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# Design range of capacitance for self excitation of a stand-alone induction generator

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## ABSTRACT

With reference to a derived mathematical model, this paper discusses the main parameters that influence the process of self excitation of an isolated induction generator feeding a 3-phase balanced load. The parameters are: the type of load impedance, the value of the load impedance, the load power factor, the terminal capacitance and the rotor speed. Results are presented to demonstrate the inter-relationships of these parameters.

The mathematical model shows that the self-excitation process will be ensured if the load impedance and the corresponding capacitance have values above a critical minimum. The rotor speed and the load power factor are shown to have a significant influence on the critical values of the capacitance required for self excitation.

## LIST OF SYMBOLS

$\omega$ = Rotor speed (elec. rad/s)

$a$ = Generated frequency (pu)

$X_1, X_2$ = Stator and rotor leakage reactance per phase referred to stator at rated frequency (Ω)

$X_m$ = Magnetizing reactance per phase (Ω)

$R_1, R_2$ = Stator and rotor resistance per phase referred to stator (Ω)

$i_D, i_d$ = Instantaneous stator and rotor direct axis currents (A)

$I_Q, i_q$ = Instantaneous stator and rotor quadrature axis currents (A)

$C$ = Terminal self-excitation capacitance per phase (F)

$p$ = Laplace operator $\frac{d}{dt}$

$f_s$ = Generated frequency (Hz)

$f$ = Rated frequency (50 Hz)

## INTRODUCTION

Wagner (1939, 1941) presented an analysis of the process of self excitation in induction generators based on equating the active and reactive power of the whole circuit of the induction generator to zero. As a result, he concluded that the voltage which the machine can excite at a certain frequency depends upon its no-load excitation characteristics at the frequency. More recently, Doxey (1963) proposed saturable reactors to be connected to the induction generator in order to improve the voltage regulation upon loading. Brennen & Abbondanti (1977) discussed the possibility of using solid state-controlled static exciters for induction generators, whilst Tandon *et al.* (1984) proposed an analytical method to calculate the capacitor values required to maintain a constant terminal voltage at varying loads. Grantham *et al.* (1989) suggested a method for accurately predicting the minimum values of capacitance necessary to initiate self excitation of a stand-alone induction generator using the D-Q axis representation. Subsequently, the theory was extended to include the transient build-up voltage during the initiation stage of self excitation. Al-Jabri & Alolah (1990) developed a method to find the minimum capacitance required for self excitation of induction generator when an inductive load is excited.

This paper represents an extension to the above. It uses a mathematical model, based on the stationary reference frame, to examine the range of capacitance required for the initiation of the self-excitation process of a squirrel cage induction generator, and to integrate the effect of both the load impedance and the load power factor, on the range of self-excitation capacitance required. The influence of speed variation on the minimum and maximum values of the capacitance is also discussed.

## DATA OF THE INDUCTION MACHINE

The induction machine used for the experimental investigation is 380 V, 50 Hz, 1 kW, 4-pole, $3\phi$ squirrel cage. Different tests were carried out to estimate the parameters of the machine.

Without prior knowledge of the terminal frequency $f_s$, it is difficult to be precise about which values of rotor parameters should be used in the analysis. Grantham (1985) stated that values of $R_2$ and $X_2$ corresponding to a rotor frequency of 0 to 2 Hz should be used. However, accurate values of rotor leakage reactance in this region are extremely difficult to obtain because the $R_2/X_2$ ratio approaches infinity and the rotor resistance dominates the leakage reactance. However, by connecting the machine to a $3\phi$ variable frequency supply with the rotor locked, the variation of $R_2$ and $X_2$ with frequency can be obtained, and are presented in Fig. 1. The values of the rotor parameters at 5 Hz were used in the theoretical analysis. The measured parameters are

$$R_1 = 8{\cdot}5\ \Omega \qquad X_1 = 15{\cdot}715\ \Omega$$

$$R_2 = 3{\cdot}95\ \Omega \qquad X_2 = 18{\cdot}06\ \Omega$$

$$X_m = 133.7\ \Omega$$

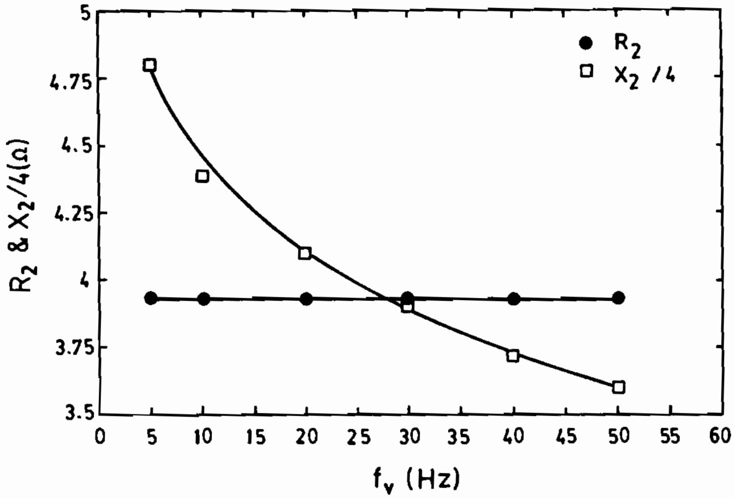


**Fig. 1.** Rotor parameters measured from variable frequency locked rotor test.

## THE SELF-EXCITATION PROCESS

Grantham *et al.* (1989) discussed the similarity between an RLC series circuit and the stator equivalent circuit of a self-excited induction generator. Particular interest has been directed to the form $K_i e^{m_i t}$ of the transient current flow in both the RLC circuit and the stator equivalent circuit. If the real part of $m_i$ is positive, the self-excitation process will start in the air-gap of the self-excited induction generator. The process of air-gap voltage build-up will continue until the iron circuit of the machine saturates. Hence, the air-gap voltage stabilizes and the current reaches its steady state, which means continuous self excitation. However, a continuously increasing transient current in the stator circuit is assured if the coefficient $K_i$ has a nonzero value. For an induction generator, this is assured by the presence of sufficient residual magnetism in the air-gap of the machine.

## THE MATHEMATICAL ANALYSIS

The initiation of the self-excitation process is a transient phenomenon and is better understood if the process is analyzed by using instantaneous values of current and voltage. Thus, the stationary reference frame will be used to represent the transient analysis of the self-excited induction generator. The stationary reference frame representation of a loaded self-excited symmetrically connected induction generator is shown in Fig. 2, where $Z$ could be one of the following cases:

(a) $Z = R + pL$ (inductive load)
(b) $Z = R$ (pure resistive load)
(c) $Z = \infty$ (no load)
(d) $Z = R + 1/pC_l$ (capacitive load)

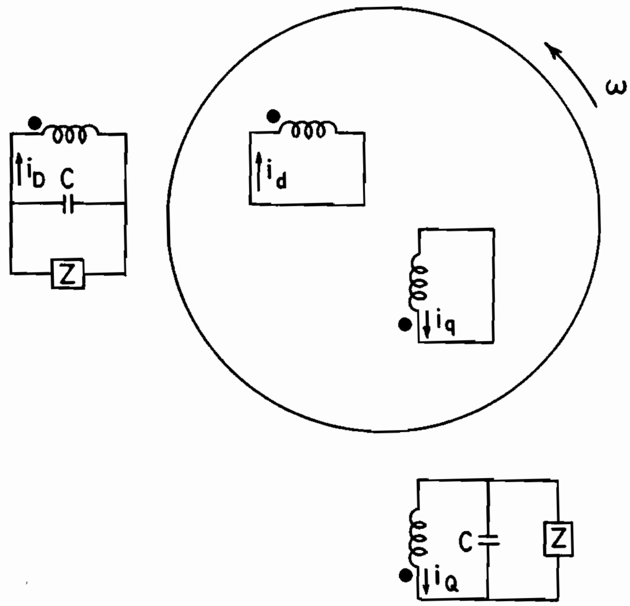


**Fig. 2.** *D-Q* axis representation of the loaded self-excited three-phase induction generator.

For a representative impedance $Z$, the voltage equations may be expressed as

$$\begin{bmatrix} v_D \\ v_Q \\ v_d \\ v_q \end{bmatrix} = \begin{bmatrix} R_1 + L_s p + \dfrac{Z}{1 + ZpC} & 0 & Mp & 0 \\ 0 & R_1 + L_s p + \dfrac{Z}{1 + ZpC} & 0 & Mp \\ Mp & -\omega_r M & R_2 + L_r p & -\omega_r L_r \\ \omega_r M & Mp & \omega_r L_r & R_2 + L_r p \end{bmatrix} \begin{bmatrix} i_D \\ i_Q \\ i_d \\ i_q \end{bmatrix} \quad (1)$$

where

$$L_s = \frac{X_1 + X_m}{2\pi f}, \qquad L_r = \frac{X_2 + X_m}{2\pi f} \qquad \text{and} \qquad M = \frac{X_m}{2\pi f}$$

Since no external voltage is applied and the rotor is short-circuited, the direct-axis stator current will be

$$i_D = \frac{0}{\left(R_1 + L_s p + \dfrac{Z}{1 + ZpC} - \dfrac{M^2 \delta p}{\Delta}\right)^2 + \left(\dfrac{M^2 \omega R_2 p}{\Delta}\right)^2} \tag{2}$$

where

$$\delta = L_r p^2 + R_2 p + \omega^2 L_r$$

and

$$\Delta = (R_2 + L_r p)^2 + (\omega L_r)^2$$

The characteristic equation which represents the self-excitation process of an induction generator and which satisfies all types of load can be obtained from Eqn (2) as

$$O = \{A_5 p^5 + A_4 p^4 + A_3 p^3 + A_2 p^2 + A_1 p + A_0\}^2 + \{B_3 p^3 + B_2 p^2 + B_1 p\}^2 \tag{3}$$

The coefficients $A$ and $B$ are listed in Appendix 1.

Thus the self-excitation process is presented by a 10th order differential equation for either the inductive or capacitive load case, while it is an 8th order differential equation in the case of a resistive load and no-load conditions. The self-excitation process will start when the polynomial presented in Eqn (3) has one root having a positive real part.

## THE COMPUTATION TECHNIQUE

The detailed description of the computing technique used to establish the range of capacitance necessary to ensure self excitation is discussed in this section. An operating speed is assumed initially and a search is carried out to test the values of the complex roots of the polynomial which represents the process of self excitation. Assuming a root is obtained in the form $\alpha_i + j\omega_i$, as the capacitance $C$ is increased, $\alpha_i$ will change its sign from negative to positive through a zero value. The corresponding value of capacitance $C$ at $\alpha_i =$ zero, is the minimum value $C_{min}$ required to start the process of self excitation, while $\omega_i$ is the corresponding angular velocity of the rotor. As the capacitance $C$ is increased further, the sign of $\alpha_i$ changes from positive to negative passing through another zero value. At this new zero value of $\alpha_i$, the corresponding capacitance is the maximum possible capacitance $C_{max}$ necessary to induce self excitation. It has been found in computer simulations that only the complex pair is affected by the change in $C$. This may be interpreted in view of the fact that the term $A_0^2$ (which represents the product of the roots) is actually independent of $C$.

Table 1 shows the typical range of capacitances necessary to achieve the initiation of self excitation for the 1 kW induction generator used for the investigation.

The comparison between the theoretical values and the corresponding experimental results (Fig. 3) supports the utility of both the computer technique and the

**Table 1.** Typical range of capacitance for the self-excitation process

| Machine rating kW | Voltage V | Speed RPM | Typical range of capacitance $\mu$F | | | |
|---|---|---|---|---|---|---|
| | | | Pure resistance load $R_L = 80\ \Omega$ | | Inductive impedance $Z_l = 100\ \Omega$ at 0·7 pf | |
| | | | $C_{min}$ | $C_{max}$ | $C_{min}$ | $C_{max}$ |
| 1 | 380 | 1250 | 85 | 152·5 | 78 | 213 |

mathematical model used. Also, the results reveal that the value of capacitance required to achieve self excitation is inversely proportional to the rotor speed, although the frequency is directly proportional to the rotor speed. However, both $C_{min}$ and $C_{max}$ will change differently when the load resistance is varied. As the load resistance is increased (Fig. 4) the value of $C_{min}$ will tend to decrease while the value of $C_{max}$ will increase.

## CRITICAL PARAMETERS FOR THE SELF-EXCITATION PROCESS

An optimization technique has been developed to search for the minimum value of capacitance and the corresponding minimum amplitude of load impedance which will ensure that the self-excitation process is induced. The search technique is started with a given rotor speed and for a known power factor. By assuming a value of load impedance $|Z|$ the corresponding values of $C_{min}$ and $C_{max}$ necessary for self excitation are computed. Then by reducing the impedance amplitude in steps of $\Delta|Z|$, the previous procedure is repeated until $|Z|_{critical}$ is reached. $|Z|_{critical}$ will be obtained when $C_{min}$ will be equal to $C_{max}$. This value of capacitance is called $C_{critical}$. The results presented in Figs 5 and 6 show that $|Z|_{critical}$ will depend on both the rotor speed and load power factor. As the rotor speed approaches its rated value, $|Z|_{critical}$

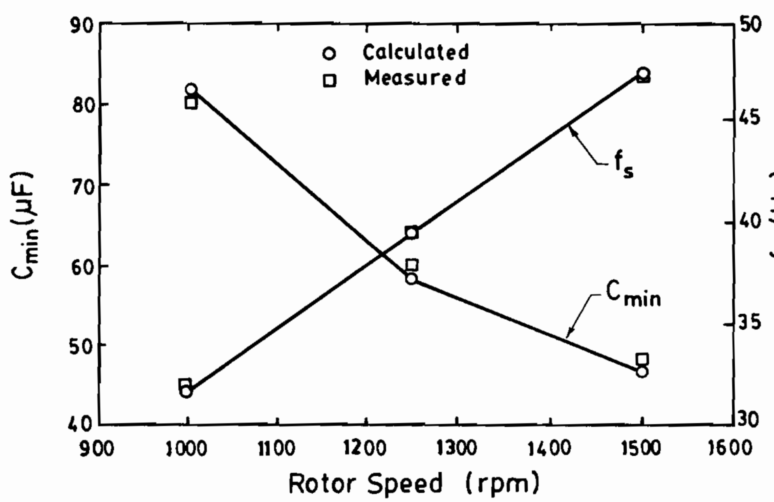


**Fig. 3.** Comparison between calculated and measured $C_{min}$ and its corresponding $f_s$ at $R = 100\ \Omega$.

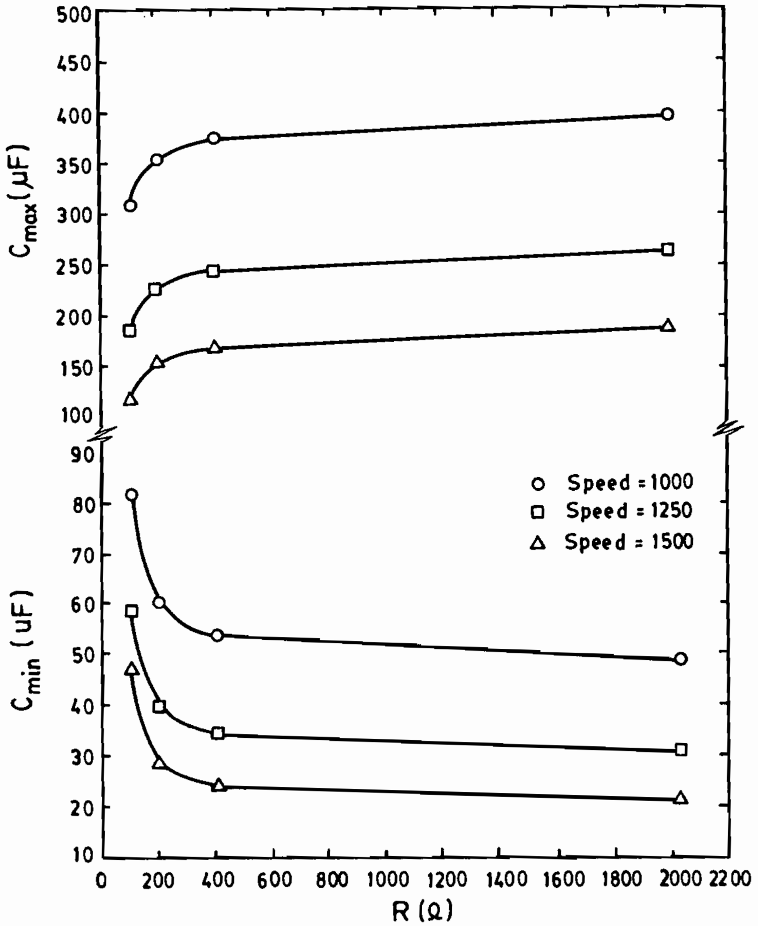


**Fig. 4.** Variation of $C_{min}$ and $C_{max}$ with $R$ and rotor speed.

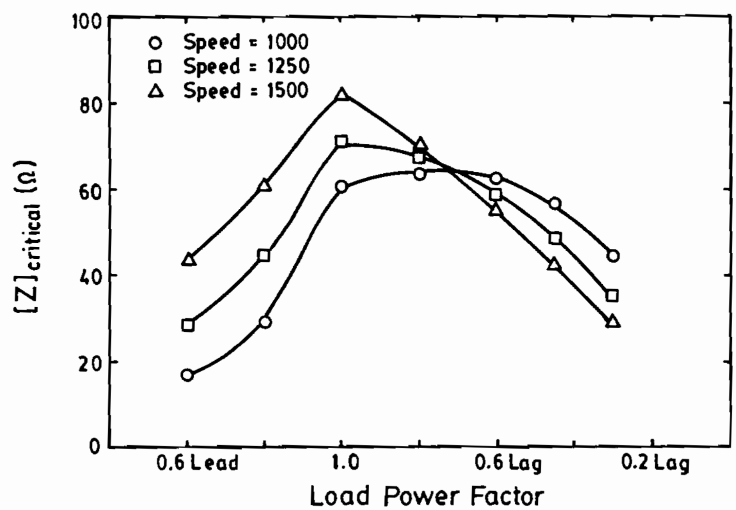


**Fig. 5.** Variation of $|Z|_{critical}$ with load power factor and rotor speed.

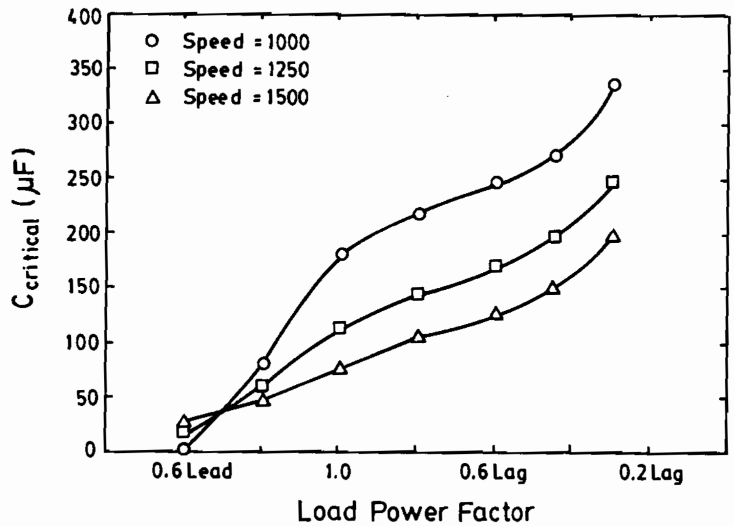


**Fig. 6.** Variation of $C_{critical}$ with load power factor and rotor speed.

tends to be symmetrical about the unity power factor (UPF) line. Although $C_{critical}$ is increased as the load power factor changes from leading to lagging values, its value is reduced as the rotor speed is increased.

## VARIATIONS OF LOAD POWER FACTOR

By assuming a load impedance $|Z| = 100\ \Omega$/phase, the minimum value of capacitance $C_{min}$ necessary for self excitation and the corresponding frequency $f_s$ are computed at different values of load power factor. The value of $Z$ was selected to ensure that the current in the stator windings did not exceed the rated value. Figs 7 and 8 show that although the self-excitation process will start at different values of $C_{min}$, the changes in the corresponding frequency $f_s$ are minor. However, minor changes are observed for $C_{max}$ when the power factor of a given load impedance is changed from 0·5 lead to 0·5 lag, as shown in Figs 9 and 10.

The results obtained show clearly that at a given drive speed and for different load impedances, the machine will act as a self-excited generator when $C_{min}$ or $C_{max}$ are connected at its terminals. However, by assuming the corresponding values of $C_{min}$ and $C_{max}$ are connected at UPF as reference, for any load power factor and as the load impedance decreases, the changes required in the value of $C_{min}$ are smaller than the corresponding changes in the value of $C_{max}$.

## CONCLUSIONS

The mathematical model and the computational technique used in the analysis are justified by the comparison between experimental and theoretical results. A range of capacitances with a minimum value $C_{min}$ and a maximum value $C_{max}$ is shown to be necessary to achieve self excitation for an induction generator. Both the experimental and theoretical results indicate that the value of capacitance required for the self-excitation process is inversely proportional to the rotor speed, although the corresponding frequency of the output signal is directly proportional to rotor speed.

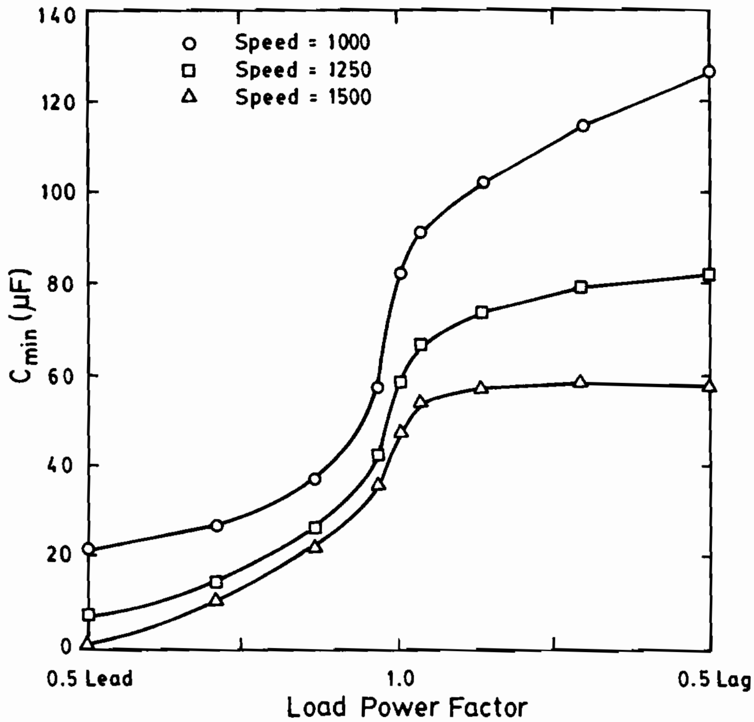


**Fig. 7.** Variation of $C_{min}$ with load power factor and rotor speed at $|Z| = 100\ \Omega$.

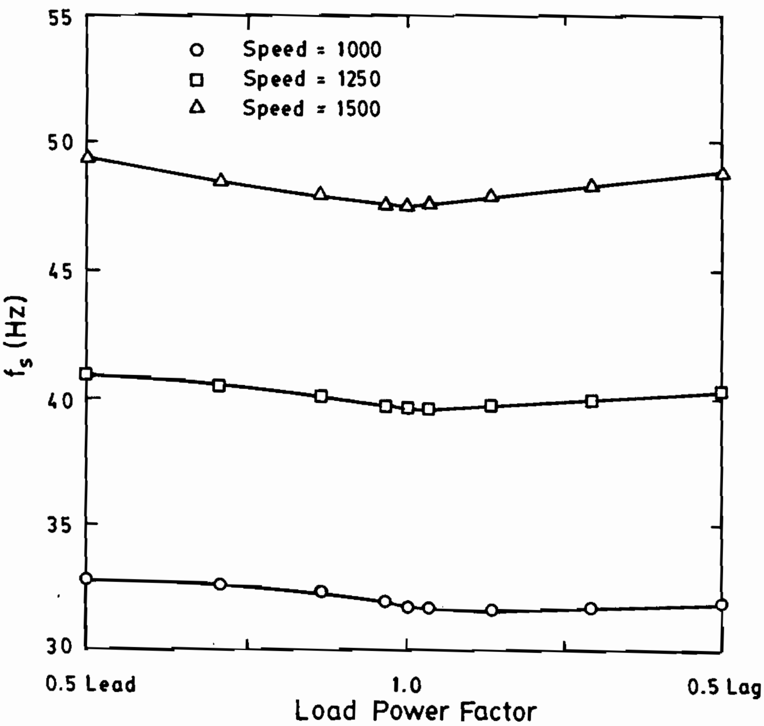


**Fig. 8.** Variation of $f_s$ corresponding to $C_{min}$ at $|Z| = 100\ \Omega$.

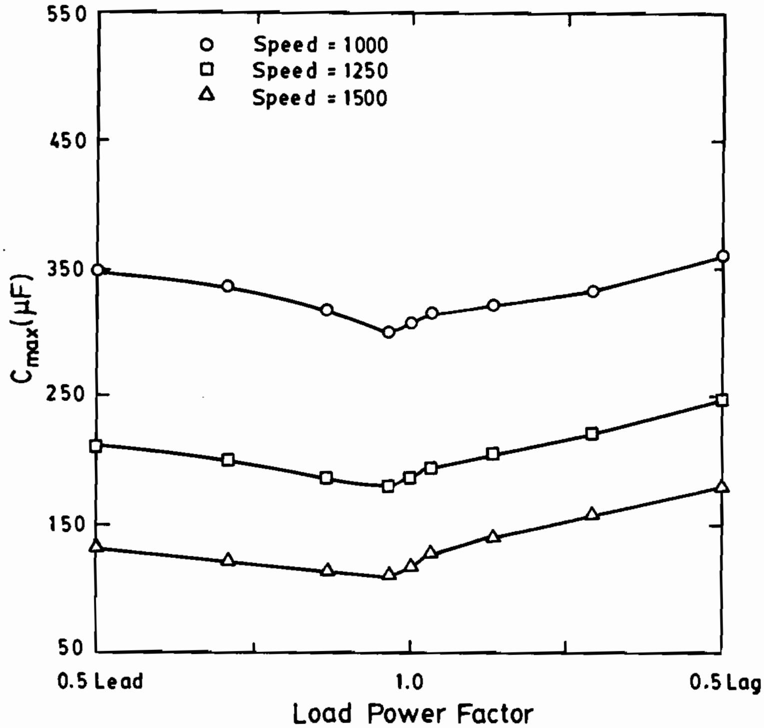


**Fig. 9.** Variation of $C_{max}$ with load power factor and rotor speed at $|Z| = 100\ \Omega$.

Variations of load resistance result in varying characteristics for both $C_{min}$ and $C_{max}$ and also for the corresponding output frequencies. Thus, the value of $C_{min}$ is reduced while its corresponding value of output frequency is increased as the load resistance is increased. The same characteristics are obtained when both the critical resistance and the critical capacitance are computed at various rotor speeds. However, the value of $C_{max}$ tends to increase while its corresponding output frequency is reduced as the load resistance is increased. By assuming the value of $C_{min}$ at unity power factor as a reference, the change in $C_{min}$ reveals a set of *V*-curves when the load power factor is varied. The highest change in $C_{min}$ is obtained at the lowest load impedance. However, the change in $C_{max}$ gives a considerably higher value compared to that of $C_{min}$. For a load impedance of fixed amplitude and variable lagging power factor, the values of $C_{min}$ required to achieve self excitation will exhibit minor changes while the corresponding values of $C_{max}$ are continuously varied.

In summary, for practical applications and also to obtain a higher frequency for the output, the results obtained reveal the following:

(1) The value of capacitance used for the self-excitation process must be close to $C_{min}$.
(2) The machine must be driven at a speed close to its rated synchronous speed.
(3) The load impedance connected to the terminals of the machine must be of a value higher than that corresponding to $|Z|_{critical}$.

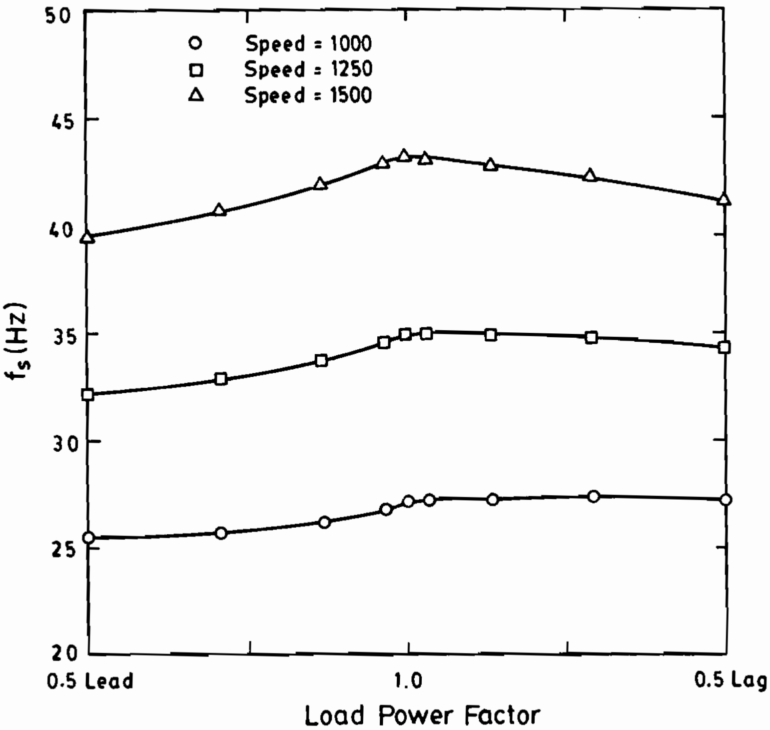


**Fig. 10.** Variation of $f_s$ corresponding to $C_{max}$ at $|Z| = 100\,\Omega$.

(4) Loads of lagging power factor are recommended since an almost constant value of capacitance will be required to ensure self excitation of the generator.

(5) The role of the magnetizing reactance on the performance of a stand-alone induction generator is thoroughly investigated in a forthcoming paper by the author.

## ACKNOWLEDGEMENT

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## APPENDIX 1

### THE A AND B COEFFICIENTS

For inductive load ($Z = R + pL$)

$$B_1 = M^2\omega R_2 \quad (1)$$

$$B_2 = M^2\omega R_2 RC \quad (2)$$

$$B_3 = M^2\omega R_2 LC \quad (3)$$

$$A_0 = (R + R_1)(R_2^2 + \omega^2 L_r^2) \quad (4)$$

$$A_1 = 2R_2 L_r(R + R_1) + (R_2^2 + \omega^2 L_r^2)(RCR_1 + L_s + L) - M^2\omega^2 L_r \quad (5)$$

$$A_2 = L_r^2(R + R_1) + 2R_2 L_r(RCR_1 + L_s + L) + (R_2^2 + \omega^2 L_r^2)(RCL_s + LCR_1) - \omega^2 L_r RCM^2 - R_2 M^2 \quad (6)$$

$$A_3 = L_r^2(RCR_1 + L_s + L) + 2R_2 L_r(RCL_s + LCR_1) - R_2 RCM^2 - L_r M^2 + LCL_s(R_2^2 + \omega^2 L_r^2) - LCM^2\omega^2 L_r \quad (7)$$

$$A_4 = L_r^2(RCL_s + R_1 LC) + 2R_2 L_r LCL_s - LCM^2 R_2 - L_r RCM^2 \quad (8)$$

$$A_5 = L_r^2 LCL_s - L_r LCM^2 \quad (9)$$

$A$ and $B$ coefficients for resistive load can be obtained from Eqns (1) to (9) by substituting $L = 0$. For the no-load case, the new set of coefficients can be obtained by multiplying the respective $A$ and $B$ coefficients for the resistive case and taking the limit as $R$ tends to infinity.

For capacitive load $\left(Z = R + \dfrac{1}{pC_l}\right)$

$$B_1 = 0 \quad (1)$$

$$B_2 = M^2\omega R_2(C + C_l) \quad (2)$$

$$B_3 = M^2\omega R_2 CRC_l \quad (3)$$

$$A_0 = R_2^2 + \omega^2 L_r^2 \quad (4)$$

$$A_1 = R_1(C + C_l)(R_2^2 + \omega^2 L_r^2) + 2R_2 L_r + RC_1 R_2^2 + \omega^2 L_r^2) \tag{5}$$

$$A_2 = (C + C_1)(2R_2 L_r R_1 + L_s[R_2^2 + \omega^2 L_r^2] - M^2\omega^2 L_r) + CRC_l R_1(R_2^2 + \omega^2 L_r^2) + (L_r^2 + 2R_2 L_r RC_l) \tag{6}$$

$$A_3 = RC_1 L_r^2 + CRC_l(2R_2 L_r R_1 + L_s[R_2^2 + \omega^2 L_r^2] - M^2\omega^2 L_r) + (C + C_l)(R_1 L_r^2 + 2R_2 L_r L_s - M^2 R_2) \tag{7}$$

$$A_4 = CRC_l(R_1 L_r^2 + 2R_2 L_r L_s - M^2 R_2) + (C + C_l)(L_s L_r^2 - M^2 L_r) \tag{8}$$

$$A_5 = CRC_l(L_s L_r^2 - M^2 L_r) \tag{9}$$

# حدود السعة اللازمة لعملية الإثارة الذاتية لمولد تأثيري مستقل التحميل

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**خلاصة**

يعتمد هذا البحث على نموذج رياضي تم اشتقاقه بهدف دراسة تأثير العناصر الأساسية على بدء عملية الإثارة الذاتية لمولد تأثيري مستقل التحميل ويغذي حملا ثلاثي الطور ومتزنا.

ويحدد النموذج الرياضي هذه العناصر كالتالي: نوعية الحمل الكهربائي، قيمة معاوقة الحمل الكهربائي، قيمة معامل قدرة الحمل، قيمة السعة المتصلة بنهايات المولد، وكذلك سرعة دوران المولد. ولقد أوضحت النتائج التي تم الحصول عليها عند معالجة هذا النموذج الرياضي بواسطة الحاسب الآلي، العلاقات المشتركة والمتداخلة بين هذه العناصر.

ويبين النموذج الرياضي أن عملية الإثارة الذاتية الضرورية لبدء عمل المولد التأثيري يمكن التأكد من تحقيقها بشرط أن تكون قيمة كل من معاوقة الحمل الكهربائي وسعة الإثـارة عند نهايات المولد أعلى من القيمة الحرجة الصغرى لكل منها. كما تبين النتائج أيضا تأثير كل من سرعة دوران المولد ومعامل قدرة الحمل على بقية العناصر الأخرى التي تؤكد بدء عملية الإثارة الذاتية للمولد.