. Figure 7 shows that for minimum copper loss, higher values of capacitance are

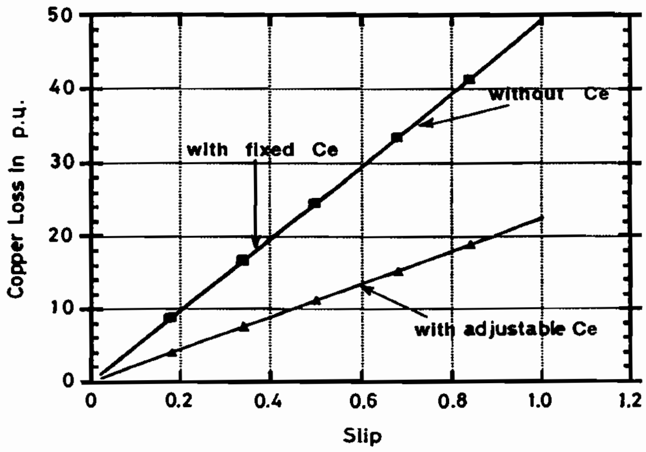

Fig. 4. Copper loss for 50 hp motor.

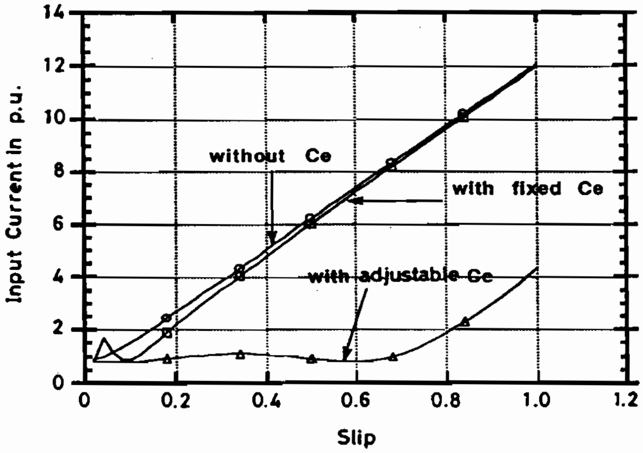

Fig. 5. Input current characteristics for 50 hp motor.

needed as the source frequency is reduced. Based on the previous results, the value of capacitance C_{ef} needed at any source frequency f can be represented in terms of the rated supply frequency f_r and its corresponding capacitance $C_{e_{rated}}$ required to ensure minimum copper loss, as

$$C_{ef} = \left(\frac{f_r}{f}\right)^2 C_{e_{rated}}$$

Figure 8 shows that the minimum copper loss obtainable increases approximately linearly with source frequency and that when the optimum value of capacitance is connected in series with the rotor, the copper loss is reduced to almost 50% of that when no capacitance is used.

CONCLUSION

The paper discusses the steady-state performance of a wound rotor induction motor when a continuously adjustable capacitance is connected in series with its rotor. The

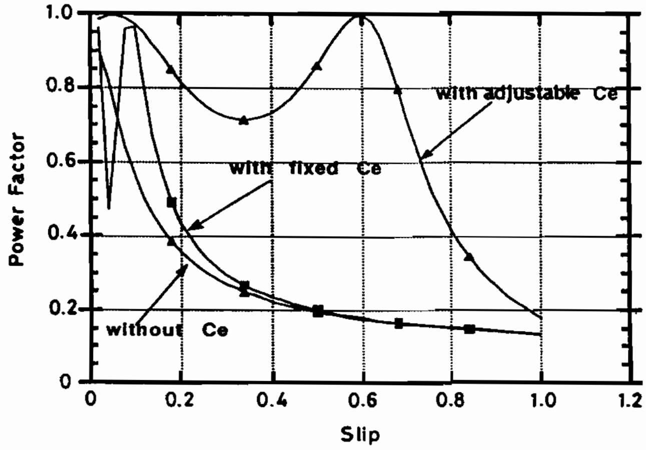

Fig. 6. Power factor characteristics for 50 hp motor.

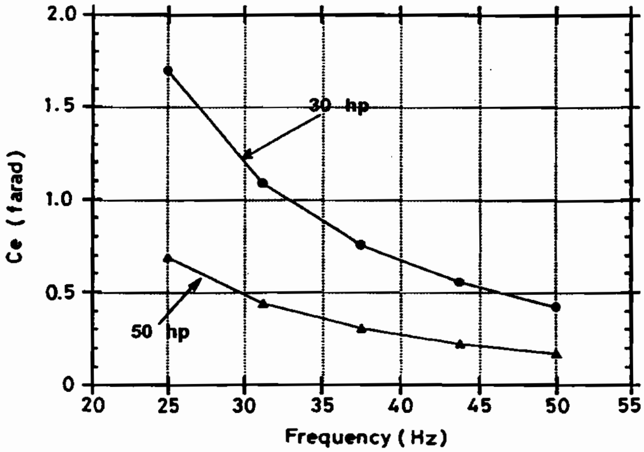

Fig. 7. Capacitance, referred to stator, for minimum copper loss.

optimization of the mathematical model indicates that any additional inductance or resistance to the rotor circuit will tend to increase the minimum copper loss obtained when an adjustable capacitance is only used. Compared to the normal operating conditions, the following features are realized when a continuously adjustable capacitance is connected in series with the rotor circuit:

1. Almost 50% reduction in the copper loss over the whole range of rotor speed is obtained.
2. The power factor is improved substantially.
3. The input power is reduced and has an almost constant value over the whole range of speed.
4. The input current fluctuates about its full-load rated value.

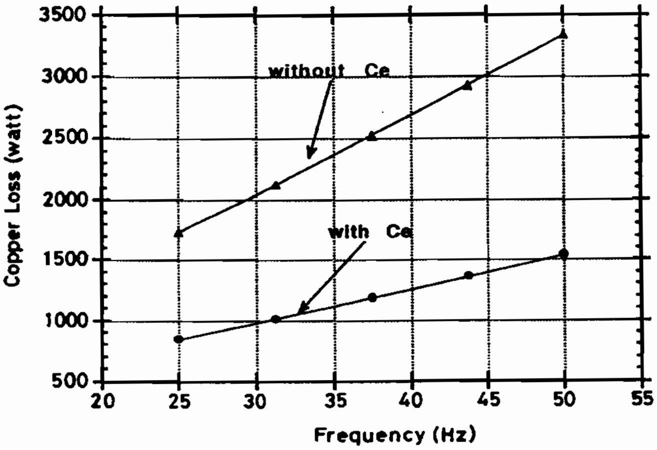

Fig. 8. Copper loss for 50 hp motor.

ACKNOWLEDGEMENT

The authors would like to acknowledge the support of Kuwait University Research Administration through project Grant No. EE 053.

REFERENCES

Baghzouz, Y. & Tan, O.T. 1989. Optimal efficiency speed control of induction motors by variable rotor impedance. IEEE Transactions on Energy Conversion, **4:** 216–23.

Galler, D. 1980. Energy efficient control of AC induction motor driven vehicles. Proceedings of IAS Annual Meeting: 301–308.

Ioannides, M.G. & Tegopoulos, J.A. 1988. Optimal efficiency slip-power recovery drive. IEEE Transactions on Energy Conversion, 3: 342–48.

Kirschen, D.S., Novotny, D.W. & Lipo, T.A. 1987. Optimal efficiency control of an induction motor drive. IEEE Transactions on Energy Conversion, EC-2: 70–76.

Kusko, A. & Gallar, D. 1983. Control means for minimization of losses in AC and DC motor drives. IEEE Transactions on Industry Applications, IA-19: 561–70.

Mohan, N. 1980. Improvement in energy efficiency of induction motors by means of voltage control. IEEE Transaction on Power Apparatus and Systems, PAS-99: 1466–71.

Tsivitse, P.J. & Klingshirn, E.A. 1971. Optimum voltage and frequency for polyphase induction motors operating with variable frequency power supplies. IEEE Transactions on Industry and General Applications, IGA-7: 480–7.

APPENDIX 1

LIST OF SYMBOLS

R_r, R_e	Internal and external rotor circuit resistance referred to stator side, ohm.
L_r, L_e	Leakage and external inductances of rotor circuit referred to state side, henry.
C_e	External rotor capacitance referred to stator side, farad.
R_1, X_1	Stator winding resistance and leakage resistance, ohm.
V_ϕ	Phase voltage applied to stator windings, volts.
s	Slip, per unit.
X_m	Magnetizing reactance, ohm.
ω	$2\pi f$
n_s	Synchronous speed, r/min.

APPENDIX 2

DATA AND PARAMETERS OF TEST MACHINES

a. First induction motor

1.5 KW, 380 V, 50 Hz, 4.5 A, 2 poles, wound rotor

$R_1 = 3.4\Omega$

$X_1 = 3.5\Omega$

$R_r = 0.43\Omega$

$X_r = 0.35\Omega$

$X_m = 133\Omega$

turns ratio = 4.31

b. Second induction motor
30 hp, 325 V, 50 Hz, 4-pole, wound rotor

$R_1 = 0.126\Omega$
$X_1 = 0.4\Omega$
$R_r = 0.118\Omega$
$X_r = 0.424\Omega$
$X_m = 18.3\Omega$

turns ratio = 2.19

c. Third induction motor
50 hp, 600 V, 50 Hz, 4 pole, wound rotor

$R_1 = 0.167\Omega$
$X_1 = 0.82\Omega$
$R_r = 0.148\Omega$
$X_r = 1.45\Omega$
$X_m = 44.6\Omega$

turns ratio = 1.67

المحتوى الكيميائي وعلاقته بالكساء الطحلبي النامي في خزانات المياه في جنوب غرب المملكة العربية السعودية

علي الحميدان
قسم النبات والميكروبيولوجي، كلية العلوم، جامعة الملك سعود
ص. ب ٢٤٥٥، الرياض ١١٤٥١
المملكة العربية السعودية

خلاصة

أجرى هذا البحث على عينات من المياه مأخوذة من خمسة خزانات في جنوب غرب المملكة العربية السعودية في شهر أغسطس ١٩٩١، وتم عمل التحليل الكيميائي لها، وتحديد محتواها من الطحالب، وقد أظهرت الدراسة أن تركيز النيتروجين غير العضوي كان عاليا في كل العينات، بينما كانت نسبة الكاتيونات الأحادية إلى الكاتيونات الثنائية منخفضة. وكانت مستويات المغذيات الدقيقة مرتفعة في بعض المواقع (العينات). وقد وجدت الطحالب الخضراء بوفرة في كل العينات، وتمثلت بثلاث رتب هي: *Orders: Chlorococcales, Cladophorales and, Zygnematales.* أما الطحالب الخضراء المزرقة فقد وجدت في أربعة من المواقع الخمسة، واتضح أن أنواعا بعينها تواجدت في موقع معين.