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Effects of bridge deck roughness on the dynamic response of single span bridges

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ABSTRACT

A single span simply supported beam model is used to investigate the maximum acceleration and maximum deflection responses of single span bridges due to a moving vehicle. The vehicle is modeled as a single degree of freedom sprung mass moving with a constant speed. The dynamic responses of the beam due to varying the natural frequency, damping ratio, unevenness height and length, vehicle speed, weight and spring stiffness are studied. The effect of each parameter is shown and analyzed. Recommendations are given for consideration in real life practice.

INTRODUCTION

The effect of roughness on bridge response was considered by Fryba (1992) and Inbanathan & Wieland (1987). They found that roughness magnified the dynamic response due to moving loads. A study to control the effect of unevenness for simply supported bridges was made by Abdel-Rohman & Leipholz (1980). Nonlinear oscillations of single-span hinged bridges with hinged supports due to a moving concentrated load on a smooth deck, and methods to control these oscillations, were studied by Abdel-Rohman & Nayfeh (1987a,b). The nonlinearity can produce responses with amplitudes higher or lower than the linear response at specific forced frequencies.

Roughness in bridge decks exists in all bridges due to braking action of vehicles, wearing of the pavement or due to moving heavy vehicles with varying speeds as outlined by Coussy, Said, & Van Hoove (1989). Humar & Kashif (1993) identified the controlling parameters that govern the dynamic response of bridges due to moving vehicle loads as speed, vehicle weight, bridge stiffness, etc. The effect of bridge deck roughness on the dynamic response was investigated by Abdel-Rohman & Al-Duaij (1996), and it was shown that roughness increased dynamic response. The dynamic responses, considering the effect of vehicle and bridge parameters, were obtained and compared between simply supported and hinged supported bridges. Maximum deflection responses and the effect of varying parameters was studied by Al-Duaij & Abdel-Rohman (1997). The roughness parameter was the most

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significant on the deflection amplitude. Maximum acceleration responses due to the effects of varying parameters were also studied by Al-Duaij *et al.* (1999), who also reported that roughness had a significant effect on the maximum acceleration amplitude.

In this study the maximum dynamic responses due to roughness parameters in the bridge deck are shown. The maximum acceleration and maximum deflection for single-span simply-supported steel bridges with smooth or rough decks for various parameters are shown in order to determine the influence of these parameters on the maximum dynamic responses.

EQUATION OF MOTION OF SINGLE SPAN BRIDGE WITH A SMOOTH DECK

The steel bridge is modeled as a simply-supported beam as shown in Fig. 1(a). Considering a single concentrated load moving with a constant speed, the equation of motion is given by:

$$mW_{tt} + cW_t + EIW_{xxxx} = P\delta(x - vt) \tag{1}$$

where P is the load due to the moving vehicle; $W(x, t)$ is the transverse deflection at distance x; W_x is the partial derivative of W with respect to x; W_t is the partial derivative of W with respect to the time t; v is the speed of the vehicle; m is the bridge mass per unit length; EI is the flexural rigidity; c is the damping in the bridge; and $\delta(\text{-})$ is a Dirac-delta function.

The solution of Eq. 1 is assumed in the form

$$W(x, t) = \sum_{n=1}^{\infty} \eta_n(t) \sin\left(\frac{n\pi x}{L}\right) \tag{2}$$

in which $\eta_n(t)$ is the generalized coordinate of vibration mode n; l is the span; and $\sin(n\pi x/L)$ is the shape of this mode which satisfies the boundary conditions.

Substituting Eq. 2 into Eq. 1 and applying the orthogonality conditions, considering the dominant first mode only, one obtains the following equation of motion:

$$\ddot{\eta} + 2\mu\dot{\eta} + \omega^2\eta = F \sin \Omega t \tag{3}$$

where $\mu = \xi\omega$; ξ is the damping ratio in the bridge; and F and Ω, and ω are given by

$$F = 2P/mL; \quad \Omega = \pi v/L; \quad \text{and} \quad \omega^2 = \frac{\pi^4 EI}{mL^4}. \tag{4}$$

EQUATION OF MOTION WITH AN UNEVEN BRIDGE DECK

The vehicle was modeled as an independent one degree of freedom sprung mass with damping as shown in Fig. 1(b). The roughness is expressed as a deterministic function $r(x)$. The load applied on the bridge is given by $P(t)$:

$$P(t) = K[z - W(\bar{x}, t) - r(\bar{x})] + C_v[\dot{z} - \omega(\bar{x}, t) - \dot{r}(\bar{x})] + m_v g. \tag{5}$$

The equations of motion of the bridge coupled with the vehicle motion are given by

$$mW_{tt} + cW_{tt} + EIW_{xxxx} = P(t)\delta(x - vt) \tag{6}$$

$$m_v\ddot{z} + C_v[\dot{z} - \dot{r}(\bar{x}) - \dot{w}(\bar{x}, t)] + K[z - r(\bar{x}) - W(\bar{x}, t)] = 0 \tag{7}$$

in which K is the stiffness of the tires and suspension springs; m_v is the mass of the vehicle; C_v is the damping in the vehicle; $\bar{x}$ is the position of the vehicle from the left support ($\bar{x} = vt$); g is the acceleration of gravity; z is the displacement of the vehicle in the vertical direction with respect to its static equilibrium position; and $\dot{z}$ is the derivative of z with respect to time. $r(\bar{x})$ is a deterministic cosinusoidal

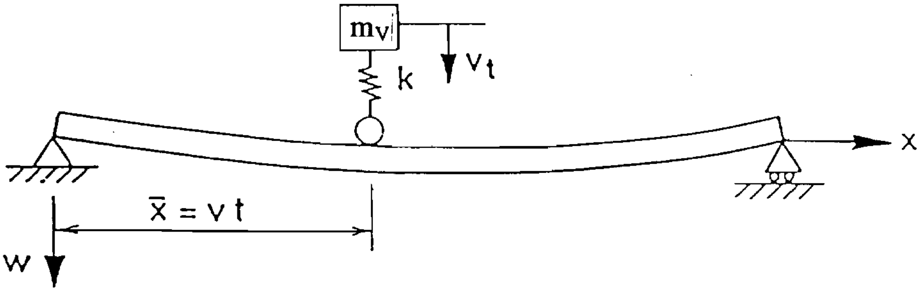


Fig. 1a Simply supported bridge with smooth deck.

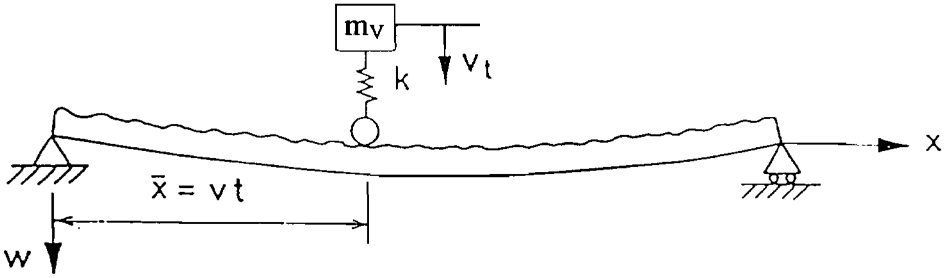


Fig. 1b Simply supported bridge with rough deck.

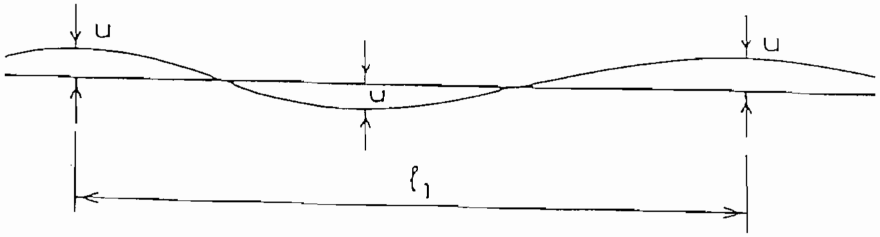


Fig. 2 Unevenness profile.

function given by

$$r(\bar{x}) = u\left(1 - \cos\frac{2\pi\bar{x}}{l_1}\right) \tag{8}$$

in which u is the amplitude of the unevenness in a wavelength l_1 as shown in Fig. 2.

Using Eq. 2 and considering the first mode only, one obtains

$$\begin{aligned}\ddot{\eta} + 2\mu\dot{\eta} + \omega^2\eta = {} & \frac{2K}{mL}(z - r(\bar{x}) - \eta\sin\Omega t)\sin\Omega t \\ & + \frac{2C_v}{mL}[\dot{z} - \dot{r}(\bar{x}) - \dot{\eta}\sin\Omega t]\sin\Omega t + \frac{2m_v g}{mL}\sin\Omega t\end{aligned} \tag{9}$$

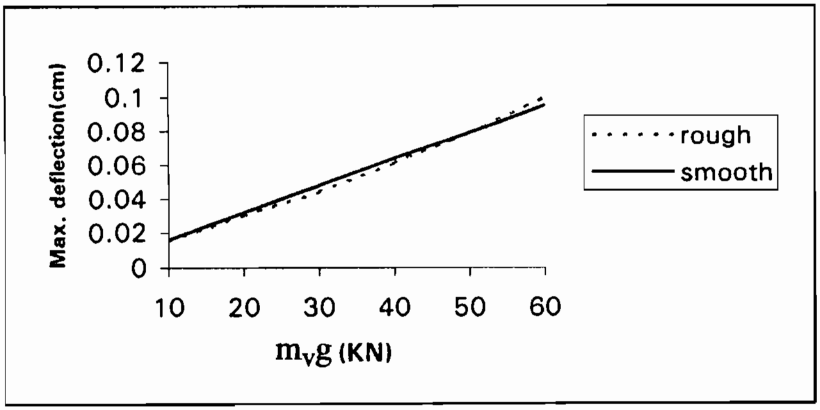
$$m_v\ddot{z} + C_v[\dot{z} - \dot{r}(\bar{x}) - \dot{\eta}\sin\Omega t] + K[z - r(\bar{x}) - \eta\sin\Omega t] = 0 \tag{10}$$

in which $\bar{x} = vt$.

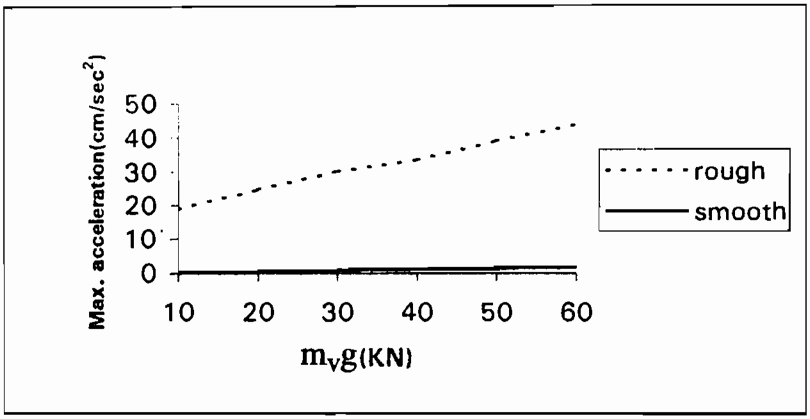
Equations 9 and 10 are coupled ordinary linear differential equations with time varying coefficients. The solutions of Eqs. 9 and 10 can be obtained numerically using the Runge-Kutta algorithm. It is obvious from Eqs. 9 and 10 that the dynamic response of the bridge is influenced by the bridge parameters ω, μ, u and l_1, in addition to the vehicle parameters m_v, C_v, v, and K. The effects of some of these parameters on the dynamic response are discussed next.

NUMERICAL INVESTIGATIONS

A single-span steel bridge model is considered in order to investigate the effect of the unevenness in the bridge deck on its dynamic response at mid-span. The basic data used for the bridge are the damping ratio, $\zeta = 5\%$; mass, $m = 0.612$ Kg.s^2/cm^2; flexural rigidity, $E^I = 10^{15}$ N.cm^2; and span, $L = 4000$ cm. From these data, the parameters ω and μ were calculated as $\omega = 7.877$ r.p.s. and $\mu = 0.394$. The basic vehicle parameters are assumed as $m_v g = 40$ kN, $m_v = 40.8$ N.S^2/cm, damping ratio in vehicle $\xi_v = 20\%$ and $K = 10$ kN/cm. The basic road profile is assumed in Eq. 8 as $l_1 = 100$ cm and $u = 10$ cm. The basic vehicle speed is assumed in Eq. 11 as $v = 2500$ cm/sec (90 Km/hr).



(a) Maximum Deflection Response



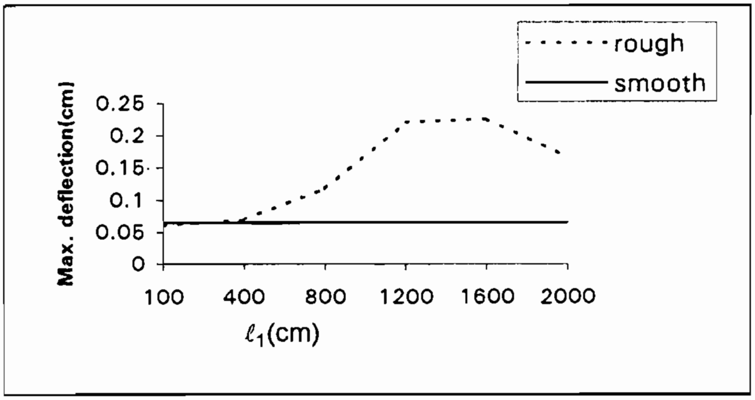
(b) Maximum Acceleration Response

Fig. 3 Maximum responses for various values of $m_v g$.

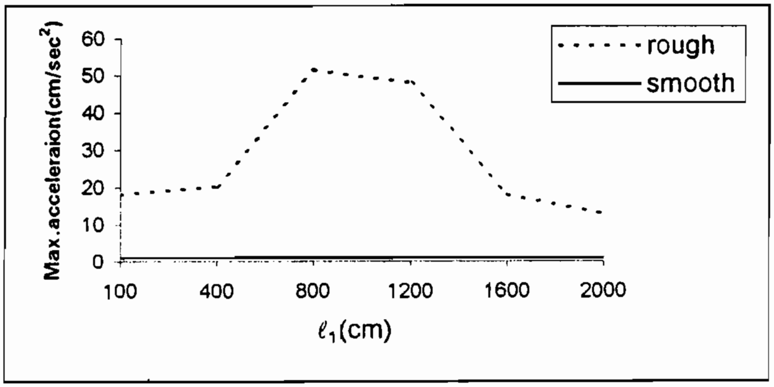
Using the above data, the maximum acceleration and deflection for the simply supported bridge model with smooth and rough decks were studied. The effects of the parameters $m_v g$, l_1, K, ζ, ω, and ξ_v on the maximum responses are shown.

Figures 3 to 9 show a comparison of the maximum deflection and acceleration responses at midspan for bridges with smooth or rough decks when $u = 10$ cm and $v = 2500$ cm/sec. For most of the cases the rough decks showed higher acceleration and deflection values than the smooth decks. However, a different behavior was noted for the maximum deflection values due to the change in the parameters K, ζ, and ξ_v.

Figures 3 shows the peak deflection and acceleration responses for increasing values of the weight of moving vehicle P. The deflection responses showed almost



(a) Maximum Deflection Response



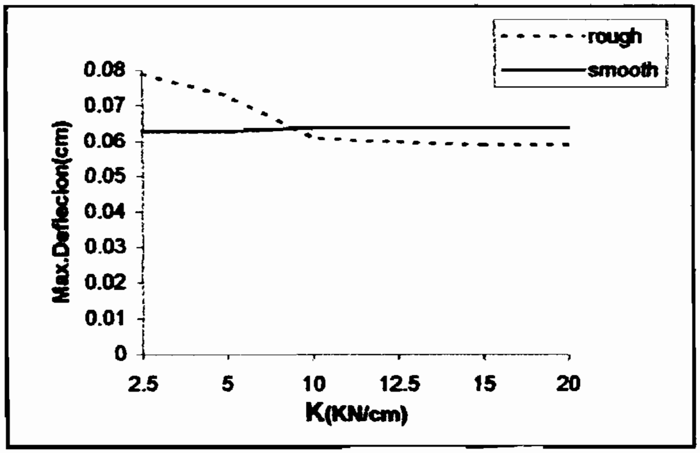
(b) Maximum Acceleration Response

Fig. 4 Maximum responses for various values of l_1.

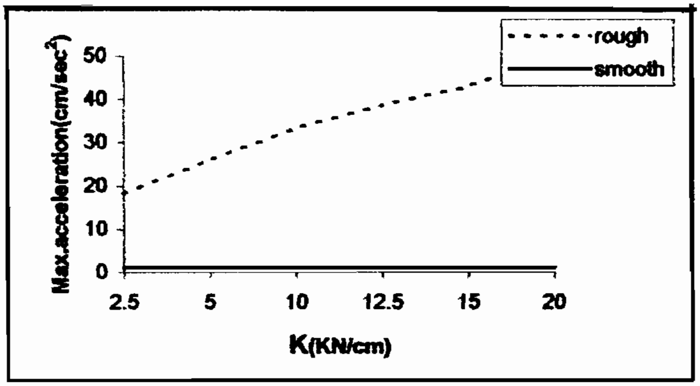
no difference between the bridge with a smooth deck and that with a rough deck, whereas the acceleration response showed a large difference for all vehicle weights. With increasing vehicle weight, the acceleration responses for bridges with rough decks increased more than the bridges with smooth surfaces.

Figure 4 shows the maximum responses against the unevenness profile l_1. A large increase in the maximum deflection and acceleration values occurred for the l_1 values in the range 400 to 1600 cm, where the bridge's natural frequency coincides with the frequency of the roughness.

In Fig. 5, the variation of the vehicle tire and suspension stiffness K is shown. For small values of K ($K = 2.5$ kN/cm), the maximum deflection values on bridges with rough decks were higher than those with smooth decks which remained almost



(a) Maximum Deflection Response



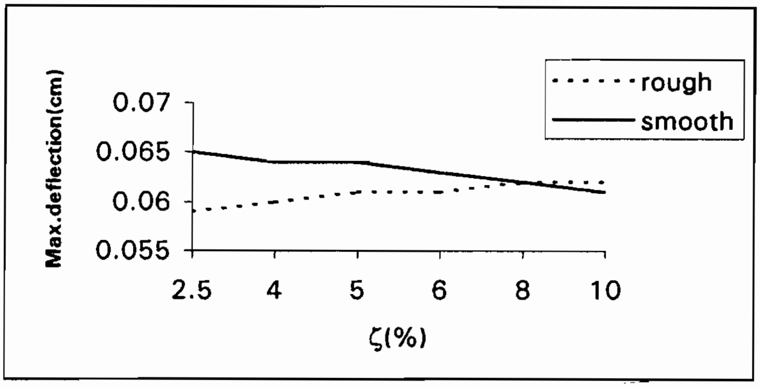
(b) Maximum Acceleration Response

Fig. 5 Maximum responses for various values of K.

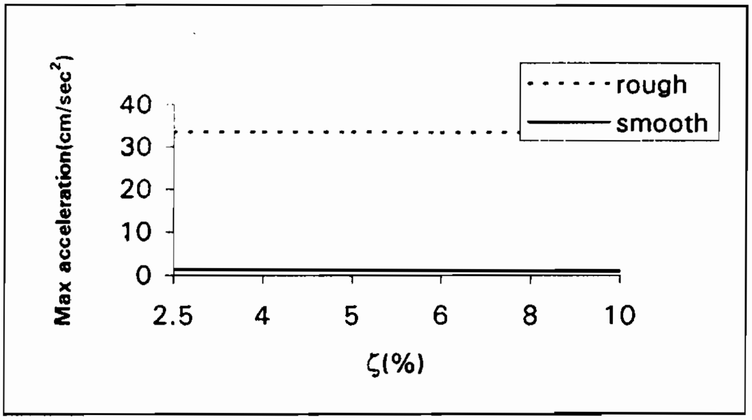
constant. In contrast, increasing K, the bridges with rough decks showed a decrease in the deflection values, and at $K \geq 10$ kN/cm, the deflections on rough decks were found to be lower than those with smooth decks. For the acceleration responses the bridges with rough decks remained higher with increasing magnitude whereas the bridges with smooth decks remained almost constant with increasing K. This observation could be due to the combined effects of vehicle speed and tire stiffness.

A small change in the deflection values were noted between the rough and smooth decks with an increase of ξ, the damping ratio. The maximum acceleration responses remained constant for both types of bridges with increasing ξ, though a large difference was seen between the rough and smooth decks.

Figure 6 shows the responses against the damping ratio ζ. A small change in the deflection values was noted between rough decks which were lower, and smooth



(a) Maximum Deflection Response



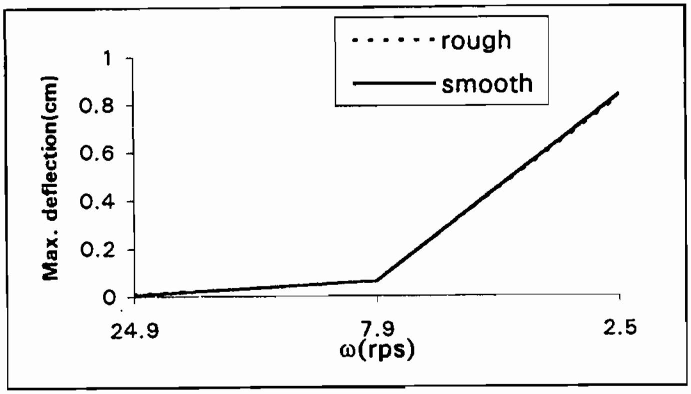
(b) Maximum Acceleration Response

Fig. 6 Maximum responses for various values of ζ.

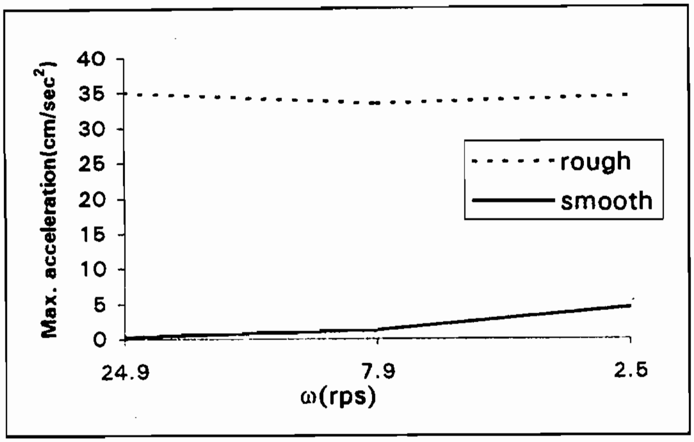
decks with the increase of ζ. The maximum acceleration responses remained constant for both types of bridges with increasing ζ, though a large difference was seen between the rough and smooth decks. For values of 8%, the maximum deflection of the smooth deck became lower than that of the rough one.

In Fig. 7, the maximum acceleration and deflection responses are shown against the bridge natural frequency ω. Identical behavior was noticed for both kinds of bridge decks with respect to the maximum deflection. A steeper increase in this response was noted for $\omega \geq 7.9$ r.p.s. A large deviation was noticed between both kinds of bridge decks in the acceleration response.

Figure 8 shows the responses plotted against ξ_v, the damping ratio in the vehicle. The deflection response of smooth bridges remained constant with increasing ξ_v, and was higher than the deflection response on rough bridges which increased with ξ_v. For the maximum acceleration responses, the rough decks showed higher values



(a) Maximum Deflection Response



(b) Maximum Acceleration Response

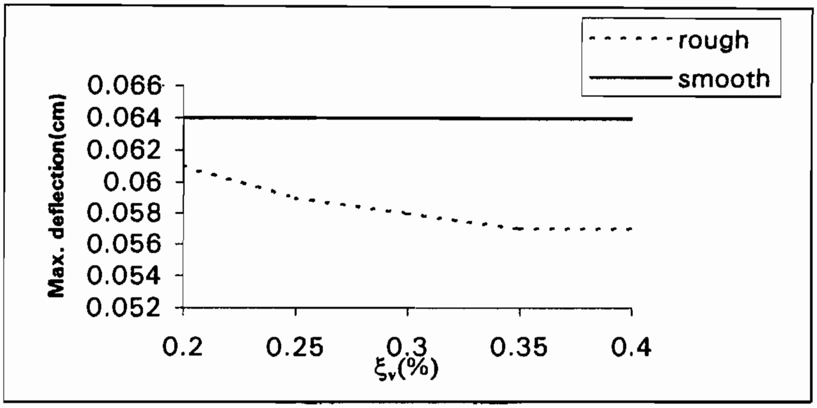
Fig. 7 Maximum responses for various values of ω.

than smooth decks, which remained almost zero.

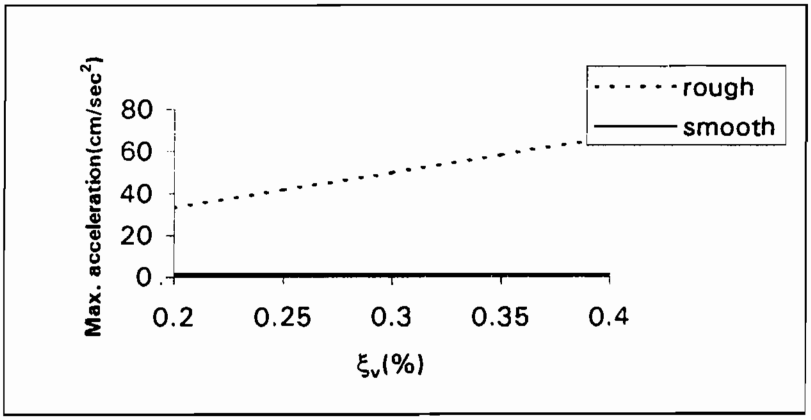
Figure 9 shows the responses as the vehicle weight varied for a high value of l_1 which was taken as 1200 cm. Here the bridges with rough decks showed a higher response than that of smooth decks for both responses, which showed a changing behavior in the maximum deflection as compared with $l_1 = 100$ as in Fig. 3. The bridges with smooth decks showed a slight change in the values.

CONCLUSION

There was no change in the maximum responses of bridges with smooth decks due to the change in the stiffness and damping ratio of vehicles. The effect of the vehicle



(a) Maximum Deflection Response

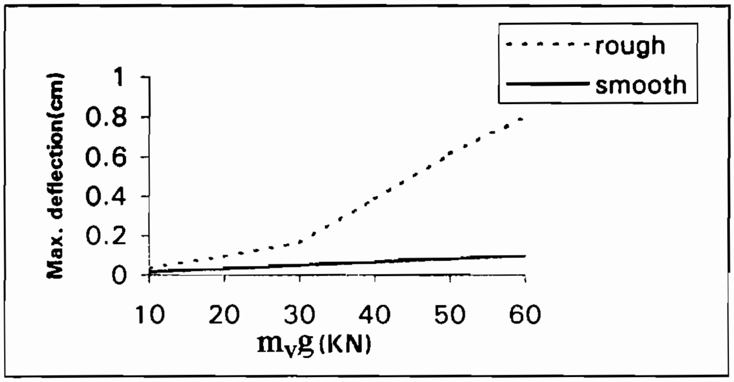


(b) Maximum Acceleration Response

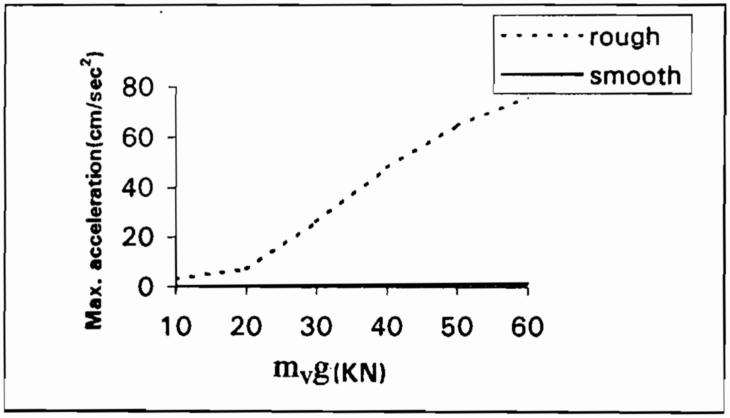
Fig. 8 Maximum responses for various values of ξ_v.

weight was seen to increase the responses for bridges with rough decks, and the deflection of the smooth deck, whereas the maximum acceleration was almost steady for the bridges with smooth decks. The increase of roughness profile affected the deflection and acceleration of the bridge tremendously within the range $l_1 = 400$ to 1600 cms. With increasing stiffness values, the maximum acceleration values showed an increase for bridges with rough decks whereas the maximum deflection values remained almost the same. Varying bridge natural frequency and bridge damping ratio caused an obvious deviation between the bridges with rough and smooth surfaces for the acceleration response only.

Design parameters and criteria are extracted if the experimental program findings coincide with these findings. An experimental model is needed for further study to justify such results.



(a) Maximum Deflection Response



(b) Maximum Acceleration Response

Fig. 9 Maximum responses for various values of $m_v g$ with $l_1 = 1200$ cm.

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أثر خشونة أسطح الجسور على اهتزازاتها الديناميكية

جمال الدعيج ، محمد عبدالرحمن و رينا بايابلي
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خلاصة

تبحث هذه الورقة في الاهتزازات الديناميكية القصوى للجسور البسيطة والمثبتة بالمفاصل من الطرفين الناتجة عن مرور عربة افتراضية عليها . هذه العربة ممثلة بكتلة أحادية تسير بسرعة ثابتة . يتم رصد ردود أفعال الجسر لمختلف المؤثرات مثل : الاهتزازات الطبيعية ، نسبة المعوقات ، درجة الخشونة ، سرعة العربة ، ووزنها بالإضافة إلى عناصر أخرى ذات ارتباط . الرسومات المرفقة تدلل على أثر كل عنصر على حدة مع تثبيت العنصرين الآخرين، وتختم الدراسة بربط النتائج بمسبباتها وكيفية الاستفادة منها في التصميم الهندسي أو الواقع العملي.