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Dynamic response of single span bridges with even and uneven decks

JAMAL AL-DUAIJ AND M. ABDEL-ROHMAN

Department of Civil Engineering, College of Engineering and Petroleum, Kuwait University, PO Box 5969, 13060 Safat, Kuwait

ABSTRACT

A single span bridge is considered in order to investigate the dynamic effect on the maximum deflection due to the effect of different parameters. The parameters considered are: type of supports (hinged-hinged or simply supported), the damping ratio, natural frequency, bridge span, stiffness of vehicles tires and suspension springs, weight of the moving vehicle, the vehicle speed, and the unevenness of the bridge deck and its profile.

INTRODUCTION

Non-linear behaviour was observed on long span flexible bridges with light damping. Non-linearity can produce large amplitude responses which may effect the safety of the structure. Non-linearity can appear due to geometry or due to the behavior of structural material. Previously, the non-linear oscillation of single span hinged-hinged bridges due to moving a concentrated load on an even deck, and the methods to control these oscillations, were studied (Abdel-Rohman & Nayfeh 1987a & b). In another study, the effect of unevenness in the bridge deck on the dynamic response of hinged-hinged single span bridges when traversed by sprung masses was investigated (Abdel-Rohman & Al-Duaij 1996). The unevenness in the bridge decks exists in all bridges due to vehicles breaking, pavement wear, the movement of heavy vehicles at varying speeds (Coussy *et al.* 1989). A study on the control of the effect of unevenness for simply supported bridges was made by Abdel-Rohman & Leipholz (1980).

Humar & Kashif (1993) attempted to identify the controlling parameters that govern the dynamic response of bridges to moving vehicle loads. Inbanathan & Wieland (1987) presented the dynamic response of a simply supported base girder bridge to a vehicle moving across the span. The results were obtained by modelling the vehicle and the vehicle-pavement interaction as a single unsprung mass and as a concentrated time-dependent force.

This study investigated the maximum dynamic deflection of single span bridges for different parameters. The bridge is modeled in four different ways. The first model is simply supported with an even deck. The second model is simply supported with an uneven deck. The third model is hinged-hinged with an even deck, and the fourth model is hinged-hinged with an uneven deck. In general it was observed that the bridges with uneven surfaces have a higher dynamic deflection than those with even surfaces. The effects of structural and vehicle parameters such as span length, natural frequency, damping ratio, vehicle speed, unevenness of the deck, and vehicle weight, are studied in this paper.

EQUATION OF MOTION OF SINGLE SPAN BRIDGE WITH EVEN DECK

The bridge is first modeled as a hinged-hinged beam of span L; a uniform mass per unit length, m; a flexural rigidity EI; a damping c; and an axial rigidity EA. Considering a single concentrated load moving with a constant speed v and taking into account the stretching of the midplane due to the immovable supports, one finds that the equation of motion is given by

$$mW_{tt} + cW_t + EIW_{xxxx} = EAeW_{xx} + P\delta(x - vt) \tag{1a}$$

where P is the weight of the moving load; $W(x, t)$ is the transverse deflection; W_x is the partial derivative of W with respect to x; W_t is the partial derivative of W with respect to the time t, and e is the stretching of the mid plane.

For the simply supported bridge, e is zero and the equation of motion becomes

$$mW_{tt} + cW_t + EIW_{xxxx} = P\delta(x - vt). \tag{1b}$$

The stretching e is defined by Nayfeh & Mook (1979) as

$$e = \frac{1}{2L}\int_0^L W_x^2\, dx. \tag{2}$$

The solution of Eq. 1 is assumed in the form

$$W(x, t) = \sum_{n=1}^{\infty} \eta_n(t)\sin\left(\frac{n\pi x}{L}\right) \tag{3}$$

in which $\eta_n(t)$ is the generalized coordinate of vibration mode n, and $\sin(n\pi x/L)$ is the shape of this mode which satisfies the boundary conditions.

Substituting Eq. 3 into Eq. 1 and applying orthogonality conditions, and considering the first mode as the dominant mode, one obtains the following equation of motion:

$$\ddot{\eta} + 2\mu\dot{\eta} + \omega^2\eta + \alpha\eta^3 = F \sin \Omega t \tag{4}$$

where $\mu = \xi\omega$; ξ is the damping ratio; $F = 2P/mL$; $\Omega = \pi v/L$; and

$$\omega^2 = \frac{\pi^4 EI}{mL^4} \tag{5}$$

$$\alpha = \frac{\pi^4 EA}{4mL^4}. \tag{6}$$

The value of α is zero for a simply supported bridge.

It is obvious that Eq. 4 includes a cubic nonlinearity due to the midplane stretching. An approximate solution to Eq. 4 can be obtained using perturbation techniques (Nayfeh 1981). One may introduce a small parameter ε as a book-keeping device to indicate the smallness of terms. To study the case of primary resonance, Eq. 4 is rewritten as

$$\ddot{\eta} + 2\varepsilon\mu\dot{\eta} + \omega^2\eta + \varepsilon\alpha\eta^3 = \varepsilon F \sin \Omega t. \tag{7}$$

The solution to Eq. 7 is given by

$$a' = -\mu a - \frac{F}{2\omega} \cos \gamma \tag{8}$$

$$a\,\gamma' = \sigma a - \frac{3\alpha a^3}{8\omega} + \frac{F}{2\omega} \sin \gamma. \tag{9}$$

The solution of Eq. 7 can also be written as (Nayfeh & Mook 1979):

$$\eta = a \cos(\Omega t - \gamma) + \dots \tag{10}$$

where a and γ are the solution of Eqs. 8 and 9.

The steady-state response is obtained when $a' = 0$ and $\gamma' = 0$. In this case one has

$$\sigma = \frac{3\alpha a^3}{8\omega} \pm \left(\frac{F^2}{4\omega^2 a^2} - \mu^2\right)^{1/2} \tag{11}$$

where $\sigma = \Omega - \omega$.

From Eq. 11 one can plot the relationship between the steady-state amplitude a of the deflection and the forced frequency Ω. This equation can also provide the linear steady-state response when $\alpha = 0$.

EQUATION OF MOTION WITH UNEVEN BRIDGE DECK

For more accurate analysis, a vehicle can be modeled as an independent, one degree of freedom sprung mass as shown in Fig. 1. The unevenness is expressed as a deterministic function $r(x)$. Then the load applied on the bridge is given by

$$P(t) = K[z - W(\bar{x}, t) - r(\bar{x})] + m_v g \tag{12}$$

in which K is the stiffness of the tires and suspension springs; m_v is the mass of the vehicle; $\bar{x}$ is the position of the vehicle from the left support ($\dot{x} = vt$); g is the acceleration of gravity; and z is the displacement of the vehicle with respect to its static equilibrium position.

The equations of motion of the bridge coupled with the vehicle motion are given by

$$mw_{tt} + Cw_t + EIW_{xxxx} = EAeW_{xx} + P(t)\delta(x - vt) \tag{13}$$

$$m_v\ddot{z} + c_v\dot{z} + K[z - r(\bar{x}) - W(\bar{x}, t)] = 0 \tag{14}$$

where c_v is the damping in the vehicle, $P(t)$ is given by Eq. 12, and $r(\bar{x})$ is a deterministic cosinusoidal function given by

$$r(\bar{x}) = u\left(1 - \cos \frac{2\pi\bar{x}}{1_1}\right) \tag{15}$$

in which u is the amplitude of the unevenness in a wave length 1_1 as shown in Fig. 2.

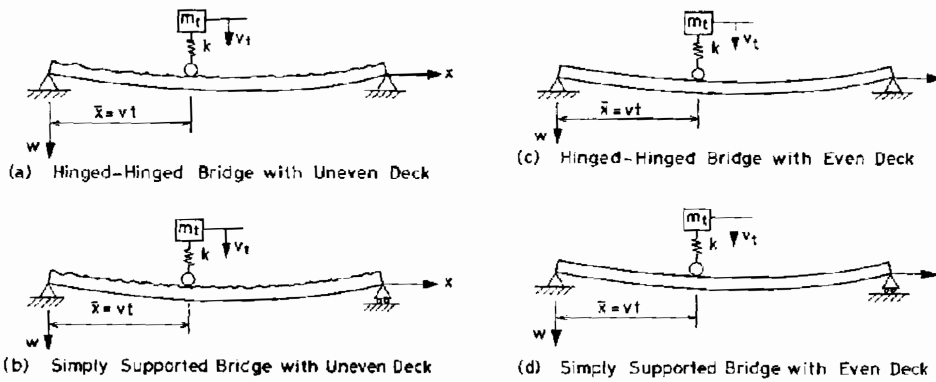

Fig. 1. (a) Hinged-hinged bridge with uneven deck. (b) Simply supported bridge with uneven deck. (c) Hinged-hinged bridge with even deck. (d) Simply supported bridge with even deck.

Using the solution of Eq. 3 and considering the first mode only, one obtains

$$\ddot{\eta} + 2\mu\dot{\eta} + \omega^2\eta + \alpha\eta^3 = \frac{2K}{mL}(v_t - r(\bar{x}) - \eta \sin \Omega t) \sin \Omega t + F \sin \Omega t \qquad (16)$$

$$m_v \ddot{z} + c_v \dot{z} + K[z - r(\bar{x}) - \eta \sin \Omega t] = 0 \qquad (17)$$

in which $\bar{x} = vt$.

Equations 16 and 17 are coupled ordinary nonlinear differential equations with time-varying coefficients. The analytical solution of these equations is difficult to obtain in a simple form as in Eq. 11. Moreover, the time used to cross a bridge by a vehicle is not enough to produce steady-state motion. Thus the solution shall be obtained numerically using the Runge-Kutta algorithm. It is obvious from Eqs. 16 and 17 that the dynamic response of the bridge is influenced by the bridge parameters ω, μ, α, u, and 1_1, in addition to the vehicle parameters m_v, C_v, v, and K. The effect of some of these parameters on the dynamic response is shown in the next section.

NUMERICAL INVESTIGATIONS

A single span bridge is considered in order to investigate the effect of support restraints and the unevenness in the bridge deck on its dynamic response at mid span. The basic data used for the bridge are the damping ratio, $\zeta = 5\%$, $m = 0.612$ Kg.s^2/cm^2, $EI = 10^{14}$ Kg.cm^2, $EA = 10^{11}$ Kg and $L = 4000$ cm. From these data the parameters ω, μ, and α were calculated using Eqs. 5 & 6; $\omega = 7.877$ rps, $\mu = 0.1$ and $\alpha = 0.0155$. The basic vehicle parameters are assumed to be $P = 4000$ Kg, $m_v = P/g = 4.08$ Kg.s^2/cm, $C_v = 0$ and $K = 1000$ kg/cm. The road profile is assumed to

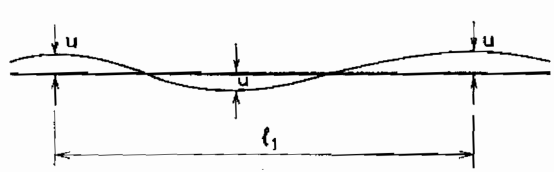

Fig. 2. Unevenness profile.

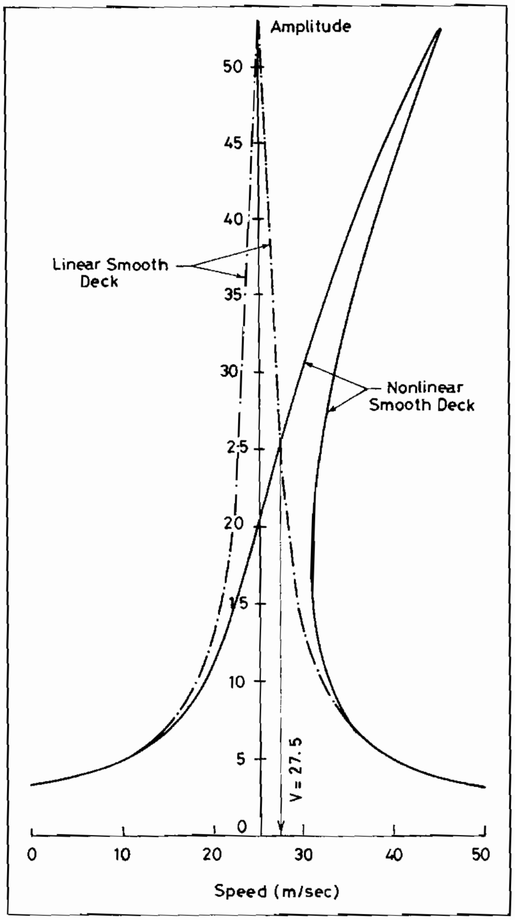

Fig. 3. Steady amplitude due to moving load.

be as in Eq. 14 where $l_1 = 100$ cm and $u = 10$ cm. The numerical investigations are focused on the vehicle speeds $v = 2500$ cm/sec (90 Km/hr) and $v = 1666.7$ cm/sec (60 Km/hr).

Using the above data, the maximum deflection for the hinged-hinged and simply supported bridges of even and uneven bridge decks were studied. Four different cases were studied for $u = 10$ cm, $u = 5$ cm, $v = 2500$ cm/sec and $v = 1666.7$ cm/sec. The other parameters, $P = 4000$ Kg, $\zeta = 5\%$, $\omega = 7.877$ rps, $l_1 = 100$ cm, $K = 1000$, $\alpha = 0.0155$ and $L = 4000$ cm, were kept as the standard case for reference.

The variation in the maximum dynamic deflection values for the simply supported and hinged-hinged bridges were studied. The effect of unevenness in the bridge deck was also studied considering the variation of the parameters P, ζ, ω, l_1, k, α, and L taken one by one. The results are plotted to show the effect of varying the parameters on the maximum deflection for the basic four cases which are abbreviated in the following way:

(HR) for hinged-hinged with rough surface
(SR) for simply supported with rough surface

(HS) for hinged-hinged with smooth surface
(SS) for simply supported with smooth surface.

Figure 4 shows the maximum deflection against the weight of moving load P. It is noted that the maximum deflection increases as P increases. The maximum deflection is smaller for the even bridge deck, as expected. Studies were done by changing u to 5 cm and v to 1666.7 cm/sec. The variation between the even and uneven cases decreased with decreasing u. A decrease in v did not cause any change. The difference in the response between hinged-hinged and simply supported bridges is not noticeable due to the small values of the deflection and the nonlinear coefficient, α, which make the contribution of the nonlinear term be very small.

Figure 5 shows the maximum deflection values against the unevenness profile l_1. It is noted that the maximum deflection on rough surfaces increases starting at $l = 400$ cm and reaches a maximum for $l_1 = 1200$ cm and then decreases.

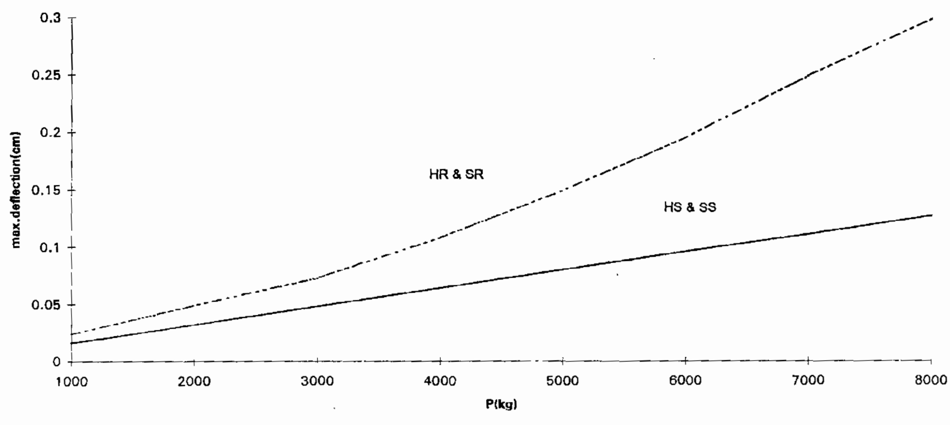

Fig. 4. Maximum deflection for various values of P.

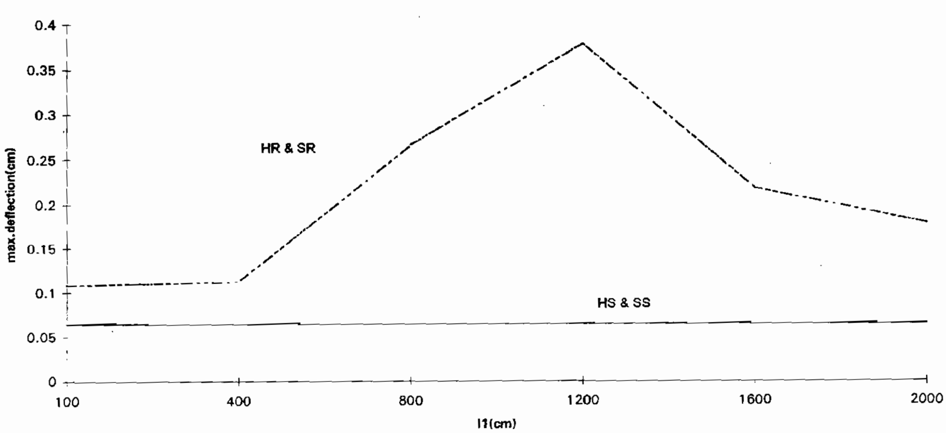

Fig. 5. Maximum deflection for various values of l_1.

Figure 6 shows the maximum deflection values against the vehicle tire and suspension stiffness K. For smooth surfaces, the maximum deflection did not change as K varied. The rough surfaces affected the deflection in an uneven pattern but showed some stabilization after $K = 1000$ kg/cm. The change of u to 5 cm caused the effect of unevenness to decrease, but changing v to 1666.7 cm/sec caused little change.

Figure 7 shows the maximum deflection values against the damping ratio ξ. Increasing the damping ratio did not cause much change in the maximum deflection. The variation between the even and uneven cases decreased with decreasing u. The decrease of v caused no change.

Figures 8 and 9 show the maximum deflection values against the bridge span L and the natural frequency of the bridge ω respectively. As ω increases the deflection

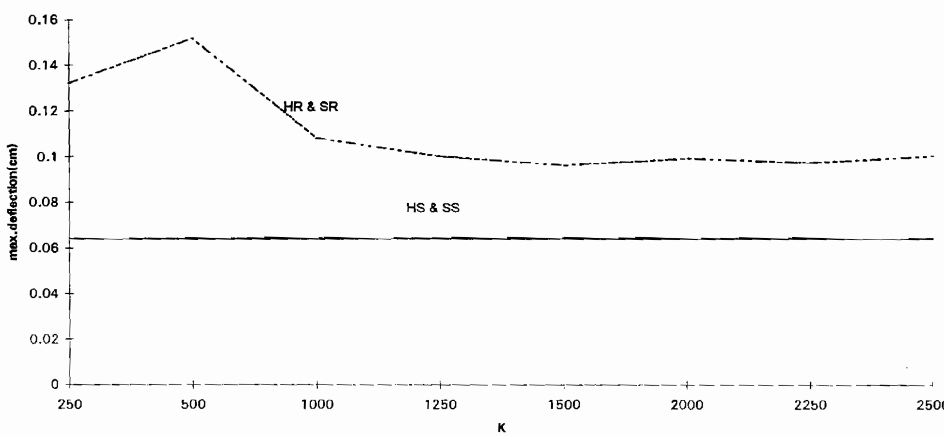

Fig. 6. Maximum deflection for various values of K.

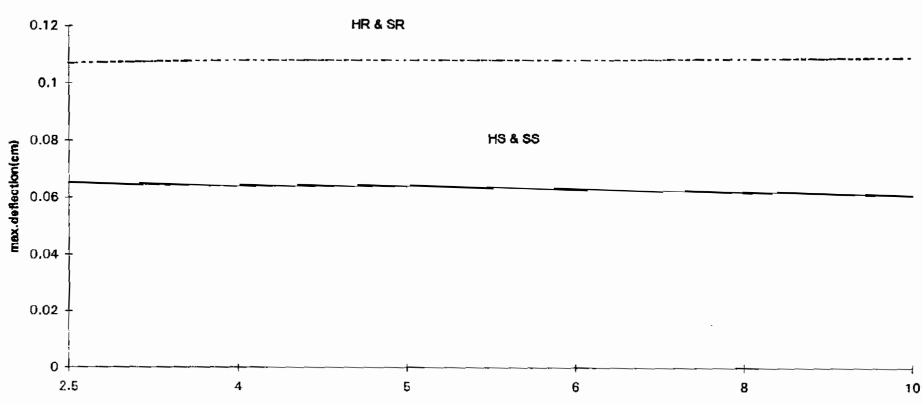

Fig. 7. Maximum deflection for various values of ceta.

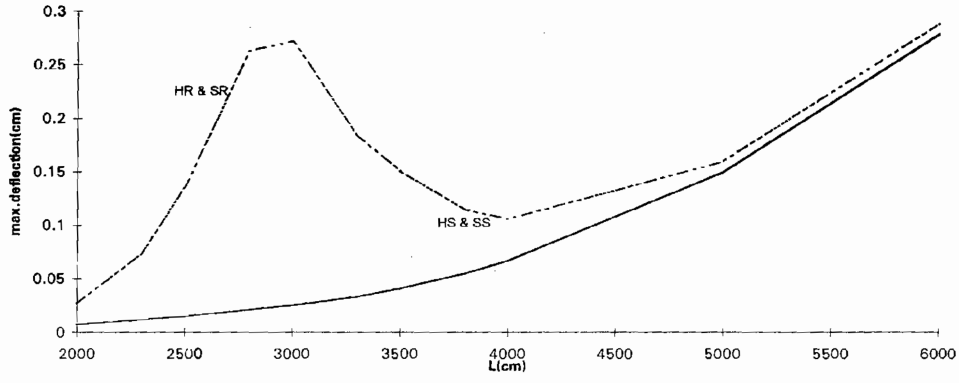

Fig. 8. Maximum deflection for various values of *L*.

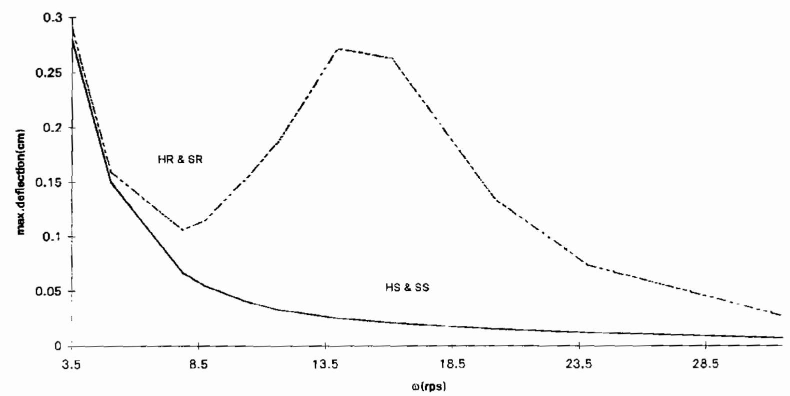

Fig. 9. Maximum deflection for various values of ω.

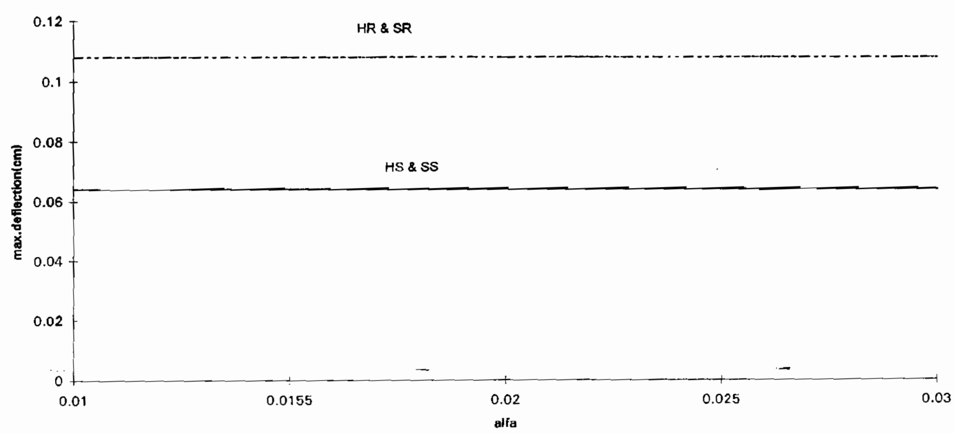

Fig. 10. Maximum deflection for various values of alfa.

decreases. However but near $\omega = 13.5$ rps, the difference between rough bridges and smooth bridges is much greater. This occurs also near $L = 3000$ cm. The decrease of u to 5 cm caused the variation between the smooth and rough to decrease. A decrease in v caused the variation between the smooth and rough decks to increase slightly.

Figure 10 shows the maximum deflection values according to the value of the nonlinearity α. No change in maximum deflection was observed with increasing α. Since the value of deflection is small, by using it the contribution of the nonlinear term becomes negligible. Change of u to 5 cms decreased the variation between the smooth and rough cases.

CONCLUSION

The maximum deflection for hinged-hinged and simply supported bridges on smooth and uneven bridge decks was studied. For each varying parameter, the effects were plotted for four different cases of u and v ($u = 10$ cm; $u = 5$ cm, $v = 2500$ cm/sec and $v = 1666.67$ cm/sec).

Maximum deflections were found to increase with P, the weight of a moving vehicle on both smooth and rough surfaces. Increasing l_1 had no effect on smooth surfaces. An increased maximum deflection was observed on rough surfaces.

Maximum deflections were found to decrease and stabilize with an increase in K on rough surfaces. On smooth surfaces, an increase in K did not cause any change.

Changes in ξ and α produced no change in maximum deflections.

Decreases in L, thereby increasing ω, caused the maximum deflection to decrease. Maximum deflections on rough surfaces increased near $L = 3000$ cm.

Decreasing u caused the maximum deflections to decrease. The difference in maximum deflections between the smooth and rough decks was found to vary less with the decrease of u. Decreases in v had little effect. The pattern of groups also did not change with changing u and v.

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دراسة ديناميكية للجسور المثبتة بالمفاصل من الطرفين ومن طرف واحد على السطوح الملساء أو الخشنة

جمال عبدالله الدعيج و محمد عبدالرحمن

قسم الهندسة المدنية بجامعة الكويت، ص. ب: 5969، الصفاة 13060، الكويت

خلاصة

تتطرق هذه الورقة لدراسة الجسور ديناميكيا بواسطة قياس انثناءاتها القصوى بسبب قوى متحركة. وتصنف الورقة أربع حالات من الجسور للدراسة لتحديد العناصر الرئيسية التي تؤثر في اهتزازات هذه الجسور العناصر هي: نسبة التبطييء، والتردد، طول الجسر، ثوابت الأجسام المتحركة، الوزن، وغيرها.

وتضع الدراسة كل ثابت بشكل منفصل للتعرف على تأثيره عند تغيره وفي نفس الوقت التدرج في تغيير باقي الثوابت وذلك فقط للانثناءات القصوى والتي معروضة في رسومات خاصة.