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A numerical approach to depth determination from magnetic data

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ABSTRACT

We have developed a numerical approach to depth determination from magnetic data. By defining a few characteristic points and distances on the profile, the problem of depth determination from magnetic data has been transformed into finding a solution to a nonlinear equation of the form $f(z) = 0$. Formulas have been derived which are applicable to a sphere, horizontal cylinder, dike, and a geologic contact. Procedures have also been formulated to estimate the effective magnetization intensity and the effective magnetization inclination. The method is applied to synthetic data with and without random errors. Errors in the model parameters dut to errors in the characteristic points and distances were also studied through imposing $\pm 1 - 10\%$ errors on these parameters on a synthetic profile due to a thin dike. Finally, the method is applied to two field examples from Brazil and India. In both cases examined, the depth parameter is found to be in good agreement with that determined by other methods.

INTRODUCTION

The thin dike, sphere, horizontal cylinder and geologic contact models are widely used in megnetic interpretation to find the depth, and the magnitude and direction of magnetization of intrusive igneous rocks which carry magnetic minerals and that may be associated with minerals of economic interest. Several graphical methods have been developed for interpreting residual magnetic anomalies due to simple models (Gay 1963 & 1965, Paul 1964, Radhakrishna Murthy 1967, Stanley 1977, Atchuta Roa & Babu 1980, Prakasa Rao *et al.* 1986, Prakasa Rao & Subrahmanyam 1988). Numerical methods such as Werner deconvolution (Werner 1953, Hartman *et al.* 1971) and Euler deconvolution (Hood 1965, Thompson 1982, Reid *el al.* 1990) are frequently used. In these methods. The depth determination problem is transformed into a problem of finding a solution of a system of linear equations.

In this paper, a numerical approach is presented to estimate the depth of buried models from magnetic data. Using previously published formulas for the magnetic anomalies caused by these models, the depth determination problem has been parameterized and transformed into a problem of finding a solution for a nonlinear equation of the form $f(z) = 0$. The accuracy of the result obtained by this method depends upon the accuracy to which the residual magnetic anomaly is separated from the observed magnetic data. The method has been applied to synthetic data with and without random errors, and is tested on two field exampels from Brazil and India.

THE METHOD

Following Abdelrahman and Hassanein (2000), the magnetic anomaly expression produced by most geologic structures with the center located at $x_i = 0$ can be represented by the following function:

$$T(x_i, z, \theta, q) = K\frac{(az^{2r} + bx_i^2)(\sin\theta)^m(\cos\theta)^n + cx_i z^p(\sin\theta)^n(\cos\theta)^m}{(x_i^2 + z^2)^2}, i = 1, 2, 3, \ldots, N \quad (1)$$

where z is the depth, K is the effective magnetization intensity, θ is the effective magnetization inclination in the vertical plane perpendicular to the strike of the structure, x_i is the horizontal coordinate position, and q is the shape factor. Values for a, b, c, m, n, r, p, and q are given in Table 1.

Table 1. Definition of a, b, c, m, n, p, r, and q values shown in equation (1). F.H.D. and S.H.D. are the first and the second horizontal derivatives of the magnetic anomaly, respectively.

Model	Magnetization	a	b	c	m	n	p	r	q
Sphere	Vertical	2	-1	-3	1	0	1	1	2.5
Sphere	Horizontal	-1	2	-3	0	1	1	1	2.5
Horizontal cylinder Dike (F.H.D.) Geologic contact (S.H.D.)	Total, vertical, horizontal	1	-1	2	0	1	1	1	2
Dike	Total, vertical, horizontal	1	0	1	0	1	0	0.5	1

At the origion ($x_i = 0$), equation (1) gives the following relationship:

$$K = \frac{T(0)z^{2q-2r}}{a(\sin\theta)^m(\cos\theta)^n}, \quad (2)$$

where $T(0)$ is the anomaly value at the origin (Fig. 1).

Using equation (2), equation (1) can be rewritten as

$$T(x_i, z, \theta, q) = \frac{T(0)z^{2q-2r}}{a}\left[\frac{az^{2r} + bx_i^2 + cx_i z^p(\tan\theta)^{n-m}}{(x_i^2 + z^2)^q}\right]. \quad (3)$$

For all shapes (function of q), equation (3) gives the following relationship at $x_i = N$

$$(\tan\theta)^{n-m} = \frac{aT(N)(N^2+z^2)^q - T(0)z^{2q-2r}(az^{2r}+bN^2)}{cNz^pT(0)z^{2q-2r}} \tag{4}$$

where $T(N)$ is the anomaly value at the $x_i = N$ (Fig. 1).

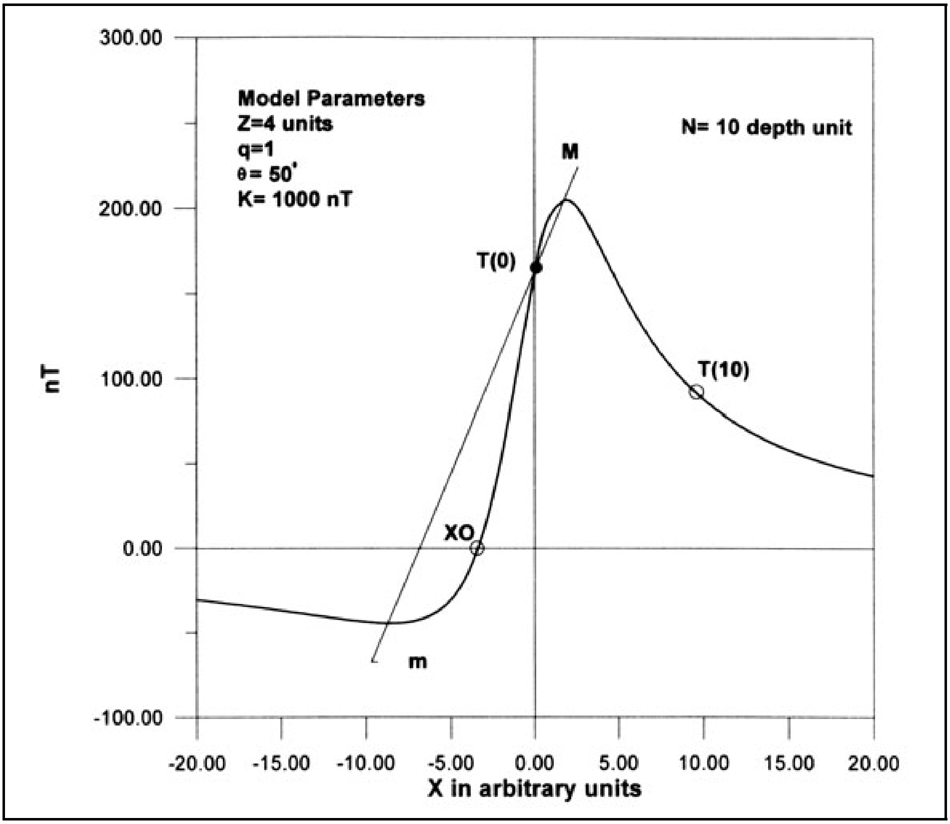

Figure 1. A typical magnetic anomaly profile over a thin dike. The anomaly value at the origin $T(0)$, the zero-anomaly distance x_0, the anomaly value $T(N)$, the position of the maximum value (M) and the minimum value (m) are illustrated. (After Stanley 1977.)

Substituting equation (4) into equation (3), we obtain the following nonlinear equation in z

$$T(x_i, x, q) = \frac{NT(0)z^{2q-2r}(az^{2r}+bx_i^2) + ax_iT(N)(N^2+z^2)^q - x_iT(0)z^{2q-2r}(az^{2r}+bN^2)}{aN(x_i^2+z^2)^q} \tag{5}$$

Setting equation (5) to zero, we obtain the following nonlinear equation in z:

$$z = \left[\frac{(ax_0T(0)z^{2q} + bN^2x_0T(0)z^{2q-2r} - bx_0^2NT(0)z^{2q-2r} - ax_0T(N)(N^2+z^2)^q)}{aNT(0)}\right]^{1/2q}, \quad (6)$$

where x_0 is the nearest zero-anomaly distance to the origin on the anomaly profile (Fig. 1).

Equation (6) can be solved for z using the standard methods for solving nonlinear equations. Here, it is solved by an iteration method (Press *et al.* 1986). The iteration form of equation (6) is given as

$$z_f = f(z_j), \quad (8)$$

where z_j is the initial depth and z_f is the revised depth. z_f will be used as z_j for next iteration. The iteration stops when $|z_f - z_j| \leq e$, where e is a small predetermined real number close to zero.

Once z is known, the effective magnetization inclination, θ, can be determined from equation (4). Finally, knowing θ, the effective magnetization intensity, K, can be determined from equation (2).

To this point, we have assumed knowledge of the axes of the magnetic profile so that $T(0)$ and x_0 can be determined. Otherwise, $T(0)$ and x_0 can be determined using methods described by Stanley (1977). As illustrated in Fig. 1, the line $M - m$ intersects the anomaly profile at $x_i = 0$. The base line of the anomaly profile lies a distance $M - T(0)$ above the minimum.

THEORETICAL EXAMPLES

Effect of random noise

Numerical results for various cases are shown in Tables 2 and 3. In each case, a search algorithm was used to determine x_0 based on linear interpolations between the observed values (Davis 1973). The search algorithm was needed to find x_0 because the true x_0 value might change due to adding random errors to the observed values. It was verified numerically that equation (6) gives accurate values of z when synthetic data with or without random noise are analyzed. The depth obtained is within ±3.5% (Tables 2 and 3). It was also shown that equations (2) and (4) give reliable values of K and θ (Tables 2 and 3). this illustrates that our method is less sensitive to errors in the magnetic anomaly than Werner and Euler deconvolution methods and therefore noisy data will still give reliable depth estimates.

Table 2. Synthetic examples for depth determination for a horizontal cylinder. Other model parameters are: $\theta = -60°$, $K = 100$ units, profile length = 100 units, sampling interval = 1 unit, $N = 10$ depth unit.

Model Depth (units)	**Using synthetic data**	**% error in z**	**% error in θ**	**% error in K**	**Using data with 10% random error**	**% error in z**	**% error in θ**	**% error in K**
2.0	2.05	2.47	-5.60	-4.55	2.04	1.93	-6.44	-5.96
2.5	2.53	1.23	-2.35	-1.68	2.54	1.73	-2.13	0.35
3.0	3.02	0.53	-0.87	-0.51	3.04	1.32	-0.91	3.84
3.5	3.50	0.13	-0.18	-0.07	3.44	-1.84	-1.22	-3.70
4.0	4.02	0.49	-0.62	-0.15	3.97	-0.71	-1.00	-2.36
4.5	4.54	0.88	-0.99	-0.03	4.64	3.19	0.30	5.18
5.0	5.05	0.93	-0.93	0.18	5.16	3.15	0.54	5.46
5.5	5.54	0.81	-0.72	0.33	5.63	2.40	-0.34	5.24
6.0	6.04	0.63	-0.49	0.38	6.05	0.89	-0.24	0.62
6.5	6.53	0.42	-0.29	0.33	6.55	0.83	-0.15	4.95
7.0	7.01	0.20	-0.12	0.19	6.83	-2.43	-1.39	-4.40
7.5	7.50	0.02	-0.01	0.02	7.53	0.42	0.18	0.29
8.0	8.03	0.34	-0.14	0.42	7.86	-1.69	-0.90	-5.56
8.5	8.54	0.49	-0.18	0.68	8.65	1.82	0.09	0.23
9.0	9.05	0.54	-0.15	0.79	9.30	3.38	1.46	5.90
9.5	9.55	0.49	-0.11	0.78	9.19	-3.26	-1.79	-5.04
10.0	10.00	0.39	-0.06	0.66	9.91	-0.89	-0.89	-1.49

Table 3. Synthetic examples for depth determination for a sphere. Other model parameters are: $\theta = -60°$, $K = 100$ units, profile length = 100 units, sampling interval = 1 unit, $N = 10$ depth unit.

Model Depth (units)	Using synthetic data	% error in z	% error in θ	% error in K	Using synthetic data with 10% random error	% error in z	% error in θ	% error in K
2.0	2.04	1.94	8.57	1.11	2.00	0.16	10.1	-2.26
2.5	2.50	0.08	0.30	0.08	2.56	2.39	-1.50	5.05
3.0	3.05	1.58	4.46	2.17	3.06	1.90	4.42	6.89
3.5	3.53	0.77	1.80	1.26	3.56	1.75	1.03	1.32
4.0	4.03	0.84	1.57	1.58	4.02	0.52	1.75	2.73
4.5	4.55	1.04	1.57	2.19	4.50	0.02	2.55	-2.67
5.0	5.01	0.11	0.13	0.26	5.15	2.94	-2.09	5.90
5.5	5.55	0.97	0.91	2.39	5.54	0.69	1.27	1.12
6.0	6.04	0.64	0.45	1.66	6.05	0.88	0.30	4.61
6.5	6.54	0.56	0.28	1.53	6.52	0.33	0.48	2.34
7.0	7.07	0.94	0.28	2.68	6.98	-0.31	1.19	-0.42
7.5	7.51	0.16	0.02	0.47	7.60	1.39	-0.95	4.68
8.0	8.08	0.98	0.01	2.97	7.97	-0.32	1.22	2.28
8.5	8.57	0.78	-0.11	2.44	8.66	1.94	-0.99	5.86
9.0	9.05	0.59	-0.15	1.88	9.27	2.99	-2.01	8.10
9.5	9.63	1.32	-0.48	4.31	9.72	2.37	-1.29	6.94
10.0	10.0	0.31	-0.13	1.02	9.98	-0.21	0.31	-2.93

Effect of error in $T(0)$ and x_0

In studying the error response of the iterative method, synthetic example of a dike (profile length = 100 depth units, $z = 3$ units, $\theta = -45$ degrees, $K = 100$ units and sampling interval = 1 unit) were considered in which errors of $\pm1, \pm2, \pm3 \ldots \pm$ 10% were imposed in both $T(0)$ and x_0 when $T(N)$ has errors of 0, +10, and -10% respectively. Following the interpretation method, and using $N = 10$ unit, values of the depth parameter (z) were computed and the percentage of errors in the depth parameter were mapped (Figs. 2, 3, and 4).

In cases where in $T(N)$ is noise free and x_0 and $T(0)$ both have errors of equal magnitude and of the same signs simultaneously, the interpreted depth will not differ much from the true values (Fig. 2). On the other hand, when $T(0)$ and x_0 possess errors of equal magnitude of opposite signs simultaneously, the interpreted depth values will vary widely from the actual values. The maximum error in depth is ±10%.

In cases where $T(N)$ has +10% error, the maximum error in depth is about +18% when x_0 and $T(0)$ both have errors of +10% and -10%, respectively, (Fig. 3). Finally, in cases where $T(N)$ has -10% error, the maximum error in depth is about -16% when x_0 and $T(0)$ both have errors -10% and +10%, respectively, (Fig. 4). In all cases examined, the percentage error in depth is smaller than the sum of errors imposed on $T(N)$, $T(0)$, and x_0.

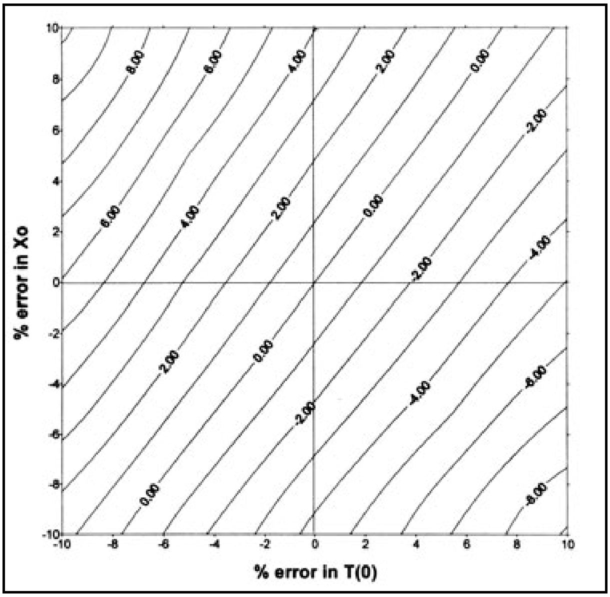

Figure 2. A map showing error response in depth estimates when $T(N)$ is noise free, $N = 10$ depth units. Abscissa: percentage of error imposed in $T(0)$. Ordinate: Percentage of error imposed in x_0. $C.I. = 1\%$.

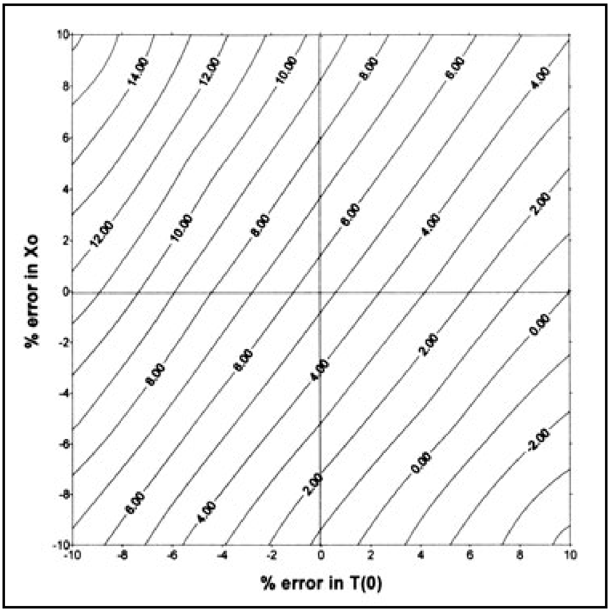

Figure 3. A map showing error response in depth estimates when $T(N)$ has $+10\%$ error, $N = 10$ depth units. Abscissa: percentage of error imposed in $T(0)$. Ordinate: Percentage of error imposed in Xo. $C.I. = 1\%$.

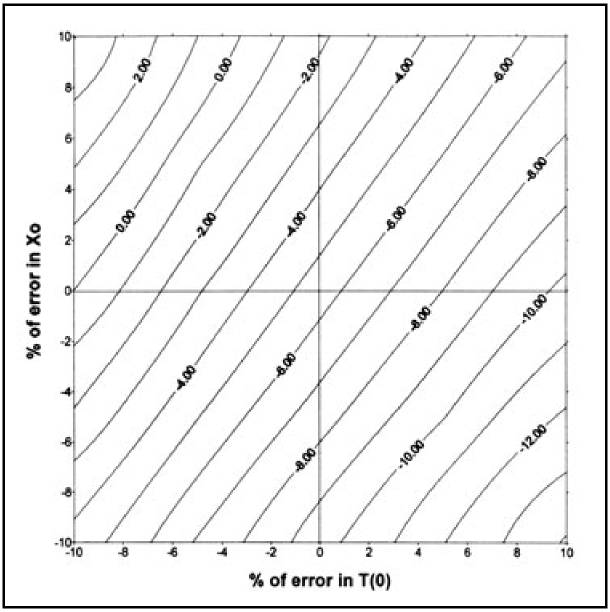

Figure 4. A map showing error response in depth estimates when, $T(N)$ has -10% error, $N = 10$ depth units. Abscissa: percentage of error imposed in $T(0)$. Ordinate: Percentage of error imposed in x_0. $C.I. = 1\%$.

FIELD EXAMPLES

To examine the applicability of the present method, the following two field examples are presented.

Parnaiba dike anomaly

Figure 5 shows a total magnetic anomaly above a Mesozoic diabase dike intruded into Paleozoic sediment from the Parnaiba Basin, Brazil (Sliva 1989, Fig. 10). The depth to the outcropping dike is 1.9m. The zero crossings were determined using Stanley's (1977) method. This anomaly profile of 26.4m length was digitized at an interval of 2.2m. The values of $T(0)$ and x_0 used in the iterative method are 42 nT and -5.94m, respectively. Equations (6), (4), and (2) were used to determine the depth, the effective magnetization inclination, and the effective magnetization intensity, respectively, using all possible cases of N (Table 4).

Table 4. Interpreted model parameters as computed from Parnaiba dike anomaly, Brazil.

N (in units)	Computed depth using equation (6) (m)	Computed θ using equation (4) (degrees)	Computed K using equation (2) (in units)	Sum-squared error $((nT)^2)$
-5	3.52	30.66	-78.23	73.11
-4	4.36	36.30	-103.50	375.20
-3	6.27	46.55	-174.30	2478.00
1	4.05	34.26	-93.56	207.30
2	3.50	30.49	-77.54	71.51
3	3.37	29.55	-73.95	69.54
4	3.19	28.35	-69.57	83.47
5	3.15	27.92	-68.08	94.21
6	**3.34**	**29.60**	**-74.20**	**68.76**

The goodness of fit was measured between the observed and the calculated magnetic data for each N. The simplest way to compare two magnetic profiles is to compute the sum squared error. The model parameters which give the least sum-squared error are the best. Table 4 shows that the best-fit model parameters are $z = 3.34$m, $\theta = 29.6$ degrees, and $K = -74.2$ units (Fig. 5). Sliva (1989) applied a method to the same magnetic data implying $M-$ fitting technique and obtained a depth of about 3.5m. The depth to the top is overestimated by the two methods. This is reasonable because the upper part of the dike was weathered and the magnetite present was oxidized, losing most of its magnetization (Silva 1989).

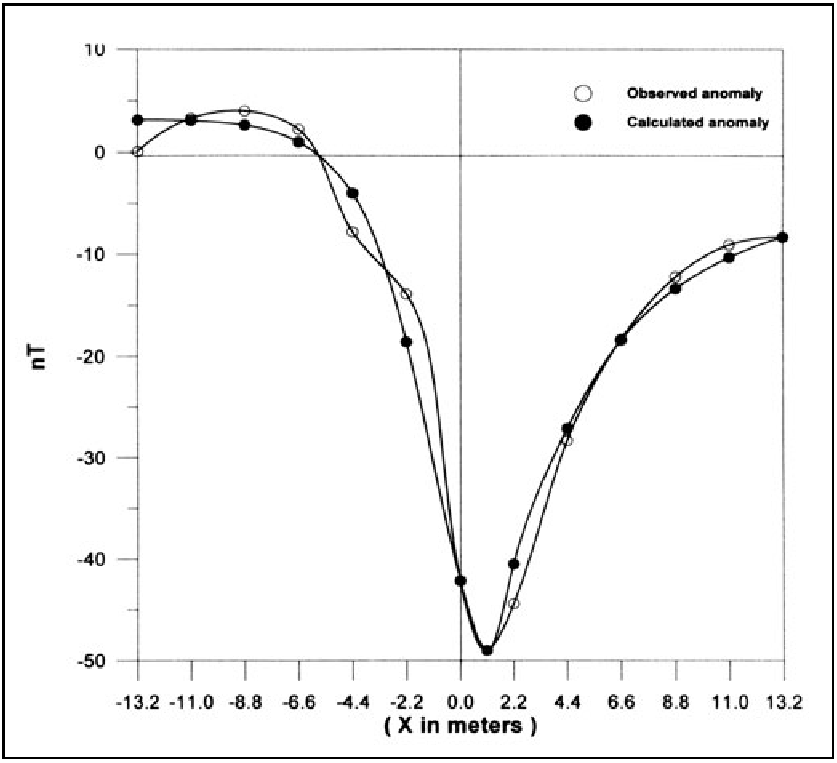

Figure 5. Total magnetic anomaly over an outcropping dike in the Parnaiba Basin, Brazil (Silva 1989). The base line and the zero crossings are determined using Stanley's (1977) method.

Bankura anomaly

Figure 6 shows a vertical anomaly profile from the Bankura area, West Bengal, India (Verma & Bandopadhyay 1975). It represents a vertical anomaly due to a spherical mass of gabbroic composition. The principal profile shown in Fig. 6 was digitized at an interval of 0.42km. The values of $T(0)$ and x_0 used in the present method are 1105 nT and 1.05km, respectively. Applying the same interpretive technique shown above, the model parameters determined are: $z = 1.47$km, $\theta = 55$ degrees, and $K = 29082$ units (Table 5 and Fig. 6). The depth agrees well with those obtained in Verma and Bandopadhyay (1975), Rao *et al.* (1973), Praksa Rao & Subrahmanyan (1988), and Abdelrahman (1994) (Table 6).

Table 5. Interpreted model parameters as computed from Bankura anomaly, West Bengal, India.

N (in units)	Computed depth using equation (6) (km)	Computed θ using equation (4) (degrees)	Computed K using equation (2) (in units)	Sum-squared error $((nT)^2)$
-5	1.47	55.10	28997	2185
-4	1.47	55.11	28978	2197
-3	**1.47**	**55.00**	**29082**	**2040**
-2	1.48	54.88	29485	2173
-1	1.50	54.24	30988	3433
3	2.07	48.34	50627	501264
4	1.41	53.48	32964	235971
5	1.49	54.43	30567	3138

Table 6. comparative results of Bankura field example.

	Using Verma & Bandopadhyay method (1975).	Using Rao *et al* method (1973).	Using Parkasa Rao & Subrahmanyan method (1988).	Using Abdelrahman method (1994).	Using present method.
Depth	1.32km	1.32km	1.52km	1.41km	1.44km

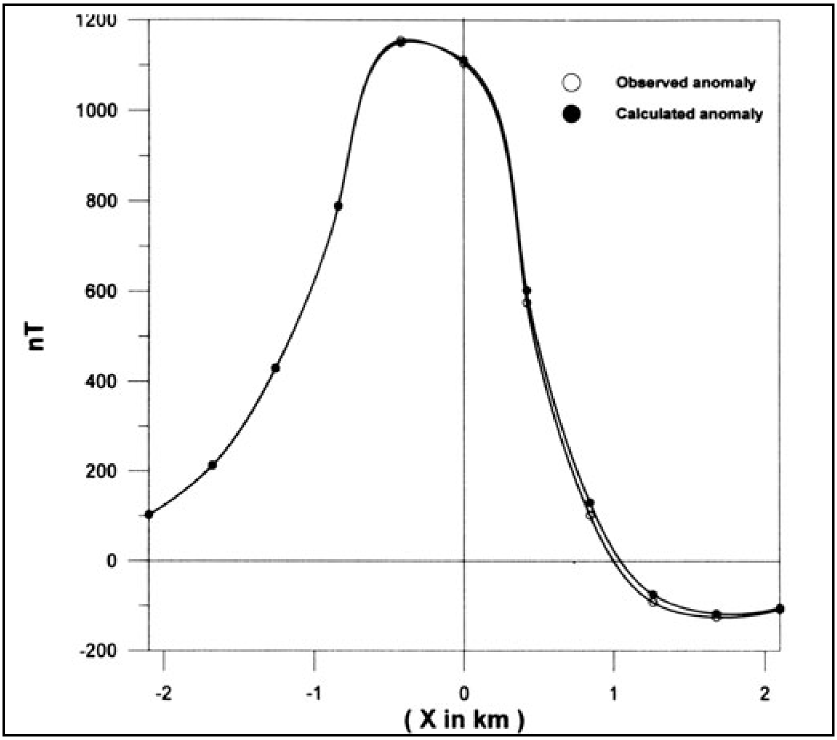

Figure 6. Vertical magnetic profile over a spherical future in the Bankura area, West Bengal, India. The base line and the zero crossing shown are the same as those given by Verma & Bandopadhyay (1975).

CONCLUSIONS

The problem of determining the depth of a buried structure from the magnetic anomaly has been transformed into a problem of finding a solution to a nonlinear equation. The method presented is very simple to execute. The advantages of the present method over previous techniques, that also use few points and distances, standardized curves, and nomograms are: (1) the method is automatic, and (2) the method may be less sensitive to errors in the magnetic anomaly.

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طريقة عددية لتعيين العمق من الشذات المغناطيسية

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خلاصة

نقدم في هذا البحث طريقة عددية لإيجاد العمق للتراكيب الجيولوجية من البيانات المغناطيسية بمعلومية بعض النقط المميزة على طول البروفيل وقد تم تحويل مشكلة تعيين العمق إلى إيجاد حل لمعادلة غير خطية د (ع) = صفر. وقد أمكن إعطاء معادلة لجميع المركبات (الرأسية والأفقية والكلية) لجسم أسطواني أفقي ولجدة رفيعة والمركبات الرأسية والأفقية للكرة، وقد تم كذلك استنباط صيغة لتقدير قوة كل من الميل والعزم المغناطيسيين لجميع التراكيب الجيولوجية.

تم تطبيق الطريقة على نماذج نظرية بها خطأ عشوائي بنسبة 10٪ وبدون خطأ عشوائي. كما تم تطبيق الطريقة على مثالين حقليين من البرازيل والهند ووجد أن العمق للخامات الجيولوجية والمقدر بالطريقة الجديدة يتفق مع نتائج الحفر والنتائج التي توصل إليها العديد من العلماء السابقين.