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# Analytical solution to cylindrical panel with higher order theory

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## ABSTRACT

A previously unavailable analytical solution to a cylindrical panel of rectangular planform fabricated with an isotropic material and subjected to static loadings is presented. A variationally consistent higher order shell theory that generates five highly coupled fourth order partial differential equations in five unknowns is utilized. A mixed set of double Fourier series-based solution functions (boundary continuous and boundary discontinuous) is assumed to solve such equations in conjunction with the admissible boundary conditions (fully-restrained simply supported). The numerical results thus presented constitute the study of convergence of displacements, and moments; and spatial variations of them are presented in the form of contour plotting for various parametric effects. These, previously unavailable, analytically obtained numerical results should serve as base-line solutions for future comparisons of popular approximate methods such as finite element, finite difference, Galerkin approach, Rayleigh-Ritz method, collocation method, least-squares method, experimental results, etc.

## INTRODUCTION

For decades researchers have sought analytical solutions to boundary value problems of plate/cylinder/shell structures so as to understand their complex behavior. This complexity increases with the innovation of new shell theories. The shell theory that takes care of parabolic variation of transverse shear stress across the thickness of the shell, known as higher order shear deformation theory (HOSDT) is a recent production (Reddy & Liu 1985), which is variationally consistent. Interested readers may find details of this theory in Reddy & Liu (1985). Nelson & Lorch (1974), Levinson (1980), and Murthy (1981) have tried HOSDT without vanishing transverse shear stresses at the top and bottom faces of the shell panels. HOSDT does not need any shear correction factors as does its previous shell theory (Reissner 1944, Mindlin 1951), shear deformation theory (RMSDT). In contrast to the aforementioned theories, the classical shell theory (CST) developed by Love & Kirchhoff

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(Donnell 1933, Flugge 1960, and Seide 1975) completely negates the effects of transverse shear deformations.

As the through-thickness theory is ameliorated for the shell structures, its corresponding analytical solutions become more arduous, if not impossible. An extensive literature search has revealed that analytical solutions based on HOSDT (strong form or differential form) for cylindrical panels of rectangular planform are available for selected boundary conditions, e.g. SS3-type (Hoff & Rehfield 1965, Reddy & Liu 1985), and mixed type boundary conditions where two opposite edges are of type SS3, remaining two edges are combinations of simply supported or clamped conditions (Librescu *et al.* 1989).

The definition of SS3-type simply supported boundary conditions at all edges is given in Reddy (1984) and Hoff & Rehfield (1965), and not elaborated here for the sake of brevity. The attempts in obtaining analytical solutions to the other boundary conditions are not seen in the current literature as researchers were mainly depending on two popular solution methods: (*i*) Navier's approach (Reddy 1984), and (*ii*) Levy-type approach (Librescu *et al.* 1989) for decades. These methods have been applied for all three shell theories as discussed earlier by various researchers. In the situation where these solution methods were inconsistent, researchers have sought the solutions in the realm of approximate approaches. Recently, Kabir (1990, 1994) and Kabir & Chaudhuri (1992), have reported an analytical solution approach based on boundary continuous solution functions for all-edge clamped boundary conditions. The through-thickness theory that they have used is based on RMSDT. The utilization of their approach with other shell theories has not yet been reported in the current literature. Chaudhuri (1989) has given an unique prescription to the solution of a set of boundary conditions for doubly curved shells with RMSDT, without presenting any numerical work. Later Chaudhuri (1989) and Chaudhuri & Kabir (1993) have presented numerical results for some boundary conditions. The numerical results for other boundary conditions are yet to be reported in literature. Further, this solution approach, unlike Navier's and Levy-type methods, is not yet utilized or tested in the domain of HOSDT for cylindrical panels. The RMSDT produces five second-order highly coupled partial differential equations while HOSDT yields the same number of equations but of the order four, as a result solutions to boundary conditions other than Navier, Levy, Chaudhuri (1989), and Kabir (1990) types are not seen in the current literature for the case of, e.g., cylindrical panels with HODST (Reddy & Liu 1985).

Therefore, the prime objective here is to develop an analytical solution to a cylindrical panel with the boundary conditions not yet solved. As an example, SS4-type simply supported boundary conditions (Hoff & Rehfield 1965) at all edges are considered. The second objective is to study convergence and the spatial variations of displacement, normal force, and moment for various parametric effects. The third objective is to compare the present results with the available RMSDT-based finite-element solutions (NISA-II 1992).

## THEORETICAL FORMULATION

A cylindrical panel of span $a \times b$, and radius $r_1$ is shown in Fig. 1, with its midplane being taken as the reference surface, defined by a set of curvilinear orthogonal coordinates $x_i$, such that $x_3$ is normal to the reference surface. The spans $a$ and $b$

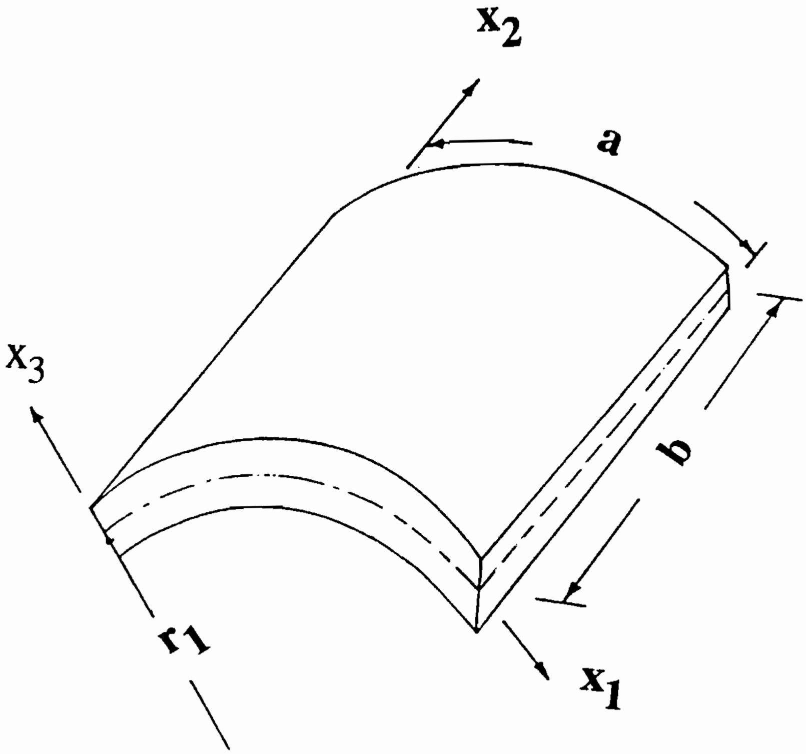


**Fig. 1.** A cylindrical panel.

are along $x_1$ and $x_2$ axes, respectively. The total thickness of the panel is defined as $h$. The strain-displacement relations based on variationally consistent (a detail of which is available in Reddy & Liu 1985) HOSDT are

$$\varepsilon_1 = \varepsilon_1^0 + x_3(\kappa_1^0 + x_3^2 \kappa_1^2) \tag{1}$$

$$\varepsilon_2 = \varepsilon_2^0 + x_3(\kappa_2^0 + x_3^2 \kappa_2^2) \tag{2}$$

$$\varepsilon_4 = \varepsilon_4^0 + x_3^2 \kappa_4^1 \tag{3}$$

$$\varepsilon_5 = \varepsilon_5^0 + x_3^2 \kappa_5^1 \tag{4}$$

$$\varepsilon_6 = \varepsilon_0^0 + x_3(\kappa_6^0 + x_3^2 \kappa_6^2) \tag{5}$$

where

$$\varepsilon_1^0 = u_{1,1} + \frac{u_3}{r_1} \tag{6}$$

$$\kappa_1^0 = \varphi_{1,1}$$

$$\kappa_1^2 = -\frac{4}{3h^2}(\varphi_{1,1} + u_{3,11}) \tag{7}$$

$$\varepsilon_2^0 = u_{2,2} \tag{8}$$

$$\kappa_2^0 = \phi_{2,2} \tag{9}$$

$$\kappa_2^2 = -\frac{4}{3h^2}(\varphi_{2,2} + u_{3,22}) \tag{10}$$

$$\varepsilon_4^0 = \phi_2 + u_{3,2} \tag{11}$$

$$\kappa_4^1 = -\frac{4}{h^2}(\varphi_2 + u_{3,2}) \tag{12}$$

$$\varepsilon_5^0 = \phi_1 + u_{3,1} \tag{13}$$

$$\kappa_5^1 = -\frac{4}{h^2}(\varphi_1 + u_{3,1}) \tag{14}$$

$$\varepsilon_6^0 = u_{2,1} + u_{1,2} \tag{15}$$

$$\kappa_6^0 = \phi_{2,1} + \phi_{1,2} \tag{16}$$

$$\kappa_6^2 = -\frac{4}{3h^2}(\phi_{2,1} + \phi_{1,2} + 2u_{3,12}) \tag{17}$$

where $\varepsilon_i$ ($i = 1, 2$) represent inplane strains along $x_i$ ($i = 1, 2$), respectively, at any point in the shell thickness. $\varepsilon_4$ and $\varepsilon_5$ are transverse shear strains along $x_2 - x_3$ and $x_1 - x_3$ planes, respectively. $\varepsilon_6$ represents inplane shear strain at any point in the shell thickness. $\varepsilon_i^0$ ($i = 1, 2, 4, 5, 6$) denote the corresponding strains at the reference surface of the shell panel. $u_i$ ($i = 1, 2, 3$) indicate middle surface displacements of the panel along $x_1$, $x_2$, and $x_3$ axes, respectively. $\psi_1$ and $\psi_2$ represent rotations of normal about $x_1$ and $x_2$ axes, respectively.

The governing partial differential equations, in a conventional form are

$$N_{1,1} + N_{6,2} = 0 \tag{18}$$

$$N_{6,1} + N_{2,2} = 0 \tag{19}$$

$$Q_{1,1} + Q_{2,2} - \frac{4}{h^2}(K_{1,1} + K_{2,2}) + \frac{4}{3h^2}(P_{1,11} + P_{2,22} + 2P_{6,12}) - \frac{N_1}{R} = q \tag{20}$$

$$M_{1,1} + M_{6,2} - Q_1 + \frac{4}{h^2}K_1 - \frac{4}{h^2}(P_{1,1} + P_{6,2}) = 0 \tag{21}$$

$$M_{6,1} + M_{2,2} - Q_1 + \frac{4}{h^2}K_2 - \frac{4}{h^2}(P_{6,1} + P_{2,2}) = 0 \tag{22}$$

where $q$ represents a uniformly distributed load. The stress resultants are defined as

$$(N_i, M_i, P_i) = \int_{-h/2}^{h/2} \sigma_i(1, x_3, x_3^2)\, dx_3; \quad i = 1, 2, 6 \tag{23}$$

$$(Q_1, K_1) = \int_{-h/2}^{h/2} \sigma_5(1, x_3^2)\, dx_3 \tag{24}$$

$$(Q_2, K_2) = \int_{-h/2}^{h/2} \sigma_4(1, x_3^2)\, dx_3 \tag{25}$$

where $N_i$ ($i = 1, 2$) are resultants of normal stresses along $x_1$ and $x_2$ axes, respectively. $N_6$ is the resultant inplane shear stress. $M_1$ and $M_2$ are moments about $x_2$ and $x_1$ axes, respectively. $M_6$ represents a twisting moment. $P_i$ ($i = 1, 2, 6$) denotes moments of corresponding moments. $Q_1$ and $Q_2$ represent resultants of transverse shear stresses in $x_1$–$x_3$, and $x_2$–$x_3$ planes, respectively. $\sigma_i$ ($i = 1, 2$) represent normal stress along $x_1$ and $x_2$ axes, respectively. $\sigma_6$ denotes inplane shear stress. $\sigma_4$ and $\sigma_5$ represent transverse shear stresses in $x_2$–$x_3$ and $x_1$–$x_3$ planes, respectively. The physical interpretations of $K_1$ and $K_2$ are difficult, but may be viewed as inertia of transverse shear stresses.

The governing differential equations (18–22) may be expressed in terms of displacements and their derivatives in the following form as:

$$\mathbf{Lu} = \mathbf{f} \tag{26}$$

where **L** is an operator containing derivatives and constant terms; **u** is an unknown displacement vector; and **f** represents the load vector.

The above equations are to be solved in conjunction with the following SS4 boundary conditions:

$$u_1\Big|_n = u_1\Big|_t = 0 \tag{27}$$

$$u_2\Big|_n = u_2\Big|_t = 0 \tag{28}$$

$$u_3 = 0 \text{ at all edges} \tag{29}$$

$$\varphi_1(x_1, 0) = \varphi_1(x_1, b) = 0 \tag{30}$$

$$\varphi_2(0, x_2) = \varphi_2(a, x_2) = 0 \tag{31}$$

$$M_1(0, x_2) = M_1(a, x_2) = 0 \tag{32}$$

$$M_2(x_1, 0) = M_2(x_1, b) = 0 \tag{33}$$

$$P_1(0, x_2) = P_1(a, x_2) = 0 \tag{34}$$

$$P_2(x_1, 0) = P_2(x_1, b) = 0 \tag{35}$$

where $|_n$ and $|_t$ denote normal and tangential to the boundaries, respectively.

For a clarification of various types of boundary conditions, SS3-type is stated below:

$$u_1|_t = 0 \tag{36}$$

$$u_2|_t = 0 \tag{37}$$

$$u_3 = 0 \text{ at all edges} \tag{38}$$

$$u_1|_n = 0 \tag{39}$$

$$u_2|_n = 0 \tag{40}$$

The remaining boundary conditions are similar to those stated for the case of SS4-type.

## Solution Methodology

The solution functions chosen are of the following mixed form:

$$u_1 = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} A_{mn} \cos(\alpha_m x_1) \sin(\beta_n x_2) \quad 0 < x_1 < a; 0 \leqslant x_2 \leqslant b \tag{41}$$

The above assumed solution function is of the mixed type, in the sense that part of it satisfies the geometric boundary conditions (along $x_2 = 0$, b) while the other part does not (along $x_1 = 0, a$). Similar characteristics are also assumed for the function

$$u_2 = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} B_{mn} \sin(\alpha_m x_1) \cos(\beta_n x_2) \quad 0 \leqslant x_1 \leqslant a; 0 < x_2 < b \tag{42}$$

The above type of mixed functions (to the best of the knowledge of the authors) are, for the first time, applied using higher-order theory for an isotropic cylindrical panel. The remaining solution functions are of the Navier type for the case of geometric boundary conditions and are as shown below:

$$u_3 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} \sin(\alpha_m x_1) \sin(\beta_n x_2) \quad 0 \leqslant x_1 \leqslant a; 0 \leqslant x_2 \leqslant b \tag{43}$$

$$\varphi_1 = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} D_{mn} \cos(\alpha_m x_1) \sin(\beta_n x_2) \quad 0 \leqslant x_1 \leqslant a; 0 \leqslant x_2 \leqslant b \tag{44}$$

$$\varphi_2 = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} E_{mn} \sin(\alpha_m x_1) \cos(\beta_n x_2) \quad 0 \leqslant x_1 \leqslant a; 0 \leqslant x_2 \leqslant b \tag{45}$$

As $u_1$ does not satisfy the boundary conditions at edges $x_1 = 0$ and $x_1 = a$, its differentiation with respect to $x_1$ is not possible. In this situation $u_1$ can be forced to vanish at the respective edges. This step will generate the following algebraic equations:

$$\sum_{m=1}^{\infty} \omega'_m A_{mn} = 0 \tag{46}$$

$$A_{on} + \sum_{m=1}^{\infty} \omega''_m A_{mn} = 0 \tag{47}$$

for all values of $n = 1, 2, 3, \ldots$ where $(\omega'_m, \omega''_m) = 0$, if $m$ is odd; and $(\omega'_m, \omega''_m) = 1$, if $m$ is even.

A similar situation arises for the case of $u_2$ at edges $x_2 = 0$ and $b$. Using the above step in the relevant edges of $u_2$, one obtains

$$\sum_{n=1}^{\infty} \omega'_n B_{mn} = 0 \tag{48}$$

$$B_{mo} + \sum_{n=1}^{\infty} \omega''_n B_{mn} = 0 \tag{49}$$

for all values of $m = 1, 2, 3, \ldots$ .

Therefore, $u_{1,1}$ or $u_{2,2}$ can easily be obtained as

$$u_{1,1} = -\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\alpha_m A_{mn}\sin(\alpha_m x_1)\sin(\beta_n x_2) \qquad 0 < x_1 < a \quad 0 \leqslant x_2 \leqslant b \quad (50)$$

$$u_{2,2} = -\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\beta_m B_{mn}\sin(\alpha_m x_1)\sin(\beta_n x_2) \qquad 0 < x_1 < a \quad 0 < x_2 < b \quad (51)$$

An ordinary discontinuity exists in $u_{1,1}$ at $x_1 = 0$ and $a$, and in $u_{2,2}$ at $x_2 = 0$ and $b$. Further differentiation of them are not possible in the ordinary sense. They are then expanded in the form suggested by Hobson (1926), Green (1944), Whitney (1970), Chaudhuri (1989), and Kabir (1994). For purposes of illustration, only $u_{1,1}$ is considered:

$$u_{1,11} = \frac{1}{2}\sum_{n=1}^{\infty} g'_n \sin(\beta_n x_2) + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(-\alpha_m^2 A_{mn} + g'_n\omega'_m + g''_n\omega''_m) \times \cos(\alpha_m x_1)\sin(\beta_n x_2) \quad (52)$$

where

$$g'_n = \frac{4}{ab}\int_0^a [u_{1,1}(a, x_2) - u_{1,1}(o, x_2)]\sin(\beta_n x_2)\, dx_2 \quad (53)$$

$$g''_n = -\frac{4}{ab}\int_0^a [u_{1,1}(a, x_2) - u_{1,1}(o, x_2)]\sin(\beta_n x_2)\, dx_2 \quad (54)$$

The above operation is applied to the situations of ordinary discontinuities as they appear in other derivatives. Finally, equations (18–22) and equations (32–35) will supply as many equations as the number of unknowns.

A computer code AFSANA-CP (**A** **F**ourier **S**eries **ANA**lysis-**C**ylindrical **P**anel) is developed to solve the above equations in a SUN workstation using FORTRAN-77.

## NUMERICAL RESULTS AND DISCUSSIONS

The numerical results are presented for a cylindrical panel with various aspect ratios. For the sake of convenience of presentation the following dimensionless quantities are defined:

$$u_3^* = \frac{10^3 E_2 h^3 u_3}{qa^4}; \quad M_1^* = \frac{10^3 M_1}{qa^2}; \quad M_2^* = \frac{10^3 M_2}{qa^2}$$

The convergence of centrally measured $u_3^*$, $M_1^*$, and $M_2^*$ for a cylindrical panel having $r/a = 10$; $a/b = 1$ for $a/h = (5, 10, 20, \text{and } 30)$ are plotted in Figs 2–5, respectively. For the case of $a/h = 5$, a smooth convergence, for all values of $m = n$, is achieved without any noticeable monotonic oscillations, a natural characteristic of Fourier expansion. This is perhaps because one is considering thick shell theory in the formulation. However, the same trend is not found with $a/h \geqslant 5$ for the case of $M_1^*$ and $M_2^*$. The oscillations found are larger in the moderately thick case ($a/h = 20$) than in the thin range ($a/h = 30$). All of them show a good convergence attitude.

Variations of centrally calculated $u_3^*$, $M_1^*$ and $M_2^*$ with respect to span-to-thickness ratios for $r/a = 10$; $b/a = 1$, and $m = n = 9$ are presented in Fig. 6. They show

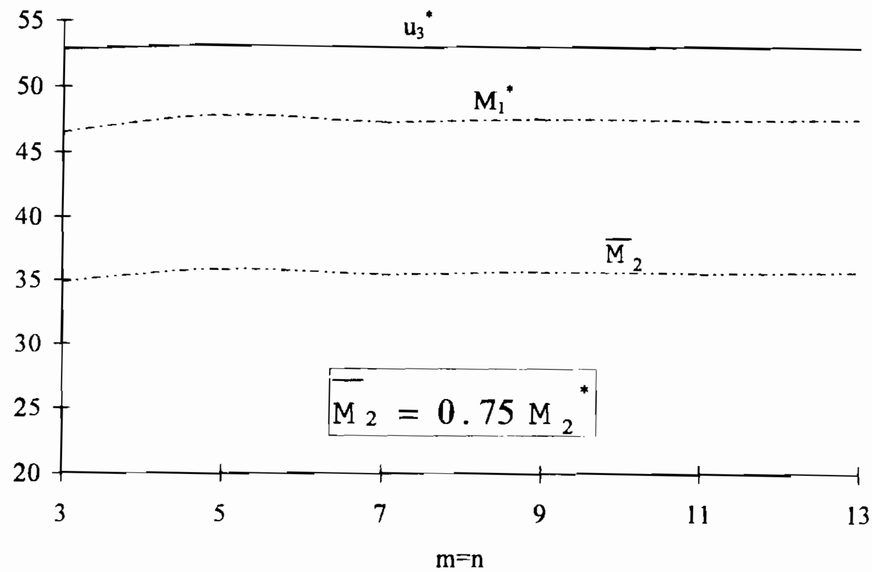


**Fig. 2.** Convergence of central values of $u_3^*$, $M_1^*$ and $M_2^*$ for a cylindrical panel with $a/h = 5$, $r/a = 10$, $b/a = 1$.

a convincing trend as $a/h$ increases. Contour plottings of $u_3^*$, $M_1^*$ and $M_2^*$ for $a/h = 5$ and 10 are shown in Figs. 7–9, and 10–12, respectively, for $r/a = 10$, $b/a = 1$ with $m = n = 9$. All of them show a consistent behavior in the domain.

The present analytical solutions are compared with the results obtained from a finite-element method (FEM). A four-node RMSDT-based element that uses

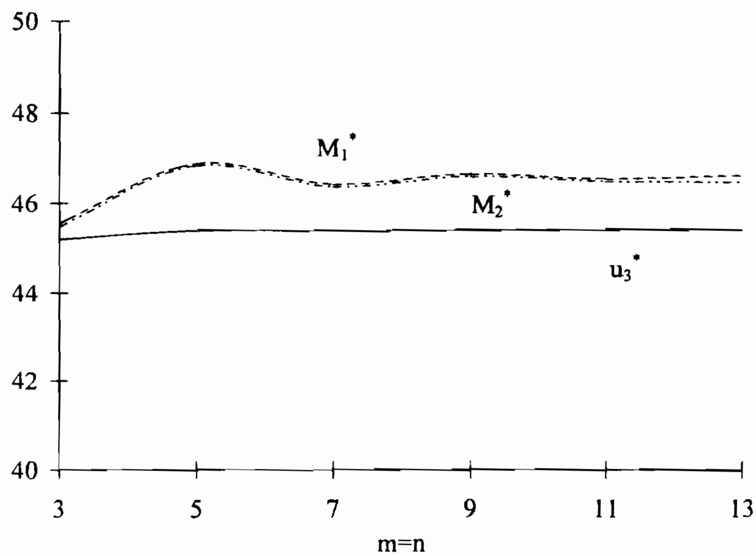


**Fig. 3.** Convergence of central values of $u_3^*$, $M_1^*$ and $M_2^*$ for a cylindrical panel with $a/h = 10$, $r/a = 10$, $b/a = 1$.

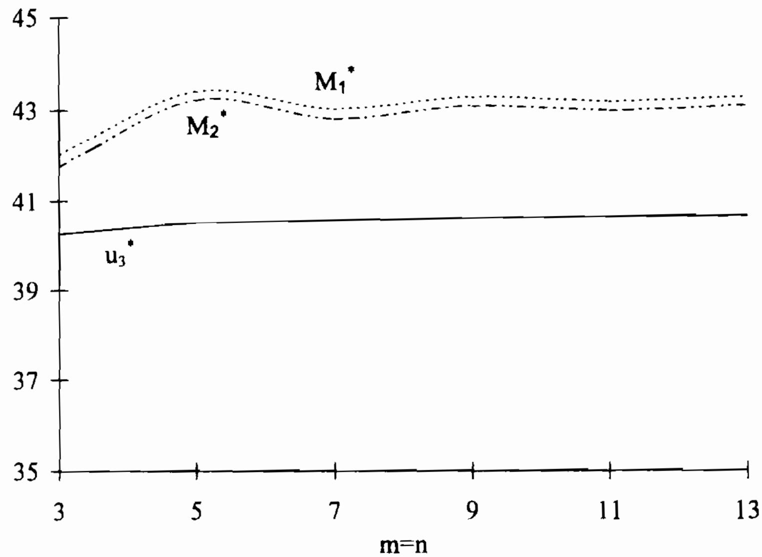


**Fig. 4.** Convergence of central values of $u_3^*$, $M_1^*$ and $M_2^*$ for a cylindrical panel with $a/h = 20$, $r/a = 10$, $b/a = 1$.

reduced integration scheme is considered. The element is implemented in the NISA (1992) family of finite elements. This element, however, did not consider the shell-curvature effects. Interested readers may find more about this element in Bathe & Dvorkin (1985), and the NISA User's Manual (1992); and this information is not provided here for the sake of brevity. Figure 13 presents normalized $u_3^*$, $M_1^*$ and $M_2^*$

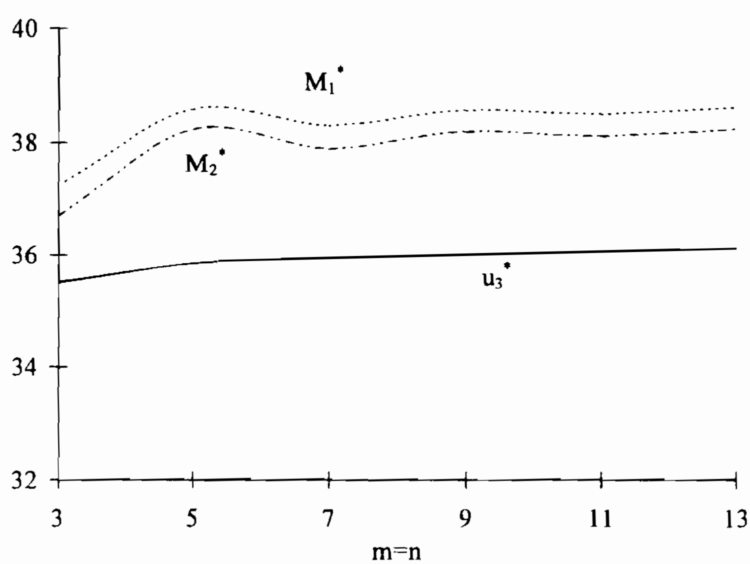


**Fig. 5.** Convergence of central values of $u_3^*$, $M_1^*$ and $M_2^*$ for a cylindrical panel with $a/h = 30$, $r/a = 10$, $b/a = 1$.

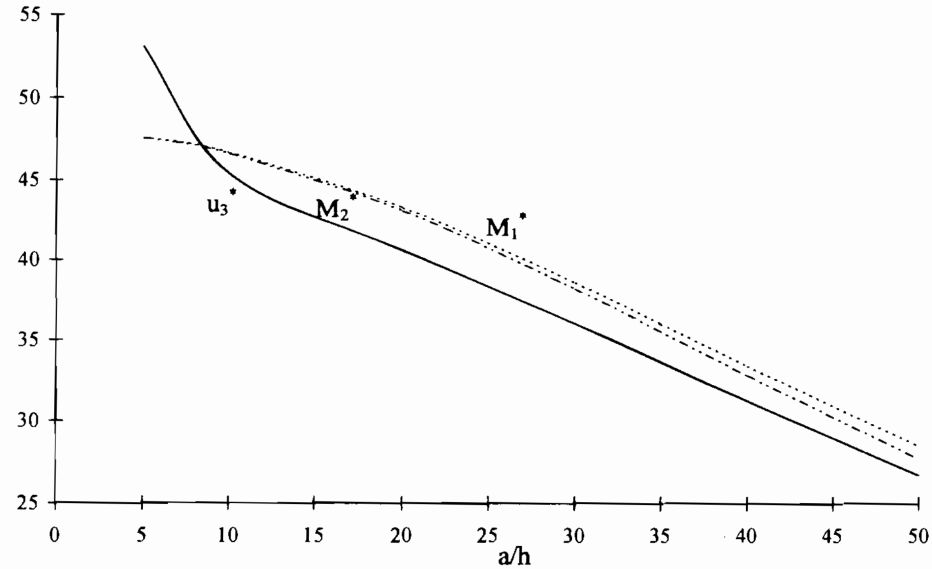


**Fig. 6.** Variations of central values of $u_3^*$, $M_1^*$ and $M_2^*$ with respect to various $a/h$ for a cylindrical panel with $r/a = 10$, $b/a = 1$, and $m = n = 9$.

at $x_2 = b/2$ along $x_1$ for a cylindrical panel with $b/a = 1$, $a/h = 10$, and $r/a = 10$. The analytical results are obtained for $m = n = 9$, while converged finite element results are for $20 \times 20$ elements. The normalized central values of moments $M_1^*$ and $M_2^*$ obtained using the FEM are 12% and 18%, respectively, more than the present

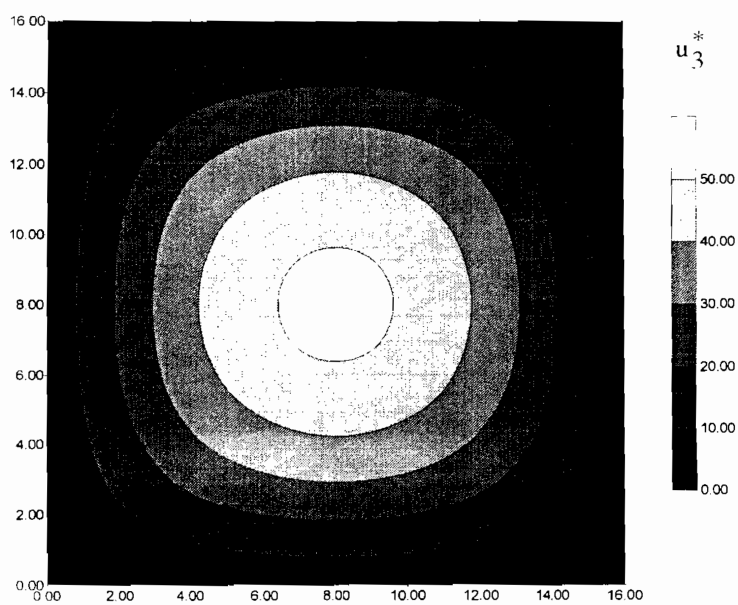


**Fig. 7.** Contour plotting for spatial variations of $u_3^*$, for a cylindrical panel with $r/a = 10$, $a/h = 5$, $b/a = 1$, and $m = n = 9$.

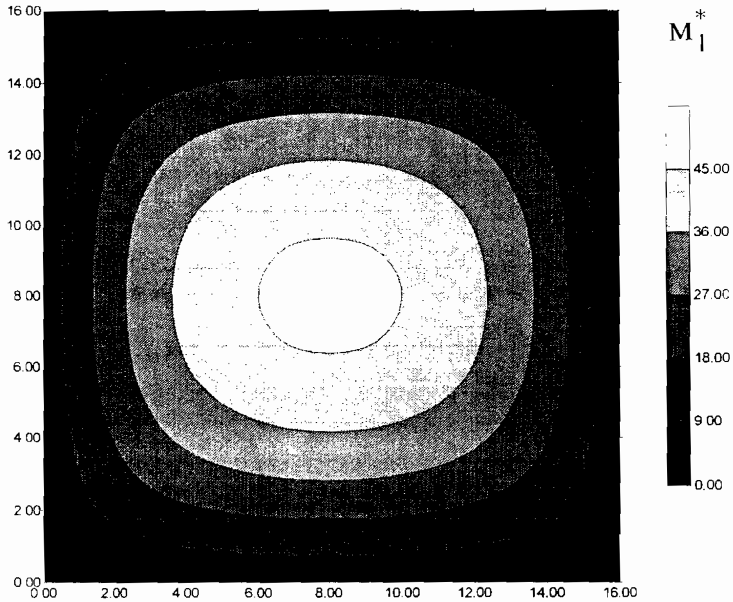


**Fig. 8.** Contour plotting for spatial variations of $M_1^*$ for a cylindrical panel with $r/a = 10$, $a/h = 5$, $b/a = 1$, and $m = n = 9$.

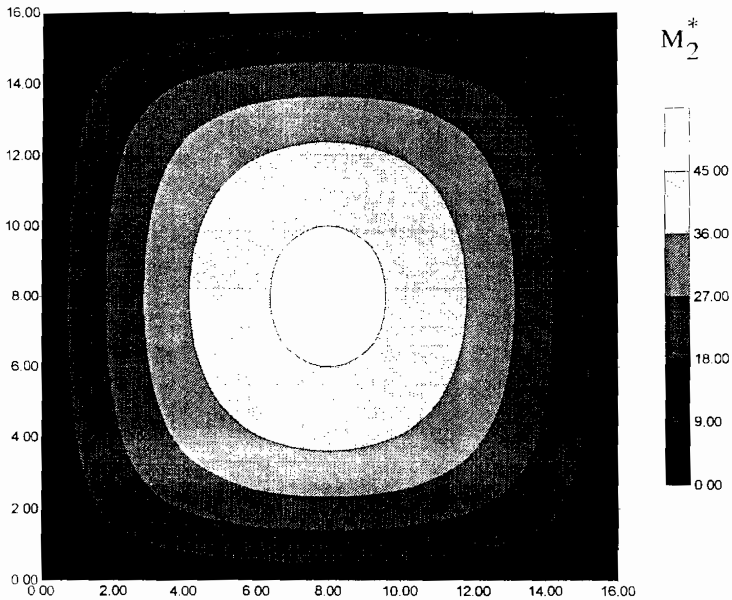


**Fig. 9.** Contour plotting for spatial variations of $M_2^*$ for a cylindrical panel with $r/a = 10$, $a/h = 5$, $b/a = 1$, and $m = n = 9$.

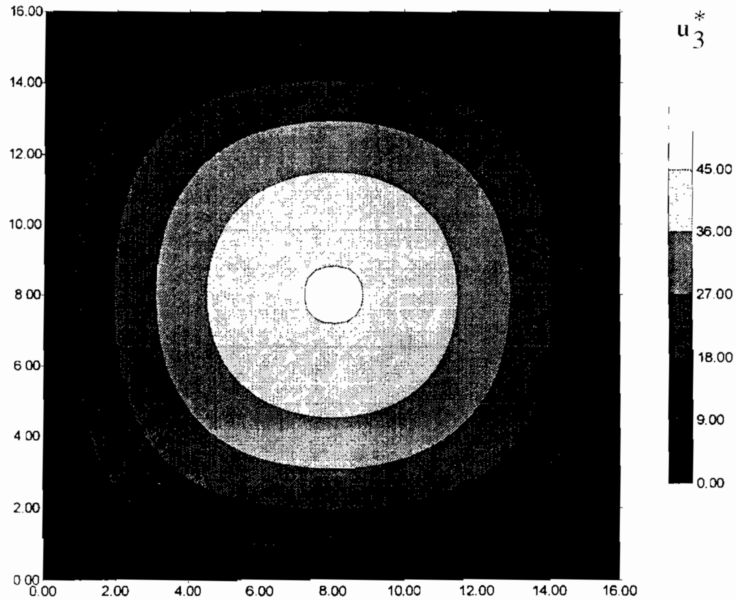


**Fig. 10.** Contour plotting for spatial variations of $u_3^*$, for a cylindrical panel with $r/a = 10$, $a/h = 10$, $b/a = 1$, and $m = n = 9$.

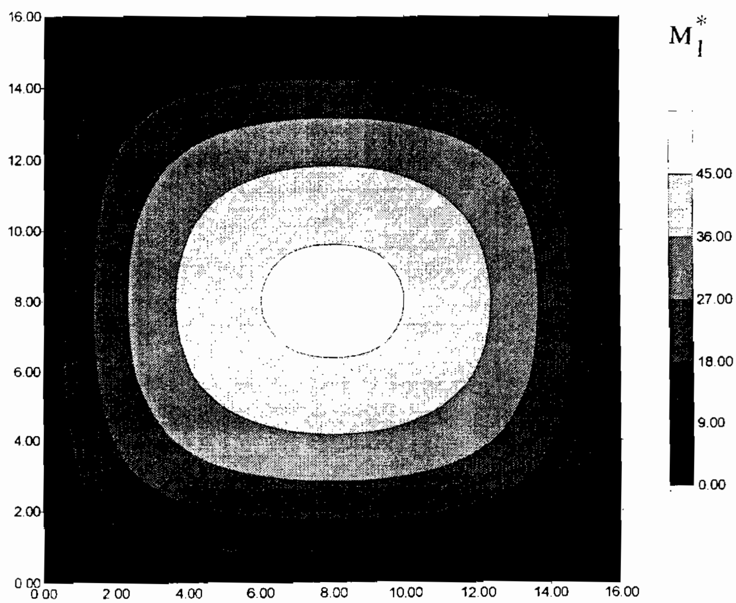


**Fig. 11.** Contour plotting for spatial variations of $M_1^*$ for a cylindrical panel with $r/a = 10$, $a/h = 10$, $b/a = 1$, and $m = n = 9$.

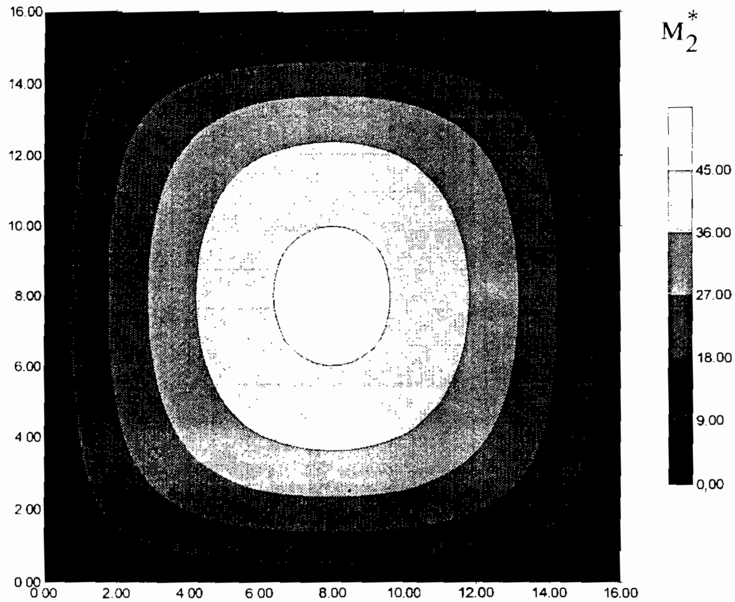


**Fig. 12.** Contour plotting for spatial variations of $M_2^*$ for a cylindrical panel with $r/a = 10$, $a/h = 10$, $b/a = 1$, and $m = n = 9$.

solution. The most interesting observation is noticed at edges, where moments at the edge due to the FEM do not vanish, while for the present solution they almost do, indicating a boundary sensitivity to the solution. The normalized central transverse displacement, due to the FEM, overpredicts the result by 13% in comparison

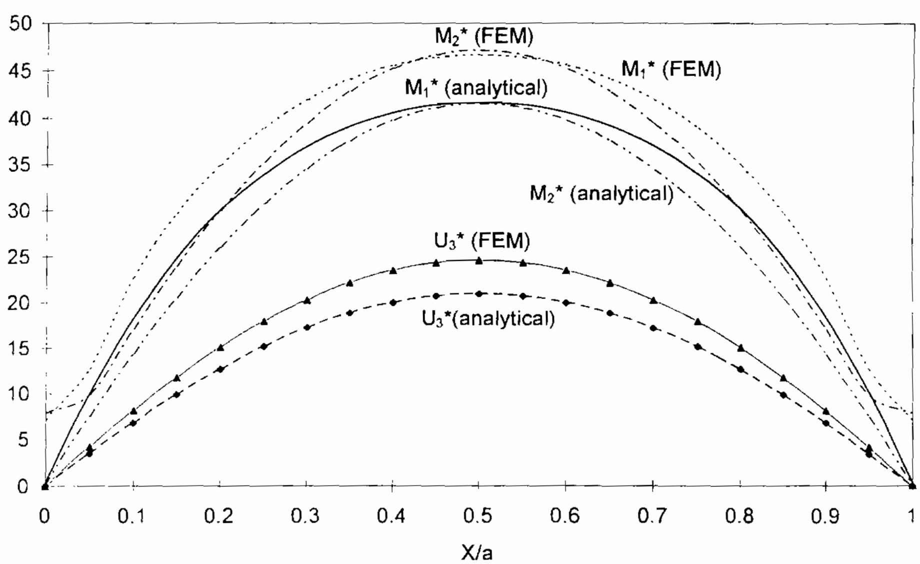


**Fig. 13.** Variation of normalized $u_3^*$, $M_1^*$, and $M_2^*$ at $x_2 = b/2$ along $x_1$ axis for a cylindrical panel with $r/a = 10$, $a/h = 10$, and $b/a = 1$, using FEM and analytical approach.

to the present one. The above variations are expected in view of two different through-thickness theories, reduced integration scheme in the finite element, and the negation of curvature effects in the finite-element formulations.

## CONCLUSIONS

An analytical solution to a boundary-value problem of a shear-flexible shallow cylindrical panel is presented. An accurate yet computationally efficient semi-boundary discontinuous double Fourier series approach is adopted to solve highly coupled partial differential equations in conjunction with the admissible boundary conditions (SS4). The numerical results thus presented herein should serve as base-line solutions for future comparisons. This work may be extended to free vibration and buckling response studies.


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# الأسلوب التحليلي للصفائح الأسطوانية وفق نظريات عالية المستوى

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## خلاصة

تعرض هذه الورقة أسلوبا تحليليا غير متوافر للصفائح الاسطوانية الشكل والمكونة من قطع مستطيلة مصنوعة من مواد متماثلة في المقاومة لعدة اتجاهات محورية وتحت تأثير قوى ثابتة. وتطرح في هذا الأسلوب التحليلي نظرية توافقية خاصة بمنشآت القشريات ، التى تتدرج تحتها الصفائح الأسطوانية ، يستخدم فيها معادلات تفاضلية ذات مستوى عال ومرتبط فيها خمسة متغيرات. كما تعرض حلا افتراضيا يستغل تطبيقات فوريير المزدوجة مع الأخذ في الإعتبار حالات حدود مستمرة وغير مستمرة وحدود مثبتة بفواصل . وقد توصلت النتائج الحسابية إلى أرقام تصل الى الحد النموذجى عند حساب الحركة والعزم ، بالإضافة إلى دراسة أثر العناصر للمعادلات المذكورة على الحركة والعزم ، مع رصدها على شكل خطوط كونتورية. والأسلوب المقترح لحل هذا النوع من المنشآت يعتبر كأساس أو كحل مثالي لأغراض المقارنة عند تطبيق أي اجراء آخر مثل : أسلوب العناصر المتناهية ، الفوارق المتناهية ، أسلوب جاليركى ، أو غيرها.