Kuwait J. Sci. Eng. **28**(2) 2001

An approach for theoretical estimates of bulk specific deposit in deep bed filters

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ABSTRACT

A large number of filtration models have been proposed since the contributions of Iwasaki. All these studies have been made for a better description of the filtration model. The observations in the filtration experiments are generally restricted to the monitoring of headloss and filtrate concentration. Based on the these values, while the estimates of the absolute specific deposit are experimentally possible, the bulk specific deposit values are estimated by using headloss models or the bulk factor treated as constant. This study presents a novel approach based on the comparison of values obtained through computer programming with the experimental values. The regression process was made between the values of the bulk specific deposit obtained through the newly-developed computer program and the absolute specific deposit obtained from the experimental studies. The regression equations had the values of $r = 0.984$ and $r = 0.992$ for run 1 and 2, respectively. The factor was found as the coefficient of the independent variable. However, the coefficient of the independent variable was termed as the bulk factor.

Keywords: Absolute specific deposit; bulk factor; bulk specific deposit; filtration; headloss.

INTRODUCTION

During the process of filtration, particles in the influent suspension are deposited as the flow passes through the filter media. The amount of deposited material increases with time through the media. The value of porosity declines as the filtration process progresses, and the headloss increases.

Certain investigators have tested many approaches for determining the filtration mechanism. For example, Ojha and Graham (1994) examined recursive algorithms to predict filter performance. The status of fundamental models of the performance of deep bed filters have been reviewed (Tobiason & Vigneswaran 1994), and they have presented a modified model for nondispersed suspensions. Dharmappa *et al.* (1997) studied a non-linear least squares algorithm for determining the empirical parameters, and reported that the algorithm appears to be well suited for obtaining the best fit for both effluent quality and headloss. Stevenson (1997) constructed a mathematical model based on an adaptation of Kozeny equations for flow in porous media.

Ojha and Graham (1993) stated that the volume of the deposited material per unit of media volume is *the bulk specific deposit,* (vol/vol) and that the mass of the deposited material per unit of media volume is *the absolute specific deposit,* (mass/vol). Coad (1982) has emphasised that the bulk specific deposit influences the headloss development across the filter media, but it is not possible to directly detect bulk specific deposit values through observations in filtration experiments. Since experimental methods are restricted to the monitoring of headloss and filtrate concentration, it is possible only to estimate the absolute specific deposit values. Hasar (1997) studied the variation in the absolute specific deposit values with headloss and time. Since the headloss is a function of the bulk specific deposit, it is necessary that the absolute specific deposit be converted into the bulk specific deposit values, a process that is not straightforward and requires knowledge of the density of the porous deposits (Ojha & Graham 1993). Fox and Cleasby (1966) and Mohanka (1969) tried to use experimental methods to solve the problem without success.

Alternatively, it is possible to obtain theoretical estimates of the bulk specific deposit by using the observed headloss and the calculated values of the absolute specific deposit obtained experimentally. However, theoretical estimates of the bulk specific deposit are based on the choice of a headloss model. Sembi and Ives (1982) suggested using the Kozeny's headloss model with no empirical parameters. Coad (1982) has questioned the use of the Kozeny's model for estimating the bulk specific deposit. Since deposit cannot uniformly be distributed through the depth of the filter bed, Tien and Payatakes (1979) have divided the filter bed into small elements. They then developed complex models by making the calculations for each element. In the present study, a simple model was developed by assuming the deposit to be uniform throughout the depth of the bed.

In this study, the Kozeny's headloss and the spherical grain models are used for estimating the bulk specific deposit. The variations of the bulk specific deposit are estimated through a computer program. The estimates of the bulk specific deposit were compared with the absolute specific deposit values obtained experimentally, considering the same values of theoretical and experimental headloss. Then, the bulk and absolute specific deposits were correlated. In previous studies, Camp (1964), Ives and Sholji (1966), and Mohanka (1969) have defined the bulk factor such that when multiplied with the absolute specific deposit, it converts into the bulk specific deposit. The bulk factor has been assumed to be constant by these investigators. From these studies, it is not clear how the bulk factor should be obtained. Ojha and Graham (1993) studied the behaviour of the bulk factor during the filtration process by using sequential decision-making algorithms. The objectives of the current study are to obtain the theoretical estimates of the bulk specific deposit and to determine the behaviour of the bulk factor that indicates the relationship between the bulk specific deposit and the absolute specific deposit. In addition, other parameters such as headloss, porosity and grain size at any time can be obtained by means of the model.

THEORY OF FILTRATION

The theory of filtration has been studied extensively in the last few decades. As a result, it is now possible to simulate the behaviour of filter beds under various conditions (Ojha & Graham 1992). A wide variety of research on headloss in rapid sand filters has been summarised and a comparative study has been carried out by Sekerdag (1980). The relationship between the distribution of particle size and the standard variation has also been studied experimentally with various granulometries on filter media. Sekerdag (1987) has studied the effect of granulometry on the headloss in drinking water filters.

Kozeny (1927) and Fair and Hatch (1933) studied different models and underlying assumptions. They independently derived an equation for the hydraulic gradient through a bed of clean sand using Poiseuille's equation for laminar flow through a circular capillary tube. Ives (1969) has mentioned three different models. These are the Spherical Grain Model, Capillary Model, and General Model. We used the spherical model in our study.

The spherical model can be given as the flowing equation:

$$\frac{S}{S_0} = \left(\frac{d}{d_0}\right)^2 = \left(\frac{V}{V_0}\right)^{2/3} = \left(1 + \beta_v \frac{\sigma_b}{\varepsilon_0}\right)^{2/3} \tag{1}$$

where V_0 is the volume of a single grain in a clean filter, V is the volume of a single grain in a deposit filter (including deposit), ε_0 is the porosity ratio for the clean bed at the start of the run (without deposit), σ_b is the bulk specific deposit (in volumetric units), S_0 is the specific surface area in a clean filter (grain surface area in a unit volume of the bed), S is the specific surface area in a deposit filter, d_0 is the size of grain in a clean filter, d is the size of grain in a deposit filter, and β_v is the backing factor defined as $\varepsilon_0/(1 - \varepsilon_0)$.

The specific surface area disappears if the pore volume has been completely filled with deposit, that is to say according to Eq. (2), if $\sigma_b = \varepsilon_0$, so $S = 0$. When the pore volume within a filter bed is filled with the deposits or impurities of the filter influent water in a rapid sand filter, the porosity is reduced. The volume of the deposited material per unit of media volume is termed as the bulk specific deposit, σ_b, and can be expressed as:

$$\sigma_b = \varepsilon_0 - \varepsilon. \tag{2}$$

While it is possible to compute σ_a in units of mass per unit volume of media bed, its conversion to σ_b (volume of deposit/volume of bed) is not straightforward due to inherent uncertainty. We tackled this problem by assuming

$$\sigma_b = b\sigma_a \tag{3}$$

where $\sigma_a =$ the absolute specific deposit in mass per unit volume of media, and $b =$ the bulk factor that converts the mass concentration of suspension deposited into the volume occupied by deposits.

The change in particle size can be written as:

$$d = d_0 + \Delta d \tag{4}$$

where Δd represents the increase in grain size by the floc sheath and d_0 denotes the size of the clean grain. The change in porosity, $\varepsilon - \varepsilon_0$, or σ_b, as a function of

$\Delta d/d_0$, is given as:

$$\left(\frac{\Delta d}{d_0}\right)^2+\left(\frac{\Delta d}{d_0}\right)-\frac{\sigma_b}{3(1-\varepsilon_0)}=0. \qquad (5)$$

This Equation has been obtained from Eq. (1) and derived by the neglecting of the 3rd order term.

Muslu (1987) indicated that the flow phenomenon inside the filter media is extremely difficult to model theoretically. The hydraulic gradient in a bed of granular material has been investigated by many researchers. In this study, the hydraulic gradient has been calculated using the Kozeny equation. This equation is:

$$J_0=\frac{\mathrm{i}\psi^2 \upsilon}{g}\frac{(1-\varepsilon_0)^2}{\varepsilon_0^3}\frac{u}{d_0^2} \qquad (6)$$

where u represents the approach velocity or rate of filtration, J_0 is the initial (clean filter) hydraulic gradient and $\mathrm{i}\psi^2$ is the friction factor for rounded grain. For example, $\mathrm{i}\psi^2$ is 180, and υ is the kinematic viscosity of water. The headloss equation is:

$$dh=JL \qquad (7)$$

where dh is headloss at any time in the filter bed, J represents the hydraulic gradient at any time, and L is the depth of the filter media.

RESEARCH PROGRAMME

Theoretical study

In the theoretical stage of this study, a computer program was written and implemented in order to calculate the bulk specific deposit by using the spherical grain model. Diameter at any time t has been obtained iteratively from Eq. (4) by increasing as Δd as the initial filter grain diameter, d_0. Then, the bulk specific deposit and porosity have been obtained from Eq. (5) and Eq. (2), respectively.

The results explained in the next stages show that the calculations are reliable. The hydraulic gradient at the start of the filter run and at any time t have been calculated from Eq. (6). Therefore, the headloss is obtained using Eq. (7). The iteration process has been continued by increasing Δd as much as possible in order to attain a certain headloss level. The flowchart of the computer program is given in Fig. 1.

Experimental study

In the experimental stage of this study, the experiments were conducted by using the laboratory-scale filtration system in which the diameter of the filter column is 10.0 cm and its height is 1.50 m. The filter grain size of 1 mm was used. Two different series were made for the experimental studies. In experimental series 1 and 2, with a bed height of 50 cm, the filter velocities selected were 0.193 and 0.378 cm/s, respectively. The constant-rate operation was proved through the use of a rate control valve in the filter inlet and flow controller in the filter outlet. During the experiments, the water temperature was 26°C.

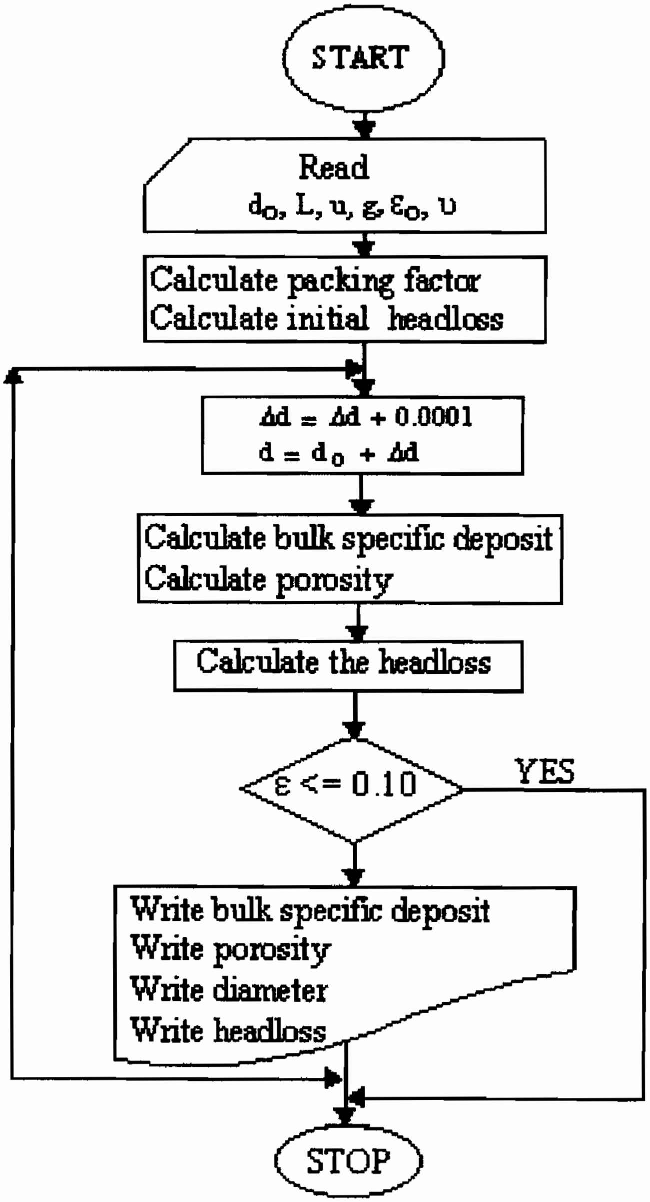

Fig. 1. Flow chart for computer program

Evaluation of the experimental and theoretical results

In the experimental studies, the variation of the headloss (*dh*) and the absolute specific deposit (σ_a) with time are obtained. However, the grain size and porosity values at the start of the filter run were known. During the later stages, it is clear that the bulk specific deposit and headloss will certainly change with time. It is not possible to know the variation of the porosity, the grain size, and the bulk specific deposit with time during the experimental studies. The variation of all these values have been theoretically calculated through the specially-developed computer program. Then, by taking into consideration approximately the same headloss values from theoretical and experimental studies, the results have been compared. Thus, the headloss, the absolute and the bulk specific deposit, the porosity and the grain size at any time have been obtained. The obtained values are presented in Tables 1 and 2.

Illustrative example

As seen in Table 1, the depth of the filter media or the bed height L is 50.0 cm, the initial filter grain size d_0 is 1.0 mm, the initial porosity ε_0 is 0.363, the filter velocity is 0.193 cm/s, and the kinematic viscosity of water is 0.008774 cm^2/s. At the start

Table 1. Data of theoretical and experimental studies for run 1

	Experimental Study			Theoretical Study		
Time min.	dh cm	σ_a g/cm^3	dh cm	σ_b cm^3/cm^3	ε	$d + \Delta d$ cm
0	12.00	0		0	0.3630	0.1000
30	14.00	0.0012	13.99	0.0058	0.3572	0.1003
60	19.30	0.0024	19.19	0.0350	0.3280	0.1018
90	26.70	0.0037	26.51	0.0631	0.2999	0.1032
120	33.50	0.0048	33.21	0.0816	0.2814	0.1041
150	40.60	0.0059	41.12	0.0983	0.2648	0.1049
180	46.00	0.0066	46.00	0.1067	0.2563	0.1053
210	53.90	0.0074	53.21	0.1173	0.2457	0.1058
240	62.10	0.0083	61.98	0.1280	0.2350	0.1063
270	70.00	0.0091	70.39	0.1366	0.2264	0.1067
300	78.00	0.0100	77.70	0.1431	0.2199	0.1070
330	89.00	0.0109	89.07	0.1519	0.2111	0.1074

$L_0 = 50$ cm $u = 0.193$ cm/s $\varepsilon_0 = 0.363$ $d_0 = 0.1$ cm $T = 26°\mathrm{C}$

Table 2. Data of theoretical and experimental studies for run 2

	Experimental Study			Theoretical Study		
Time min.	dh cm	σ_a g/cm^3	dh cm	σ_b cm^3/cm^3	ε	$d + \Delta d$ cm
0		0		0	0.370	0.100
30	24.0	0.0017	24.31	0.0019	0.368	0.1001
60	32.5	0.0031	32.26	0.0288	0.353	0.1015
90	42.0	0.0048	41.94	0.0524	0.326	0.1027
120	56.5	0.0063	57.10	0.0786	0.302	0.1040
150	68.0	0.0075	68.27	0.0930	0.285	0.1047
180	84.5	0.0088	84.77	0.1097	0.269	0.1055
195	90.0	0.0094	89.69	0.1139	0.265	0.1057

$L_0 = 50$ cm $u = 0.378$ cm/s $\varepsilon_0 = 0.370$ $d_0 = 0.1$ cm $T = 26°\mathrm{C}$

of the filtration process, the headloss for the known grain size, d_0, and porosity ε_0 values are calculated with the given values of other variables such as the depth of the filter media, kinematic viscosity of water, and the filter velocity.

d_0 is increased by the increment value of $\Delta d = 0.0001$. When the grain diameter reached 0.1018 cm, the bulk specific deposit was found to be 0.035 $\mathrm{cm}^3/\mathrm{cm}^3$ from Eq. (5). Then, the porosity ε at this time was calculated to be 0.328 from Eq. (2) through the obtained bulk specific deposit value. In the last stage of the theoretical study, the headloss value was estimated to be 19.19 cm.

In the experimental stage of this study, while the headloss value was observed to be 19.3 cm through the experimental set-up, at 60 minutes, the absolute specific deposit value obtained was 0.0024 $\mathrm{g/cm}^3$ from the filtrate concentration.

DISCUSSION

Finally, the experimental values combined with the theoretical values are utilised to obtain results. The values of the bulk specific deposit versus the absolute specific deposit are plotted to determine the relationship between the absolute and the bulk specific deposits.

As seen in Fig. 2, the correlation coefficient, r, has been found to be 0.984 and the relationship between bulk specific deposit and absolute specific deposit can be expressed as:

$$\sigma_b = 15.086\sigma_a. \tag{8}$$

This equation is obtained for the bed height of 50.0 cm and the filter approach velocity of 0.193 cm/s.

As seen in Fig. 3, the correlation coefficient, r, is 0.992, and the relationship between the bulk specific deposit and the absolute specific deposit, for the case of $L = 50$ cm and $u = 0.378$ cm/s, can be expressed as:

$$\sigma_b = 12.109\sigma_a. \tag{9}$$

Figures 2 and 3 present the relationship between the bulk specific deposit, the absolute specific deposit, and the line of best fit given by Eqs. (8) and (9). In these cases, the values of r are 0.984 and 0.992 for runs 1 and 2, respectively. Since the values of r are sufficiently close to 1.0, these values are safe. Equations (8) and (9) have been obtained in the form of $\sigma_b = b\sigma_a$, such as in Eq. (3). The coefficients of the independent variable in Eqs. (8) and (9) can be expressed as the *bulk factor*. The values of the bulk factor are 15.086 for $L = 50.0$ cm and $u = 0.193$ cm/s, and 12.109 for $L = 50.0$ cm and $u = 0.378$ cm/s.

CONCLUSIONS

The present paper indicates that the use of a model developed by assuming the deposit to be of uniform depth in a filter bed is possible. The results of the computer program and analysis yield the value of the bulk factor. The correlation between the bulk specific deposit and absolute specific deposit indicate reasonably good fits. The plots indicate that the volume fraction of the deposit is proportional to the mass fraction of deposit. However, this paper indicates that the filtration parameters such

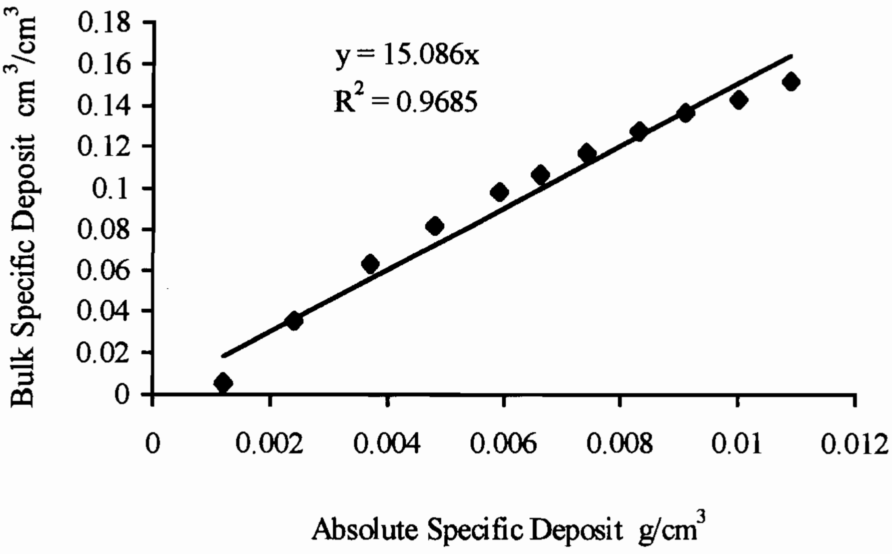

Fig. 2. The plot of bulk specific deposit versus absolute specific deposit for run 1.

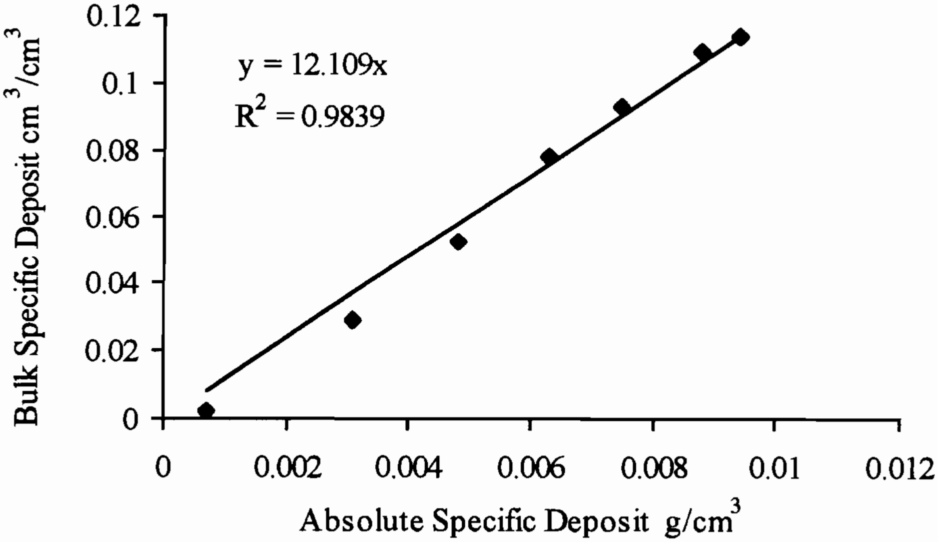

Fig. 3. The plot of bulk specific deposit versus absolute specific deposit for run 2

as the absolute specific deposit, the bulk specific deposit, the headloss, the porosity and grain size, could be easily determined by means of the experimental data and the computer program including a simple model, as a function of time. This method can be applied to solve continuity and kinetic filtration equations.

ACKNOWLEDGMENTS

This study was supported by the Research Foundation of Firat University under project No. FÜNAF-236.

APPENDIX 1—NOTATION

The following symbols are used in this paper:

d_0 = size of grain at the start a run without deposit, L
d = size of grain (including deposit) expressed as the diameter of a sphere of equal volume, L
Δd = incremental size of grain, L
dh = headloss of filter at depth L and time t, L
J_0 = initial hydraulic gradient
J = hydraulic gradient at any time t
L = depth of filter media (measured down from the top of the bed), L
S_0 = specific surface area of clean filter, L^2
S = specific surface area of clean filter, L^2
u = approach velocity of filter, $L.T^{-1}$
V_0 = volume of a single grain in clean filter, L^3
V = volume of a single grain in deposited filter, L^3
ε_0 = porosity of clean bed at the start of run
ε = porosity of filter bed (with deposit)
σ_a = bulk specific deposit, $L^3.L^{-3}$
σ_b = absolute specific deposit, $M.L^{-3}$
b = bulk factor
β_v = packing factor
υ = kinematic viscosity of water, $L^2.T^{-1}$
g = the gravity constant, $L.T^{-1}$
$i\psi^2$ = the dimensionless constant

REFERENCES

Coad, M.A. 1982. Conductimetic measurements of deposits in sand filters. **Ph.D. thesis**. University of London, London, England.

Camp, T.R. 1964. Theory of filtration. Journal of the Sanitary Engineering Division, ASCE **90**: 1–30.

Dharmappa, H.B., Parasanthi, H., Krishna, M.U. & Xiao, Y. 1997. Estimation of empirical coefficients of a granular filtration model. Water Research **31**: 1083–1091.

Fair, G.M. & Hatch, L.P. 1933. Fundamental factors governing the streamline flow of water through sand. Journal of the American Water Works Association **25**: 1551.

Fox, D.M. & Cleasby, J.L. 1966. Experimental evaluation of sand filtration theory. Journal of the Sanitary Engineering Division, ASCE **87**: 23–37.

Hasar,H. 1997. Investigation of appropriate of headloss observed experimentally in sand filter with headloss obtained from theoretical models. **M.Sc. thesis**. University of Firat, Elazig, Turkey.

Ives, K.J. & Sholji, I. 1966. Research on variables affecting filtration. Journal of the Sanitary Engineering Division, ASCE **91**: 1–18.

Ives, K.J. 1969. Theory of Filtration. International Supply Congress. Vienna, Austria.

Kozeny, J. 1927. Wasserkraft und wasserwirtschaft. **22(67)**: 227–235.

Mohanka, S.S. 1969. Theory of multilayer filtration. Journal of the Sanitary Engineering Division, ASCE **95**: 1079–1095.

Muslu, Y. 1987. Shape factor and degree of packing in fluidization. Journal of Environmental Engineering **113**: 311–329.

Ojha, C.S.P. & Graham, N.J.D. 1992. Computer-aided solutions of filtration equation. Water Research **26**: 145–150.

Ojha, C.S.P. & Graham, N.J.D. 1993. Theoretical estimates of bulk specific deposit in deep bed filters. Water Research **27**: 377–387.

Ojha, C.S.P. & Graham, N.J.D. 1994. Prediction of deep bed filter performance using recursive algorithms. Journal of Environmental Engineering **120**: 961–971.

Sembi, S. & Ives, K.J. 1982. Optimisation of size graded filters. Proceedings of the Symposium on Water Filtration. Antwerp. Belgium, 21–23 April, Pp. 5.10–5.12.

Sekerdag, N. 1980. The investigation of effect of various factors on the flow in sand filters. **M.Sc. thesis.** Istanbul Technical University, Istanbul, Turkey.

Sekerdag, N. 1987. The effect of granulometry on the headloss in the filters of drinking water. Journal of Isparta Engineering Faculty of Akdeniz University, Civil Engineering **No. 3**: 19–26.

Stevenson, D.G. 1997. Flow and filtration through granular media – the effect of grain and particle size dispersion. Water Research **31**: 310–322.

Tien, C. & Payatakes, A.C. 1979. Advances in deep bed filtration. American Institute of Chemical Engineers Journal **25**: 737–759.

Tobiason, J.E. & Vigneswaran, B. 1994. Evaluation of modified model for deep filtration. Water Research **28**: 335–342.

(Submitted 25 April 2000)
(Revised 13 January 2001)
(Accepted 20 January 2001)

طريقة نظرية لتقدير معامل الترسب النوعي الوسطي في مرشحات عميقة

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الخلاصة

قام عدد كبير من الباحثين بتقديم عدة نماذج لعملية الترشيح بعد عمل أواسكي الرائد في هذا المجال وذلك لتقديم شرح أفضل لعملية الترشيح. وتتحصر دراسات الترشيح التجريبية- بوجه عام- على ملاحظة وقياس الفقد في الضغط ونسبة تركيز العوالق. تستخدم هذه القيم في تقدير الترسب النوعي كما يمكن تقدير متوسط bulk الترسب النوعي باستخدام نماذج لفرق الضغط أو معامل وسطي يتم معاملته كقيمة ثابتة.

وتقدم هذه الدراسة طريقة جديدة تعتمد علي مقارنة القيم التي تم استنباطها تجريبيا بواسطة برنامج حاسوب. وتتيح هذه الطريقة مقارنه يتم الترسب النوعي الوسطي والتي تم حسابها من برنامج حاسوب جديد تم عمله مع قيم الترسب النوعي المطلقة والتي تم استنتاجها من التجارب المعملية. وقد وجد أن المعادلات المستنبطة بواسطة الحاسوب من نتائج مجموتي تجارب أعطيت قيم تقارب $r = 0.984$ ، $r = 0.992$. وقد تم استنتاج معامل للمتغيرات المستقلة تم تسمية المعامل الوسطي.