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Modelling subsidence deformations at the Slovak coalfields

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ABSTRACT

The modelling of subsidence development is presented as a convenient subsidence prediction technique. The proposed prediction method of mining-induced subsidence development applied at the Slovak Handlova Coalfield to mining longwall operations is based on a prediction method by Knothe. Theoretical calculations in modelling were proofed using data collected from surface monitoring, from which the appropriate site-specific parameters were obtained. The results, with accuracy analysis in subsidence development modelling, have confirmed the applicability of this method in deformation surveying practice.

INTRODUCTION

The dynamic nature of mining subsidences of the hanging walls on the earth's surface is dependent on a mining technology, and the time of its activity. The determination of subsidence development over time, being the final phase of the related deformation analysis, and a convenient subsidence prediction techniques, is the basis of the subsidence models. Many methods for determining the nature of subsidence can be used to simulate in advance the final and continuous state of the subsidence process.

The theory of modelling subsidence development follows from a method proposed by Knothe. The method is based on the Gauss distribution of effects. The method determining the time dependence of subsidence is verified by several studies of the undermined area in the Slovak brown coal deposit at Handlova (Sedlak 1992a & b).

The main methods of simulating the movement of the earth's surface caused by mining activity are generally oriented to the final asymptotic strain state. The process of development of subsidence with time does not make it possible to carry out detailed calculations of the deformation parameters in the individual intermediate stages of its development, during subsidence. Even the most efficient mining projects, which include planning of deformation measurements to prevent or minimize the damage caused by mining activity on the earth's surface, require a suitable method for determining the development of subsidence and a method enabling prediction of the procedure of formation of the process of subsidence in relation to the time factor.

The currently available solutions of the time dependence of formation of subsidence often disregard many important subsidence data in the stages from monitoring subsidence up to scientific analysis. The development of the methods of the time dependence makes it possible to use data which support the formation of several reliable and convincing technologies which can be used in predicting the nature of the subsidence process. In this paper, the existing method defining the gradual development of subsidence in relation to time is described, mainly for mining coalfields by longwall mining operations. This method is analyzed and suitably applied to the specific mining and geological conditions of the Handlova Brown Coalfield in Slovakia.

Computer modelling of subsidence development is very important from a mining surveying view. The periodic monitoring of subsidence data at each observed measurement point of the monitoring station must be adjusted and saved. A process of computer modelling of subsidence development can be simplified and sped up by some integrated measurement systems—the total geodetic stations, hardware and software (for example MicroStation). The MicroStation Development Language (MDL) can be used for modelling mine subsidences (Sedlak & Havlice 1994).

MODELLING SUBSIDENCE DEVELOPMENT

Theoretical principles

The effect of time during subsidence can be expressed by the function (Knothe 1953)

$$z(t) = 1 - \exp(-ct) \tag{1}$$

and, considering that $c(t)$ is constant, the subsidence rate $\dot{s}(t)$ can be expressed by means of the difference (Knothe 1953, 1957)

$$\dot{s}(t) = c[s_f(t) - s(t)] \tag{2}$$

where

$s_f(t)$ is the final (asymptotic) subsidence at time t.
$s(t)$ is the actual subsidence at time t.
$[s_f(t) - s(t)]$ is the subsidence potential at time t.
c is the time coefficient.

Solving this relation, the actual subsidence $s(t)$ at time t is obtained

$$s(t) = s_f(t) - \exp[-c(t)] \int_0^t \dot{s}_f(g)\exp(cg)\, dg \tag{3}$$

where

$\dot{s}_f(g)$ is the rate of the final subsidence development.
g is a coefficient which takes into account the geometrical parameters of the mined coalfield and influence radius.

The final form of the solution depends on the relation describing the final (asymptotic) state of subsidence $s_f(t)$. If the final subsidence is described using the influence function method based on the normal distribution of influences (Knothe 1957, 1984), then the specific solution for a rectangular excavation panel with one advancing side approximating longwall mining, can be developed (Fig. 1).

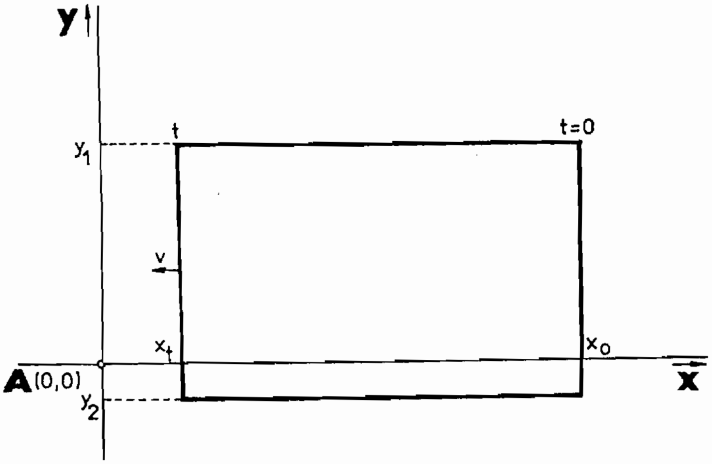

Fig. 1. Geometry of the longwall mining operation with a single moving wall.

If the simplest example of rectangular block mining operations with a single moving wall representing longwall operation is taken into account (Fig. 1), then the final subsidence can be expressed by using the method of the function of the effect using the normal distribution of subsidence (Knothe 1957, 1984):

$$s_f(x_t, x_0, y_1, z) = \frac{s_{\max}}{r_h^2} \int_{x_0}^{x_t} \int_{y_1}^{y_2} \exp\left[-\pi \frac{x^2 y^2}{r_h^2}\right] dx\, dy \tag{4}$$

where

$s_{\max}$ is the maximum subsidence, ($s_{\max} = -am$).
a is the subsidence factor.
m is the thickness of the mined layer.
r_h is the influence radius on the horizon h.

Subsidence at the longwall layers

The origin of the coordinate system is placed at the point $A(0, 0)$ at which subsidence is examined, with the mining advancing at a constant rate v in the x-axis (Fig. 1).

The time development of subsidence of the longwall layers over the mined layers, which can be indicated by periodic monitoring of the observed point network in the earth's surface, has three main subsidence phases. All phases are conditional on mining excavation technology and its working schedule. The first subsidence phase is the initial phase and begins from the start of mining activity up to the moment when the face wall moves below the observed point in the earth's surface. The second phase is defined by the time period t conditioned by the mining rate v, time coefficient c and influence radius r. The time period of this main subsidence part can be expressed by the simple Eq. 5 (Karamis *et al.* 1990):

$$t = \left(\frac{r}{v} + \frac{1}{c}\right). \tag{5}$$

The third phase, presenting the final subsidence, is limited by the end of the second phase and the time at which the observed point acquires the final subsidence value. Because in this phase the final subsidence at time $s_f(t)$ equals the resultant subsidence value of whole subsidence, then it can be used as the constant (s_f) given the following considerations.

The second main phase of subsidence development in time depends on the maximum rate of subsidence $\dot{s}_{\max}$ which is approximated by Eq. 6

$$\dot{s}_{\max} = \frac{s_f\, v}{r + \left(\frac{v}{c}\right)} \tag{6}$$

where

s_f is the final subsidence value.

In the case in which the subsidence process is continuous without interruption ($c = \infty$), the maximum rate of subsidence will be

$$\dot{s}_{\max} = \left(\frac{s_f}{r}\right) v. \tag{7}$$

Then modelling subsidence development can be possible for each observed point of a monitoring station in the earth's surface or some mine horizon as the maximum rate of subsidence at any time of mining activity.

APPLICATION OF THE METHOD OF MODELLING TO THE HANDLOVA BROWN COALFIELD

The Handlova Brown Coalfield belongs to the largest and the most economically significant coalfield in Slovakia. The longwall mine method is the typical extraction method for this coalfield. Two flat, technically faulted seams with an extraction thickness m of 2 m are found at the mining depth H of 300–400 m.

During a long-term examination of deformation changes in the earth's surface of the Handlova Brown Coalfield, we collected and analyzed subsidence data of three main subsidence lines from more than 90 measurement points in the monitoring station (Fig. 2). In all cases of the subsidence examination we selected, from the final subsidence curves, characteristic parameters for the functional method of the subsidence effect with respect to time, such as (Sedlak 1993):

- the influence radius $r = (170–200\text{ m})$;
- the final subsidence $s_f = (0.2–2.5\text{ m})$;
- the distance of the so-called "edge effect" d, i.e., the distance by which the wall must advance to ensure that the rock on the undermined region comes to assume a regular form, $d = (1/4–1/5)$ of the mined depth h, (theoretical calculated distance d (72–78 m) and the actual one d (70–87 m) of the "edge effect" were taken into account when modelling);
- the mined depth $h = (300–400\text{ m})$;
- the mining rate $v = (0.9–1.2\text{ m day}^{-1})$;
- the phase of main subsidence $t = (194–216\text{ days})$;

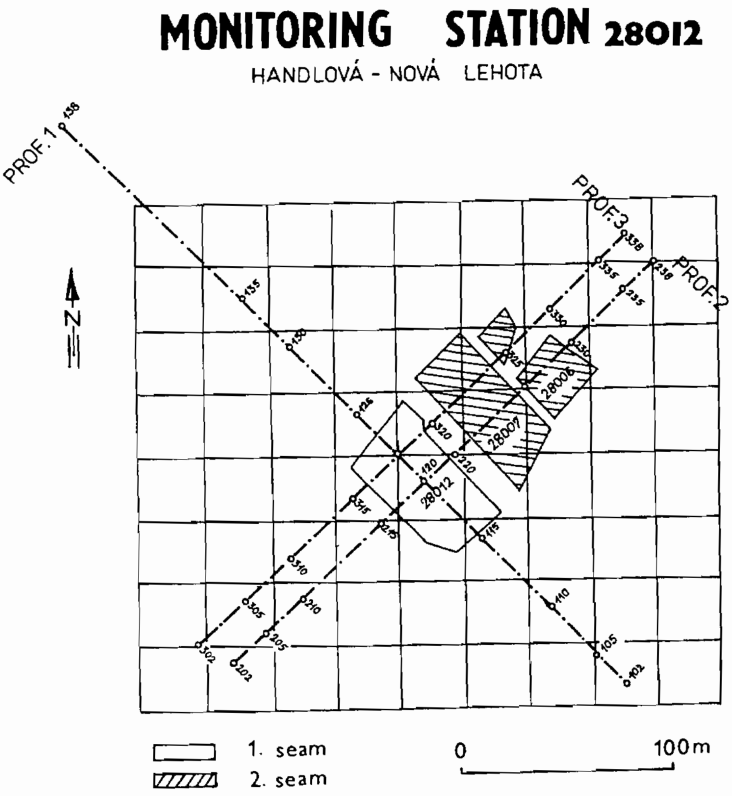

Fig. 2. Monitoring station in Handlova-Nova Lehota.

— the content of hard rock in hanging wall layers $TH = (35\text{–}40\%)$, (at the Handlova Brown Coalfield it is approximately 36%);
— the time coefficient $c = (0.036\ \text{day}^{-1}$, i.e., $1.3\ \text{year}^{-1})$.

Table 1 shows the comparison of the maximum subsidence development rate values $\dot{s}_{\max,G}$ from periodic geodetic measurements and calculated (modelling) ones $\dot{s}_{\max,M}$ from Eq. 6. The final subsidence values s_f were considered as the geodetic ones s_{f_G}, because the final subsidence values s_{f_G} were observed by using the very accurate geodetic total station (TOPCON GTS 6A).

Table 1. Subsidence development rate values.

Profile	Point	h [m]	v [m.day^{-1}]	$s_f = s_{f_G}$ [m]	$\dot{s}_{\max,G}$ [m.day^{-1}.10^{-3}]	$\dot{s}_{\max,M}$ [m.day^{-1}.10^{-3}]
1/28012	120	346	0.9	1.775	7.8	7.6
1/28012	123	337	0.8	2.281	8.5	8.8
2/28012	220	370	1.2	1.530	8.6	8.4
2/28012	223	340	1.2	0.967	5.6	5.3
3/28012	312	392	1.1	0.996	5.3	5.1
3/28012	320	313	1.1	1.324	6.7	6.8

If the subsidence over the advancing face of extraction is less than 10% of the final subsidence s_f, then subsidence at any time $s(t)$ can be approximated by the equation

$$s(t) = s_f\{1 - \exp[-c(t - \Delta t_0)]\}, \qquad \text{for} \quad t \in (\Delta t_0, \propto) \tag{8}$$

where

t is time calculated from the moment the face of excavation passes under the subsiding point.

Δt_0 is time adjustment value due to "edge effect" present at the excavation face, which creates some additional delay in subsidence development (at the Handlova Brown Coalfield it is approximately 75 days), ($d' = \Delta t_0 v$).

Figure 3 demonstrates cases of the single function (Eq. 8) approximating the time dependence of subsidence development for two choosen observed points of the monitoring station.

Modelling subsidence development of each observed point of the monitoring station at the Handlova Brown Coalfield can be derived from the presented theory about the time dependence of the subsidence development at a given moment of mining activity. In this way the predicted subsidence development of the earth's surface over a mined space can be obtained. Comparison of the measured final

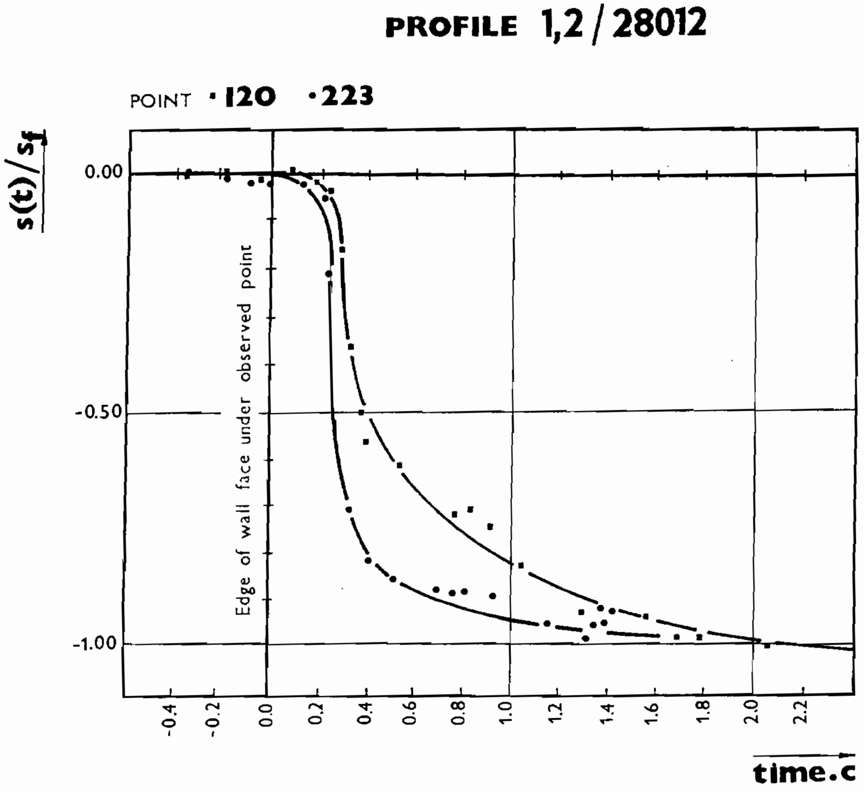

Fig. 3. Time dependence of subsidence for choosen points.

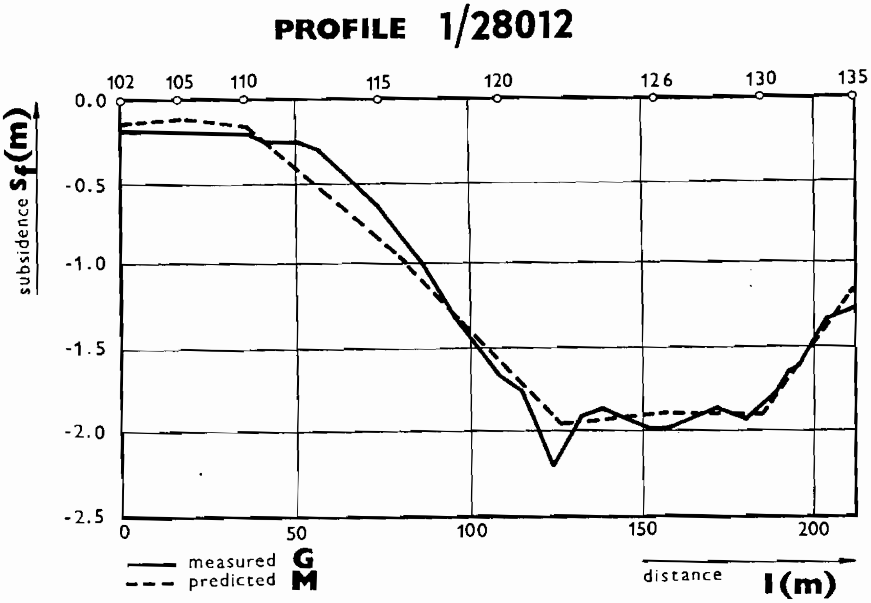

Fig. 4. Graphical comparison of measured and predicted subsidence development-PROFILE 1/28012.

subsidence development (s_{f_G}) and the calculated one (s_{f_M}-Eq. 4) at the profile No. 1/28012 of the Handlova monitoring station is shown in Fig. 4.

ANALYSIS OF SUBSIDENCE ACCURACY

Accuracy of the investigated subsidences is determined on a basis of comparing subsidences of monitoring station points, which were experimentally determined geodetically with these subsidences determined by modelling way.

Note: For simplification, we have introduced the following symbols:

— geodetic subsidences:

$$s_{GEOD} = s_{(120-120')G} = s_{G_1};\ s_{(123-123')G} = s_{G_2};\ \ldots\ s_{(320-320')G} = s_{G_n} = s_{(ij)G} = s_G$$

— modelling subsidences:

$$s_{MODEL} = s_{(120-120')M} = s_{M_1};\ s_{(123-123')M} = s_{M_2};\ \ldots\ s_{(320-320')M} = s_{M_n} = s_{(ij)M} = s_M$$

Every space distance s_{ij}; $i, j\langle 1, n\rangle$ (subsiding point makes a space distance from its start, for example point 120, to final subsidence value, point 120′) is defined by two points B_i, B_j whose coordinates x, y, z are determined in a cartesian three-dimensional (3D) rectangular system by convenient geodetic methods.

For these subsidences, let

$$s_{120-120'} = \sqrt{(x_{120'} - x_{120})^2 + (y_{120'} - y_{120})^2 + (z_{120'} - z_{120})^2}$$

$$\vdots$$

$$s_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}$$

$$\vdots$$

$$s = f(c_1, \ldots c_i, c_j, \ldots) \tag{9}$$

where

$$\boldsymbol{c}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \tag{10}$$

is the 3 × 1 vector of coordinates of a point B_i and

$$\Sigma_{c_i} = \begin{bmatrix} m_{x_i}^2 & & \\ & m_{y_i}^2 & \\ & & m_{z_i}{}^2 \end{bmatrix} \tag{11}$$

is its covariance matrix (with zero covariance components).

In order to express the variances of s, the known law of propagation of variances can be applied to Eq. 9 in the form

$$\Sigma_s = \boldsymbol{F}\Sigma_c \boldsymbol{F}^T \tag{12}$$

where

$$\Sigma_c = \begin{bmatrix} m^2_{x120} & & & & & & & & & \\ & m^2_{y120} & & & & & & & & \\ & & m^2_{z120} & & & & & & & \\ & & & m^2_{x123} & & & & & & \\ & & & & m^2_{y123} & & & & & \\ & & & & & m^2_{z123} & & & & \\ & & & & & & \ddots & & & \\ & & & & & & & m^2_{x320} & & \\ & & & & & & & & m^2_{y320} & \\ & & & & & & & & & m^2_{z320} \end{bmatrix} \tag{13}$$

is the covariance matrix of all coordinates of six choosen points and

$$F = \begin{bmatrix} \dfrac{\partial s_{G_1}}{\partial x_{120}}, \dfrac{\partial s_{G_1}}{\partial y_{120}}, \dfrac{\partial s_{G_1}}{\partial z_{120}} \cdots \dfrac{\partial s_{G_1}}{\partial x_i}, \dfrac{\partial s_{G_1}}{\partial y_i}, \dfrac{\partial s_{G_1}}{\partial z_i} \cdots \dfrac{\partial s_{G_1}}{\partial x_n}, \dfrac{\partial s_{G_1}}{\partial y_n}, \dfrac{\partial s_{G_1}}{\partial z_n} \\ \vdots \\ \dfrac{\partial s_{G_2}}{\partial x_1} \\ \vdots \\ \dfrac{\partial s_{ij}}{\partial x_1}, \dfrac{\partial s_{ij}}{\partial y_1}, \dfrac{\partial s_{ij}}{\partial z_1} \cdots \dfrac{\partial s_{ij}}{\partial x_i}, \dfrac{\partial s_{ij}}{\partial y_i}, \dfrac{\partial s_{ij}}{\partial z_i} \\ \vdots \end{bmatrix} \tag{14}$$

is the matrix of related coefficients (Jacobian matrix).

The standard errors m_x, m_y, m_z in Eq. 13 were determined by calculation of the average measured geodetic values: horizontal angles ϖ, zenit distances z and lengths l

$$m_x^2 = 12.3 \text{ mm}^2$$
$$m_y^2 = 12.4 \text{ mm}^2$$
$$m_z^2 = 2.2 \text{ mm}^2$$

have then been introduced into the matrix Σ_c Eq. 13.

The covariance matrix Σ_s indicates the variances of determined subsidences s_{ij}. The variances $m_{s_{ij}}^2$ of these subsidences are situated on the diagonal of this matrix and the covariances $m_{s_{ij}}$ off the diagonal.

$$\Sigma_s = \begin{bmatrix} m^2_{s(G_1)}, & m_{s(G_1),(G_2)} & \cdots & \\ m_{s(G_2),(G_1)} & m^2_{s(G_2)} & & \\ \vdots & & \ddots & \\ & & & m^2_{s_{ij}} \\ \vdots & & & \ddots \end{bmatrix} \tag{15}$$

The analogous numerical procedure (Gauss-Markov model) was applied for determining variances $m_{s_{ij}}^2$ of the modelling subsidences s_M.

Table 2 shows a summary of the mean square (standard) errors obtained. If real data is taken into consideration where the subsidences s_G are determined with the standard error $m_G = \pm 4.0$ mm, it means that these subsidences are not absolutely exact. Then the final standard error m_{MODEL} of the modelling determined subsidences will be given by the equation

$$m_{MODEL} = \pm\sqrt{m_M^2 - m_G^2}$$
$$m_{MODEL} = \pm 29.8 \text{ mm}. \tag{16}$$

For the purpose of a detailed accuracy analysis of the subsidences determined by modelling, it is necessary to take into account the standard errors of the parameter values mentioned earlier (the influence radius r, final subsidence s_f, edge effect d, mined depth h, mining rate v, phase of main subsidence t, content of hard rock in

Table 2. Mean square errors of the subsidences (s_{f_G}, s_{f_M}).

	Subsidence		Mean square error			
	s_{f_M}	s_{f_G}	$m_{s_{ij}}^2$ [mm^2]		$m_{s_{ij}}$ [mm]	
Point	[m]		M	G	M	G
120	1.716	1.775	972.1	15.6	31.2	3.9
123	2.260	2.281	912.9	14.8	30.2	3.8
220	1.511	1.530	860.4	10.2	29.3	3.2
223	0.872	0.967	845.7	18.3	29.1	4.3
312	0.870	0.996	984.0	16.6	31.4	4.1
320	1.269	1.324	867.2	19.5	29.5	4.4

$m_{AVERAGE}$: $m_{s_M} = 30.1$ mm $m_{s_G} = \pm 4.0$ mm hanging wall layers *TH*, time coefficient *c*, etc.). However, in mining damages the ±3% accuracy in subsidence development predicted by the model is very exact.

CONCLUSIONS

The results of modelling subsidence developments obtained in this paper are confirmed by the close correlation between measured and predicted subsidence data. This approximation can be efficiently used for the mining and geological conditions of the Handlova Brown Coalfield. An analysis of the results indicates that for mined depths of approximately 300–400 m with the wall heading advance of the face front from 0.9–1.2 $\mathrm{m.day}^{-1}$ and the content of hard rock in the hanging wall layers in the range of 35–40%, the time coefficient c is 0.036 day^{-1}, i.e., 1.3 year^{-1}.

The theoretical examination of the problem of subsidence with time, described in this paper, and the comparison of the calculated and measured results for the given example of the Handlova Brown Coalfield, confirms and validates the efficiency of the modelling method of characterisation of the time dependence of subsidence.

To determine more accurately the time coefficient *c*, it is necessary to analyze some further deformation parameters, for example tiltings, tensile and compressive strains, horizontal movements, etc. This will be part of the future research into this problem.

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نمذجة الخسف في حقول الفحم السلوفاكية

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خلاصة

يعرض هذا البحث نمذجة الخسف على شكل مبسط من تضمير الخسف. وتعتمد وسيلة التضمير المقترحة لتطور الخسف الناتج عن عمليات المناجم في حقل فحم هاندلوفا السلوفاكي المنفذ لعمليات الاستخراج الحائطي على وسيلة تضمير كنوثي. وقد برهنت الحسابات النظرية خلال النمذجة بواسطة معلومات تم جمعها أثناء المراقبة السطحية والتي نتج عنها معاملات موقعية. وأكدت النتائج المصحوبة بتحليل الدقة أثناء نمذجة تطور الخسف ملائمة تطبيق هذه الوسيلة أثناء مسح التشوهات.