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Nonexistence of entire solutions to some nonlinear elliptic equations

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ABSTRACT

We prove the nonexistence of entire solutions to elliptic equations of the form $div\,(|\nabla u|^{p-2}\nabla u) + \beta(x)f(u) = g(x),\ \ p > 1$ under some conditions on β, g, and the nonlinearity f. Typical models for f include the exponential and power functions. All Theorems proved herein generalize some previously known results, when $p = 2$.

Keywords: Entire Solutions; Nonlinear Elliptic Equations.

1. INTRODUCTION

Oleinik (19978) considers the problem of nonexistence of solutions to the semilinear equation $\Delta u + \beta(x)e^u = g(x)$ under some conditions on $\beta(x)$ and $g(x)$. This problem arises in a geometric context where one tries to find conditions on a smooth function β such that β is the Gaussian curvature of a conformal metric $\tilde{g} = e^{-2u}g$ where g is a given metric on some Riemannian manifold (See NI 1982b for details). As a result, related problems of existence or nonexistence have been studied extensively by several authors, such as in benjin & Zucki (2000), Mitidieri & Pokhozhaev (1998), Pucci & Serrin (1988a,b), Bernard (1996), Gui (1992), Lin (1985), Ni (1982a,b), Sattinger (1978), and references therein.

In this paper, we extend the results of Oleinik (1978) to the quasilinear elliptic equations

$$\Delta_p u + \beta(x)f(u) = g(x),$$

where $f \in C(\mathbb{R})$ is not identically zero and Δ_p is the p-Laplace operator:

$$\Delta_p u := div\,(|\nabla u|^{p-2}\nabla u), \quad 1 < p < \infty.$$

To further stipulate on f, let us use the notation:

$$\sigma := \inf\{\tau : f(s) > I(f),\ s > \tau\},$$

where

$$I(f) := \inf\{f(s) : s \in \mathbb{R}\} \geq 0\ .$$

We seek entire solutions of the following equation

$$\begin{cases} \Delta_p u + \beta(x)\, f(u) = g(x), & x \in \mathbb{R}^n, \\ u(x) > \sigma & x \in \mathbb{R}^n\ . \end{cases} \tag{1.1}$$

An entire solution to Equation (1.1) is a function $u \in C^2(\mathbb{R}^n)$ that satisfies the equation at every point in $\mathbb{R}^n$. The nonlinear funciton f will be assumed in $C^2(\sigma, \infty)$, satisfying the following conditions:

$$f'(s) > 0,\ f''(s) \geq 0 \text{ on } (\sigma, \infty), \text{ and } (f^{-1})'(t) \leq \mu t^{-\alpha} \text{ on } (0, \infty). \tag{$*$}$$

Here, the constants μ, $\alpha > 0$, and f^{-1} denotes the inverse of f on (σ, ∞). Typical models for f are power functions ($f(s) = |s|^q,\ q > 1,\ \sigma = 0$), and the exponential function ($f(s) = c^s,\ \sigma = -\infty$). The methods used herein are similar to those of Oleinik (1978).

2. The case when $\beta(x) \leq 0$ and $g(x) \leq 0$

We need the following Lemma for our first nonexistence result.

Lemma 2.1. *Suppose f satisfies (*) with $\alpha > (p-2)/(p-1)$, and that $\beta(x) \leq -\beta_0$ for $x \in B(x_0, R)$ and some constant $\beta_0 > 0$. If $u \in C^2(B(x_0, R))$ is a solution of (1.1) with $g(x) \geq 0$ in $B(x_0, R)$, then there is a positive constant $C = C(n, \alpha, p)$ such that*

$$f(u(x_0)) \leq C\, \beta_0^{-1/(\alpha(p-1))} R^{-\varrho} + I(f),$$

where,

$$\varrho := p\left(\frac{2}{\alpha(p-1) - (p-2)} - \frac{1}{\alpha(p-1)}\right).$$

Proof. Without loss of generality we can assume that $I(f) = 0$, for otherwise we replace f by $(f = I(f))$ in Equation (1.1), and since $g(x) - \beta(x)\, I(f) \geq 0$ on $\mathbb{R}^n$, the following argument would still be valid.

Let $v_0(x) := \gamma(r^2 - |x - x_0|^2)^{-k}$, where $k := p/[(\alpha(p - 1\_(p - 2)]$ with α as in (*), while γ is to be specified below. Notice that $k > 0$, by hypothesis. Then, since $(f^{-1})'' \leq 0$, we see that

$$\begin{aligned}\Delta_p f^{-1}(v_0) + \beta(x)v_0 &= (p-1)((f^{-1}(v_0))^{p-2}(f^{-1})''(v_0)|\nabla v_0|^p \\ &+ ((f^{-1})'(v_0))^{p-1}\ \Delta_p v_0 + \beta(x)v_0 \leq ((f^{-1})'(v_0))^{-1}\ \Delta_p v_0 + \beta(x)v_0 \qquad (2.1)\end{aligned}$$

Direct calculations yield:

$$\Delta_p v_0 = (R^2 - |x - x_0|^2)^{-m}(a_{\alpha,p}|x - x_0|^p + b_{\alpha,p}(R^2 - |x - x_0|^2)|x - x_0|^{p-2}),$$

where

$$\begin{cases} a_{\alpha,p} := 2(k+1)(p-1)(2\gamma k)^{p-1}, \\ b_{\alpha,p} := (p-2+n)(2\gamma k)^{p-1}, \\ m := (k+1)(p-1)+1. \end{cases}$$

Using this in Equation (2.1), and letting $c_{\alpha,p} := a_{\alpha,p} + b_{\alpha,p}$, we see that

$$\begin{aligned}\Delta_p f^{-1}(v_0) + \beta(x)v_0 &\leq ((f^{-1})'(v_0))^{p-1}\ (r^2 - |x - x_0|^2)^{-m} \times \\ &\times[c_{\alpha,p}R^p - \beta_0(R^2 - |x - x_0|^2)^{m-k}((f^{-1})'(v_0))^{1-p}]\ . \qquad (2.2)\end{aligned}$$

From (*), we have $(f^{-1})'(v_0) \leq \mu v_0^{-\alpha}$, and therefore we get the estimate

$$(R^2 - |x - x_0|^2)^{m-k}((f^{-1})'(v_0))^{1-p} \geq \mu^{1-p}\gamma^{\alpha(p-1)}(R^2 - |x - x_0|)^{m-k-k(\alpha(p-1))}.$$

Recalling the values of m and k, we observe that $k - k - k(\alpha(p-1)) = 0$. thus, using this in Equation (2.2), we obtain the estimate

$$\Delta_p f^{-1}(v_0) + \beta(x)v_0 \leq ((f^{-1})'(v_0))^{p-1}(R^2 - |x - x_0|^2)^{-m}\left(c_{\alpha,p}R^p - \frac{\beta_0}{\mu^{p-1}}\gamma^{\alpha(p-1)}\right).$$

We choose γ as,

$$\gamma := (c_{\alpha,p}\mu^{p-1}R^p\beta_0^{-1})^{1/(\alpha(p-1))},$$

to get the inequality

$$\Delta_p f^{-1}(v_0) + \beta(x)v_0 \le 0, \quad \forall\, x \in B(x_0, R).$$

Now, let $0 < \epsilon < 1$, and $v := f(u)$. Since $f^{-1}(t) \to \infty$ as $t \to \infty$, and u is bounded on $B(x_0, (1-\epsilon)R)$, it follows that $f^{-1}(v) < f^{-1}(v_0)$ on $\partial B(x_0, (1-\epsilon)R)$ for sufficiently small ϵ. We claim that $f^{-1}(v) \le f^{-1}(v_0)$ on $B := B(x_0, (1-\epsilon)R)$.

Suppose not. This means that at some point in B, $f^{-1}(v) > f^{-1}(v_0)$. Then $f^{-1}(v) = f^{-1}(v_0)$ on $\partial\,\Omega$ where $\Omega := \{x \in B : f^{-1}(v)(x) > f^{-1}(v_0)(x)\}$. Since f is increasing, we see that $v > v_0$ on B. Therefore for $x \in \Omega$, we have:

$$\Delta_p f^{-1}(v) - \Delta_p f^{-1}(v_0) = -\beta(x)(v - v_0) + g(x) - (\Delta_p f^{-1}(v_0) + \beta(x)v_0) \ge 0.$$

Thus, by the Comparison Principle (See Tolksdorf 1983) we have $f^{-1}(v) \le f^{-1}(v_0)$ on Ω, which is an obvious contradiction. Thus $f^{-1}(v) \le f^{-1}(v_0)$ on $B(x_0, (1-\epsilon)R)$. Since ϵ is arbitrary, and f is increasing we conclude, that

$$f(u(x)) \le \gamma((R^2 - |x - x_0|^2)^{-k}, \quad \forall\, x \in B(x_0, R).$$

Thus,

$$f(u(x_0)) \le C\beta_0^{-1/(\alpha(p-1))}\ R^{-\left(\frac{2p}{\alpha(p-1)-(p-2)} - \frac{p}{\alpha(p-1)}\right)},$$

where $C - C(\alpha, n, p)$ is a positive constant, as desired.

In the following theorem, ϱ denotes the constant in the previous Lemma.

Theorem 2.2. *Let f satisfy (*) with* $\alpha > |p - |/(p-1)$. *Suppose* $\beta(x) \le 0$ *in* $\mathbb{R}^n$, *and that* $|\beta(x)| \ge \theta(|x|)|x|^{-\varrho\,\alpha(p-1)}$ *for* $|x| \ge R_0 > 0$, *where* $\theta(t) \to \infty$ *as* $t \to \infty$, $\theta(t) > 0$ *for* $R_0 \le t < \infty$ *and* $\theta(t)t^{-\varrho\,\alpha(p-1)}$ *is a non-increasing function of t. Then Equation (1.1) has no entire solution for* $g(x) \ge 0$ *in* $\mathbb{R}^n$.

Proof. As in the previous proof, we can assume that $I(f) = 0$. Suppose $u \in C^2(\mathbb{R}^n)$ is a solution, and let $x \in \mathbb{R}^n$ with $|x| = r > R_0$. We apply Lemma 2.1 to the ball $B(x, r - R_0)$. Since $|y| \le 2|x| - R_0$ for $y \in B(x, r - R_0)$. we have $\theta(|y|)|y|^{-\varrho\,\alpha(p-1)} \ge \theta(2|x| - R_0)(2|x| - R_0)^{-\varrho\,\alpha(p-1)}$, by hypothesis. Therefore, we can estimate $\beta(y)$ as

$$\beta(y) \le -\theta(2|x| - R_0)(2|x| - R_0)^{-\varrho\,\alpha(p-1)} = -\beta_0\ .$$

We then apply Lemma 2.1 to obtain

$$f(u(x)) \leq \frac{C}{(|x| - R_0)^{\varrho}} \beta_0^{-1/\alpha(p-1)} \leq C\,\theta(2|x| - R_0)^{-1/\alpha(p-1)},$$

for large $|x|$. Thus, as $|x| \to \infty$ we see that $f(u(xx)) \to 0$. Now let $v := f(u)$. First we point out that

$$\begin{aligned} 0 \leq g(x) &= \Delta_p f^{-1}(v) + \beta(x)v \\ &= ((f^{-1})'(v))^{p-1}\ \Delta_p v + (p-1)((f^{-1})'(v))^{p-2}(f^{-1})''(v)|v|^2 + \beta(x)v, \end{aligned}$$

so that

$$((f^{-1})'(v))^{p-1}\Delta_p v \geq -(p-1)((f^{-1})'(v))^{p-2}(f^{-1})''(v)|v|^2 - \beta(x)v.$$

Since $(f^{-1})' > 0$, $(f^{-1})'' \leq 0$ by (*), $\beta(x) \leq 0$, and $v \geq 0$ by hypothesis, we conclude that $\Delta_p v \geq 0$. Therefore, by the Maximum Principle we obtain

$$\max_{B} \leq \max_{\partial B} v$$

for any ball $B \subseteq \mathbb{R}^n$. thus, if $x \in \mathbb{R}^n$ and R is large enough such that $x \in B(0, R)$, we have

$$0 < f(u(x)) \leq \max\{f(u(y)) : |y| = R\}.$$

Since $f(u(y)) \to 0$ as $|y| \to \infty$, we have a contradiction. Therefore, a solution u to Equation (1.1) cannot exist under the prescibed conditions. This establishes Theorem 2.2.

Remark 2.3. For $p = 2$, this result was obtained by Oleinik (1978), with $f(s) = e^s$, and by Lin (1985) with $f(s) = |s|^q$.

3. The case when $\beta(x) \geq 0$ and $g(x) \leq 0$

Theorem 3.1. *Suppose $\beta(x) \geq 0$ on $B(0, R_0)$ and that $\beta(x) \geq C_a|x|^{-a}$ for all $|x| \geq R_0$, for some constants $C_a > 0$, $a \in \mathbb{R}$, and $R_0 \geq 1$. Suppose further that f satisfies (*), $g(x) \leq 0$ on $\mathbb{R}^n$, and that $p = 2$. Then Equation (1.1) has no entire solution u satisfying the condition*

$$u(x) \geq f^{-1}(C_b|x|^{-b}), \quad \forall\ x \in \mathbb{R}^n \quad \text{with} \quad |x| \geq R_0, \tag{3.1}$$

for some constant $C_b > 0$, and some exponent b in $\mathbb{R}$, such that $a + \alpha b < 2$.

Proof. Upon multiplying Equation (1.1) by $f(u)^{-1}$, and integrating it on the ball $B := B(0, R)$ we find that

$$\int_B (f(u)^{-1}\ \Delta u + \beta(x))dx \leq 0.$$

Let us note that $div(f(u)^{-1}\ \nabla u) = -f(u)^{-2}\ f'(u)|\nabla u|^2 + f(u)^{-1}\ \Delta u$. Using this fact. Green's Theorem, and the the previous integral, we obtain

$$\int_B \left(\frac{f'(u)}{f(u)^2}|\nabla u|^2 + \beta(x)\right) dx \leq -\int_{\varrho B} f(u)^{-1}\ \nabla u\ .\ \eta\, d\sigma \leq \int_{\varrho B} \frac{|\nabla u|}{f(u)} d\sigma,$$

where η is an outward unit normal vector to the sphere $\partial B(0, R)$. Now, by Hölder's Inequality, we have the following estimate concerning the last surface integral,

$$\int_{\partial B} \frac{|\nabla u|}{f(u)} d\sigma \leq \left(\int_{\partial B} \frac{f'(u)}{f(u)^2}|\nabla u|^2 d\sigma\right)^{1/2} \left(\int_{\partial B} (f'(u))^{-1} d\sigma\right)^{1/2}$$

$$= \left[\left(\int_{\partial B} \frac{f'(u)}{f(u)^2}|\nabla u|^2 d\sigma\right)^{1/2} \left(\int_{\partial B} \beta(x) d\sigma\right)^{1/2}\right] \times$$

$$\left[\left(\int_{\partial B} (f'(u))^{-1} d\sigma\right)^{1/2} \left[\int_{\partial B} \beta(x) d\sigma\right)^{1/2}\right].$$

Moreover, using the Cauchy-Schwarz Inequality, together with conditions in (*), and those given in Equation (3.1), we obtain, for $R \geq R_0$,

$$\int_B \left(\frac{f'(u)}{f(u)^2}|\nabla u|^2 + \beta(x)\right) dx \ \leq \ \frac{1}{2}\left(\int_{\partial B} (\frac{f'(u)}{f(u)^2}|\nabla u|^2 + \beta(x)) d\sigma\right) \times$$

$$\left(\int_{\partial B} ((f^{-1})'(C_b^{-1}|x|^b))^{-1} d\sigma\right)^{1/2} \times (C_a^{-1} R^a)^1 |\partial B|^{-1/2} \leq$$

$$\frac{1}{2}\left(\int_{\partial B} (\frac{f'(u)}{F(u)^2}|\nabla u|^2 + \beta(x)) d\sigma\right) (\mu C_a^{-1} C_b^{-\alpha} R^(a + \alpha b))^{1/2}.$$

Thus, we get the differential inequality

$$F(R) \leq \gamma R^{(a+\alpha b)/2} F'(R), \tag{3.2}$$

where

$$F(R) := \int_{B(0,R)} \left(\frac{f'(u)}{f(u)^2} |\nabla u|^2 + \beta(x) \right) dx, \quad \text{and} \quad \gamma := \frac{1}{2} (\mu^{-1} C_a C_b^{\alpha})^{-1/2}.$$

Let us fix some $R^* > R_0$. Multiplying both sides of (3.2) by

$$-\gamma^{-1} R^{-a+\alpha b)/2} \Psi(R),$$

where

$$\Psi(R) := \exp\left(-\gamma^{-1} \int_{R^*}^{R} t^{-(a+\alpha b)/2} dt \right),$$

we obtain $\Psi'(R)F(R) \geq -\Psi(R)F'(R)$, or $(\Psi(R)F(R))' \geq 0$. Integrating this inequality on (R^*, R). we get

$$F(R^*) \leq \Psi(R)F(R), \qquad R_0 < R^* < R. \tag{3.3}$$

We now estimate $F(R)$ for $R > R^*$. To this end, let us pick $0 \leq \phi \in C^\infty(\mathbb{R}^n)$ with $\text{supp}(\phi) \subseteq B(0, R+1)$, $\phi \equiv 1$ on $B(0, R)$ and satisfying $|\nabla \phi|^2 \leq M \phi$ for $f(u)^{-1} \phi$ and integrating the resulting expression we find that

$$\int_{B(0,R+1)} \left(\frac{f'(u)}{f(u)^2} |\nabla u|^2 \phi - f(u)^{-1} \ \nabla u \ . \ \nabla \phi + \beta(x)\phi - g(x)\, f(u)^{-1} \phi \right) dx = 0.$$

Proceeding in the manner established above, we find

$$\int_{B(0,R+1)} \left(\frac{f'(u)}{f(u)^2} |\nabla u|^2 + \beta(x) \right) \phi \, dx \leq \int_{B(0,R+1)} f(u)^{-1} |\nabla u| |\nabla \phi| dx$$

$$\leq \frac{1}{2} \int_{B(0,R+1)} \frac{f'(u)}{f(u)^2} |\nabla u|^2 \phi \, dx + \frac{1}{2} \int_{B(0,R+1)} \frac{|\nabla \phi|^2}{f'(u)} \phi^{-1} dx$$

so that

$$\begin{aligned}\int_{B(0,R+1)}\left(\frac{f'(u)}{f(u)^2}|\nabla u|^2+\beta(x)\right)\phi\,dx &\leq \int_{B(0,R+1)}\frac{|\nabla\phi|^2}{f'(u)}\phi^{-1}dx\\ &\leq M\int_{B(0,R+1)}\frac{1}{f'(u)}dx \leq CR^{\alpha b+n}\end{aligned}$$

Once more, we have used conditions (*) and (3.1) in the last inequality. Recalling that $\phi \equiv 1$ on $B(0,R)$, the last inequality gives us the estimate

$$F(R) \leq CR^{\alpha b+n}.$$

This together with the estimate in Equation (3.3) gives $F(R^*) \leq CR^{\alpha b+n}\,\Psi(R)$ for all $R \geq R^*$. Thus

$$\int_{B(0,R^*)}\beta(x)dx \leq CR^{\alpha b+n}\,\Psi(R), \qquad \text{for all} \quad R \geq R^*.$$

But $R^{\alpha b+n}\,\Psi(R) \to 0$ as $R \to \infty$ provided that $a+\alpha b<2$. this contradicts the fact that β is nonzero on $\mathbb{R}^n$, thus completing the proof.

In the next theorem, we show the nonexistence of entire redial solutions to Equation (1.1) that satisfy certain conditions. This we do for any $p>1$. To this end, we introduce the following notation,

$$H_k(s)=\begin{cases}\frac{p-k}{p-1}s^{(p-1)/(p-k)} & if \;\; k\neq p\\ \ln s & if \;\; k=p\end{cases}, \qquad s>0$$

We then have the following result.

Theorem 3.2. *Suppose $\beta(x)\geq 0$ on $B(0,R_0)$ and that $\beta(x)\geq C_a|x|^{-a}$ for all $|x|\geq R_0$, for some constants $C_a>0$, $a\in\mathbb{R}$, and $R_0\geq 1$. Suppose also that f satisfies the following conditions:*

$$u(x)\geq f^{-1}(C_b|x|^{-b}), \qquad \forall\, x \;\; \text{with} \;\; |x|\geq R_0, \tag{3.4}$$

for some constant $C_b>0, b$ a real exponent with $b+a<n$, and such that

$$\lim_{R\to\infty}\left(f^{-1}(C_bR^{-b})+\left(\frac{C_bC_b}{n-(a+b)}\right)^{1/(p-1)}H_{a+b}(R)\right)=\infty. \tag{3.5}$$

Then, when $g(x) \leq 0$ on $\mathbb{R}^n$, no entire radial solution of Equation (1.1) can be found.

Proof. Let u be a radial solution of Equation (1.1). Then, it is well known that u satisfies the ODE

$$(r^{n-1}|u'|^{p-2}u')' + r^{n-1}\beta(x)f(u) = r^{n-1}g(x), \qquad u'(0) = 0.$$

From this we note that $u'(r) < 0$ for all $r > 0$. Thus, the above equation can be written as

$$-(r^{n-1}(-u'^{p-1})' + r^{n-1}\beta(x)f(u) = r^{n-1}g(x), \quad u'(0) = 0.$$

By condition (3.4) along with the hypothesis on $\beta(x)$ and $g(x)$ we conclude that

$$-(r^{n-1}(-u')^{p-1})' + C_aC_br^{n-1-(a+b)} \leq 0, \qquad u'(0) = 0.$$

Integrating this on $(0, r)$ and rearranging leads to

$$-u'(r) \geq r^{(1-n)/(p-1)}\left(\int_0^t s^{n-1-(a+b)}ds\right)^{1/(p-1)}.$$

Integrating once more on (r, R) we find

$$u(R) + (C_aC_b)^{1/(p-1)} \quad \int_0^r t^{(1-n)/(p-1)} \int_0^t s^{n-1-1(a+b)}dx\Big)^{1/(p-1)} \quad dt \leq u(r).$$

Once again using condition (3.4), and recalling that $a + b < n$ we obtain:

$$f^{-1}(C_bR^{-b}) + \left(\frac{C_aC_b}{n-(a+b)}\right)^{1/(p-1)}(H_{a+b}(R) - H_{a+b}(r)) \leq u(r), \quad r < R.$$

In view of condition (3.4), this last inequality does not hold for any $R > r$.

Remark 3.3. If $p = 2$, then the word "radial" can be removed from the statement of Theorem 3.2.

This can be seen as follows. For $u \in C(\mathbb{R}^n)$, we denote by $\bar{u}$ the spherical mean of u, namely:

$$\bar{u}(r) := \frac{1}{\omega_n r^{n-1}}\int_{|x|=r} u(y)d\sigma, \qquad \text{for } \; r > 0, \qquad \text{and} \qquad \bar{u}(0) = u(0).$$

By taking the spherical mean of both sides of Equation (1.1), we obtain the following Cauchy problem:

$$\Delta\bar{u} + \bar{G}(r) = 0, \quad \bar{u}(0) = u(0), \quad \bar{u}'(0) = 0,$$

where $G(x) := \beta(x)\, f(u(x)) - g(x) \geq 0$. That is

$$(r^{n-1}\bar{u}')' + r^{n-1}\bar{G} = 0, \quad \bar{u}(0) = u(0), \quad \bar{u}'(0) = 0.$$

Proceeding as in the previous proof we recover the conclusion of Theorem 3.2.

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عدم وجود حلول تامة لبعض المعادلات اللاخطية الناقصية

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خلاصة

في هذا البحث نعتبر المعادلات الناقصية التي على الصورة

$$div\Big(|\nabla u|^{p-2}\ \nabla u\Big) + \beta(x)\, f(u) = g(x), \quad p > 1$$

حيث نبرهن أنه لا يوجد حلولاً تامة لهذه المعادلات وذلك ضمن الشروط على β و g وبفرض أن الدالة f ليست خطية. بعض الأمثلة النموذجية للدالة f تشمل الدالة الأسية ودالة القوة. تعتبر النتائج التي حصلنا عليها تعميماً لبعض النتائج المعروفة سابقاً عندما $p = 2$.