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Comparison of coarse slurry pipeline models

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ABSTRACT

The most successful models for predicting the pressure drop for the flow of heterogeneous mixtures of solids and liquids in pipelines are based on force balances. The distribution of solids over the cross-section of the pipe must either be known or assumed and the most commonly used model for the interpretation of experimental results is the two-layer model in which the particles in the upper layer are in suspended flow and those in the lower layer are (at least partially) supported by forces arising from the interactions between the particles and the pipe wall. The model is being progressively developed and experimental results for transport of solids in a small (38 mm) horizontal pipeline are compared with predictions from three versions of the model. In addition, experimental results for a large pipe (263 mm) have been compared with two of the models. A comparison with the experimental results, shown as points, shows a satisfactory agreement with both models (Richardson *et al.* [1996] and Modified Wilson Model [Gillies *et al.* 1991]) the former being a little closer for both small and large pipeline data.

INTRODUCTION

It has long been appreciated that the only satisfactory methods of correlating data on the pressure gradient for the flow of non-homogeneous suspensions in horizontal pipelines are those which take account of the flow mechanisms. Newitt *et al.* (1955) were among the first to propose a model based on a force or momentum balance. In the early 1970's, Wilson developed this approach by introducing the concept of a two-layer model in which particles were in suspended flow in the upper layer and in which, for at least a proportion of the particles in the lower layer, solid interaction forces at the pipe wall counterbalanced part of their buoyant weight. This model has been developed progressively over the years by Wilson himself and others, notably Shook and his co-workers [Wilson (1970, 1975, 1988), Wilson *et al.* (1972), Shook *et al.* (1986) and Gillies *et al.* (1991)]. For a steady state flow, the force due to the pressure must balance the sum of the resistance forces due to fluid friction and interaction between the particles and the walls. This force balance must hold for the pipe cross-section as a whole and for the individual layers. In considering the two layers, account must be taken of the forces due to shear stress at the interface, the retarding force on the lower boundary of the upper, faster-moving, layer being exactly equal and opposite to the accelerating force on the lower layer. These force balances will be developed first, together with relations between concentrations of particles and velocities which are determined by material balances.

The solution of the resulting simultaneous equations may be approached in several ways dependent upon the simplifying assumptions which are made. Three examples will be given, and the results will then be compared with those obtained from experiments. Only a small proportion of the published results are suitable for this purpose, as basic physical data, particularly the coefficient of coulombic friction between the solids and the pipe wall, are so often not quoted.

It must be stressed that the existence of two layers within the pipeline is a concept used simply for modelling purposes and is not a physical reality, any more than a boundary layer formed during the flow of a fluid over a surface has a distinct identity.

BASIC RELATIONSHIPS

Material balances

Denoting the cross-sectional areas, volumetric fractional concentrations and velocities in the upper and lower layers by suffixes 1 and 2 respectively, and neglecting local 'slip' between liquid and solid within each layer, the material balances in terms of mean delivered concentration C are:

For the mixture: $VA = V_1A_1 + V_2A_2$ (1)

For the solid: $CAV = C_1A_1V_1 + C_2A_2V_2 = C_1AV + (C_2 - C_1)A_2V_2$ (2)

For the liquid: $(1 - C)AV = (1 - C_1)A_1V_1 + (1 - C_2)A_2V_2$ (3)

In addition, the material balances may be written in terms of the velocities of the liquid and solids (V_L and V_S), and delivered and in-line concentrations (C and C_x):

For solids: $VC = V_SC_x$ (4)

For liquids: $V(1 - C) = V_L(1 - C_x)$ (5)

It is convenient to express the areas, perimeters and chords in terms of the half angle θ subtended at the centre of the pipe, as shown in Fig. 1. From consideration of geometry:

$$A_2 = \frac{D^2(\theta - \sin\theta\cos\theta)}{4} \tag{6}$$

$$A_1 = \frac{D^2(\pi - \theta + \sin\theta\cos\theta)}{4} \tag{7}$$

$$S_2 = D\theta \tag{8}$$

$$S_1 = D(\pi - \theta) \tag{9}$$

$$S_i = D\sin\theta \tag{10}$$

using the above relations.

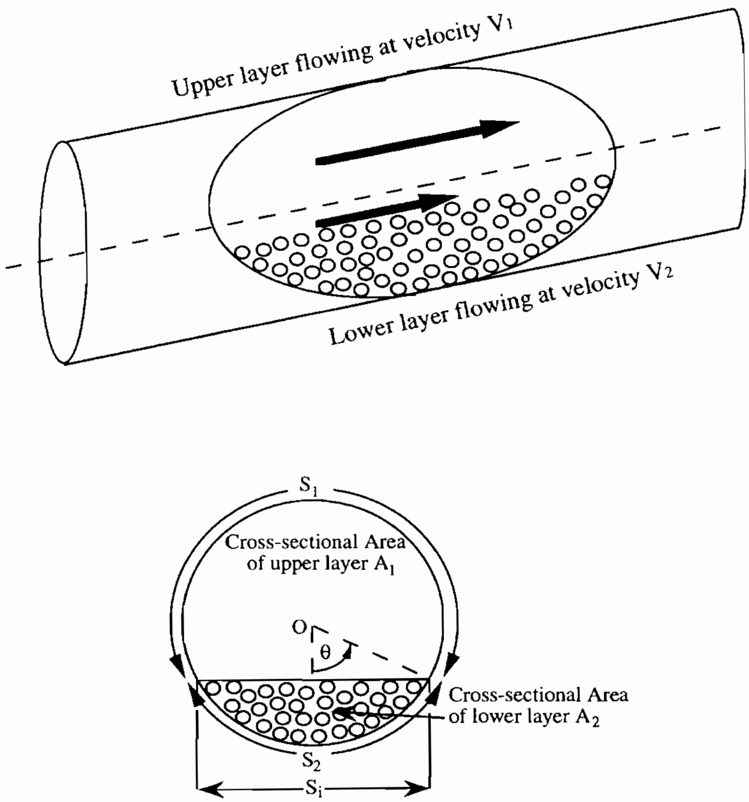

Fig. 1. Model—basic concept.

Force or momentum balance

In the lower layer, there will be impelling forces attributable to the pressure gradient and to the interfacial shear stress at the upper surface of the bed exerted normally to the surface of the pipe. At equilibrium, these forces will be counter-balanced by the combined effects of fluid friction at the walls and particle-pipe friction. In the upper layer, τ_i will operate in the opposite direction and the force balance equation per unit length is:

For the upper layer: $$-\frac{dP}{dx}A_1 = \tau_1 S_1 + \tau_i S_i \qquad (11)$$

For the lower layer: $$-\frac{dP}{dx}A_2 = \tau_2 S_2 - \tau_i S_i + \mu_F \sum F \qquad (12)$$

For the whole pipe: $$-\frac{dP}{dx}A = \tau_1 S_1 + \tau_2 S_2 + \mu_F \sum F. \qquad (13)$$

The significance of the above symbols can be understood by reference to Fig. 1, with suffixes 1, 2 and i referring, respectively, to the upper layer, the lower layer and interface, and $\sum F$ is the total force per unit length exerted normally to the surface of the pipe over area A_2. μ_F is the coefficient of coulombic friction.

Three models will now be used for the evaluation of each of the terms in the above relations. Models are being continuously modified and refined as more experimental information becomes available, but it is convenient to use versions which are in fairly general use.

SHOOK MODEL (MODIFIED WILSON MODEL)

The model which is taken is that based on work by Shook, Wilson and others and as set out in Shook & Roco's book (1991). The following assumptions are made:

(1) The concentrations and velocities within each layer (C_1, V_1, C_2 and V_2) are uniform and there is no slip between the solids and the liquids within either layer.
(2) The solids in the upper layer are fully suspended.
(3) The total concentration C_2 of solids in the lower layer is that of a packed bed and is taken as 0.6. Those solids in the lower layer are considered as constituting a suspended load of concentration C_1 and solids of concentration $C_2 - C_1$ whose buoyant weight is supported at the walls as a result of interparticle contacts. This concentration, averaged over the total cross-section of the pipe, is defined as the "contact load" C_c

$$C_c = \frac{(C_2 - C_1)A_2}{A}. \tag{14}$$

The mean in-situ concentration

$$C_x = C_1 + C_c. \tag{15}$$

(4) The relationship between C_c and C_x is given by means of an experimental correlation:

$$\frac{C_c}{C_x} = 1 - \frac{C_1}{C_x} = \exp\left[-0.124Ar^{-0.061}\left(\frac{V^2}{gd}\right)^{0.28}\left(\frac{d}{D}\right)^{-0.431}\left(\frac{\rho_s}{\rho} - 1\right)^{-0.272}\right] \tag{16}$$

where Ar is Archimedes number $\rho\, d^2(\rho_s - \rho)g/\mu^2$.

Hence, if C_x is known (or assumed if C is specified), C_c can be calculated. Thus C_1 is given by Eq. (15) and if C_2 is taken as 0.6, A_2/A is determined ((Eq. 14) and hence θ from Eq. (6). Thus all the area and length terms are given by Eqs. (6) to (10).

(5) The various state stresses τ_1, τ_2 are attributed to those of a fluid (viscosity equal to that of the liquid μ_L and density equal to that of a suspension of concentration C_1) and τ_i is treated as the rough grain boundary of relative roughness determined by the grain size.

(6) The friction factors f_1, f_2 are based on the Reynolds number of the liquid flowing at the average velocity V in the pipe i.e., $D\rho_L V/\mu_L$ and on the relative roughness of the pipe k/D. They are calculated using the Churchill (1977) equation:

$$F = 2[8\ \mathrm{Re}^{-12} + (A+B)^{-1.5}]^{1/12} \tag{17}$$

where $A = [-2.457\ \ln(7\ \mathrm{Re}^{-0.9} + 0.27k/D)]^{16}$ and $B = (37{,}530\ \mathrm{Re}^{-1})^{16}$.

Although Re should be calculated using ρ_1 and either V_1 or V_2 (which are not known at this stage), little error is introduced because friction factor f_i is a weak function of the Reynold's number.

(7) The friction factor f_i is calculated from a modified Colebrook equation using the particle: pipe diameter ratio d/D as an equivalent sand roughness (Nikuradse 1932).

$$f_i = \frac{2(1+Y)}{[4\ \log_{10}(D/d) + 3.36]^2} \tag{18}$$

where $Y = 0, d/D < 0.0015$, $Y = 4 + 1.42\ \log_{10}(d/D), 0.0015 < d/D < 0.15$ and $Ar < 3 \times 10^5$.

Considering the somewhat nebulous interface between the two layers as a surface of equivalent roughness d/D is probably the most speculative point in the analysis.

(8) The fluid shear stresses are then calculated as:

$$\tau_1 = \frac{f_1 \rho_1 V_1^2}{2}, \quad \tau_2 = \frac{f_2 \rho_1 V_2^2}{2}, \quad \tau_i = \frac{f_i \rho_1 (V_1 - V_2)^2}{2}. \tag{19}$$

V_1 is related to V_2 by Eq. (2). In addition, there is the further requirement that the values of pressure gradients as determined by Eqs. (11), (12) and (13) must be equal. An iterative solution for the values of V_1 and V_2 is therefore necessary.

(9) The term ΣF is given by Shook as:

$$\frac{D^2 g(\rho_S - \rho_L)(\sin\theta - \theta\cos\theta)}{2}\left\{\frac{(C_2 - C_1)(1 - C_2)}{1 + C_1 - C_2}\right\}. \tag{20}$$

The contribution of Bagnold (1957) stresses is neglected on the grounds that the layer of contact load solids is shallow compared to its width.

The calculation procedure used for the iterative solution of the equation for the case where the delivered concentration of solids C and the average velocity V are given is detailed in Fig. 2.

RICHARDSON, KHAN AND BLACKWELL MODEL

A simplified model (Richardson *et al.* 1996) has been proposed which has many of the features of the original two-layer model of Wilson (1970). This is shown to be applicable to transport of solids in a small (38 mm) diameter pipeline. The experimental work upon which it is based involved the transport of gravel particles (3 sizes: 3.5, 5.7 and 8.1 mm) (Pirie 1990) and measurements were made of in-line concentration of solids C_x (γ-ray absorption), mixture velocity V (electromagnetic

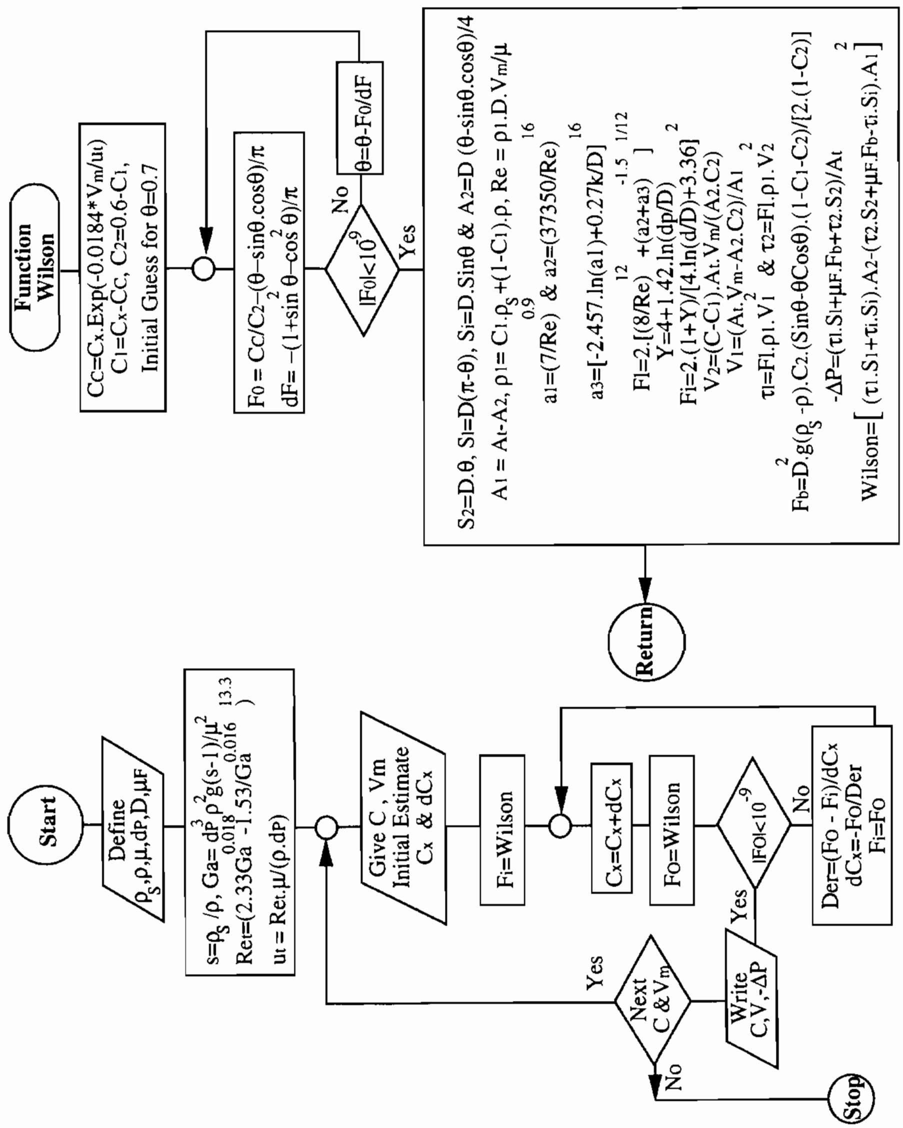

Fig.2. Flow-chart of Modified Wilson model (Shook *et al.* 1990).

flowmeter), liquid velocity V_L (salt injection technique, Pirie *et al.* [1988]) and delivered concentration C at the pipe outlet (sufficient results to check the consistency of the measurements of V, V_L and C_x).

The assumptions in the model are:

(1) All particles travel in the lower layer at a concentration $C_2 = 0.5$ (as measured for a loose packed bed). Hence $C_1 = 0$ and $C_x = C_2A_2/A$.
(2) Slip is neglected between the liquid and solids in the lower layer and hence the solids velocity $V_S = V_2$.
(3) The friction losses attributable to the solids arise from the coulombic friction at the pipe walls.
(4) The slip velocity averaged over the whole cross-section of the pipe ($V_r = V_L - V_S$) approximates to the free settling velocity V_0 of the particles. Utilization of this assumption, based on experimental measurements, eliminates the necessity of estimating the shear stress τ_i at the interface between the two layers. Measured values of the ratio V_r/V_0 were 0.85 ± 0.2 (95% confidence limits) and showed no significant trends with d, V, or C_x. A sensitivity analysis showed that the calculated pressure drop was insensitive ($\pm 5\%$) to the value over the range ($0.65 < V_r/V_0 < 1.05$). It also showed that it was insensitive to the value of C_2 over the range ($0.45 < C_2 < 0.55$). Computations were carried out therefore using $V_r/V_0 = 1$ and $C_2 = 0.5$. On this basis all of the in-line parameters (concentrations and velocities) for a given mixture velocity V and delivered concentration C may be calculated.

Equations (2) and (3) now become:

Solids: $$CAV = C_2A_2V_2 \tag{2a}$$

Liquid: $$(1 - C)AV = A_1V_1 + (1 - C_2)A_2V_2 \tag{3a}$$

Using Eqs (6) and (7):

$$V_1 = \frac{\pi(1 - C/C_2)V}{\pi - \theta + \sin\theta\cos\theta} = \frac{\pi(1 - 2C)V}{\pi - \theta + \sin\theta\cos\theta} \qquad (\text{for } C_2 = 0.5) \tag{21}$$

$$V_2 = \frac{\pi(C/C_2)V}{\theta - \sin\theta\cos\theta} = \frac{2\pi CV}{\theta - \sin\theta\cos\theta} \qquad (\text{for } C_2 = 0.5) \tag{22}$$

$$C_x = \frac{C_2A_2}{A} = \frac{CV}{V_2} = \frac{C_2(\theta - \sin\theta\cos\theta)}{\pi} = \frac{\theta - \sin\theta\cos\theta}{2\pi} \qquad (\text{for } C_2 = 0.5). \tag{23}$$

The values V_1, V_2 and C_x can be conveniently calculated in terms of θ. All of the relevant velocities (V_1, V_2, V_L and V_S), in-line concentration C_x, area ratios, and angle θ can be calculated by solving the relevant equations simultaneously. An iterative process is not necessary. The pressure gradient can be obtained by direct evaluation of the terms in Eqs. (2a), (3a), (6), (7), (21), (22) and (23) again without the necessity for first obtaining the required values of V_1 and V_2 for the pressure gradient to be the same in each of the layers. The stages in the computation can be followed by reference to Fig. 3.

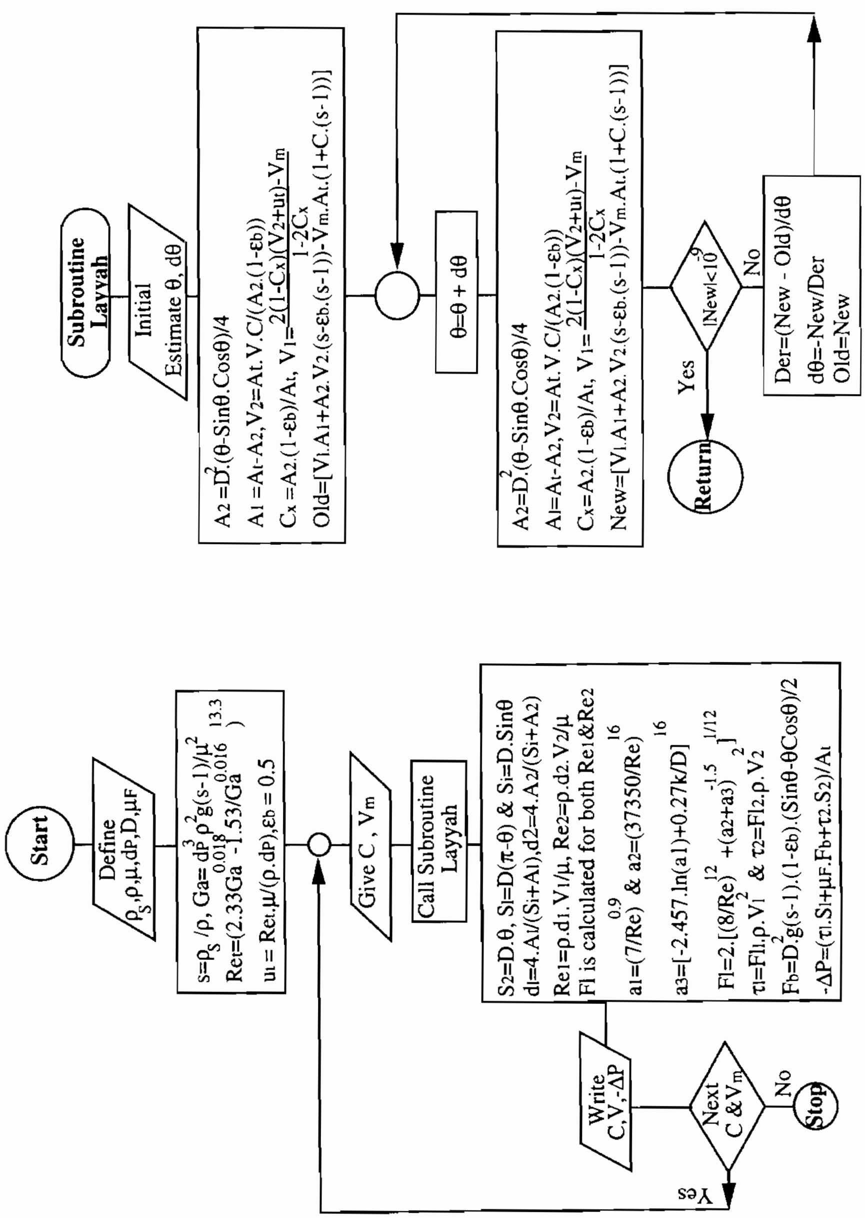

Fig.3. Flow-chart of Richardson *et al.* (1996) model.

DORON, GRANICA AND BARNEA (1987) MODEL

The authors have suggested a two-layer model with either a stationary or moving bed at the bottom of the pipe and a heterogeneous suspension in the upper portion. Consideration will be given only to the case where the bed is moving. The concentration distribution of solids in the upper layer is assumed to be that resulting from a turbulent dispersion process, with the concentration at its bottom interface equal to that of the lower layer of C_2. A value of 0.52 has been assumed for C_2. The dispersion process is governed by an equation of the form:

$$\varepsilon \frac{d^2C}{dy^2} + V_C \frac{dC}{dy} = 0 \tag{24}$$

where V_C is the velocity of settling of a particle in a suspension of concentration C. If the effect of variation of V_C which is a function of C, in the y (vertical) direction is neglected (V_C is assumed constant), integration of Eq. (24) between the surface of the bed ($C = C_2$) and a height y above the bed gives:

$$C(y) = C_2 e^{-(V_C/\varepsilon)y}. \tag{25}$$

The dispersion coefficient ε has been evaluated by Taylor (1954) to give:

$$\varepsilon = 0.026 V_1 \sqrt{\frac{f_i}{2}} D_1. \tag{26}$$

D_1 is the assumed value of the hydraulic mean diameter of the upper layer, V_1 is the mean velocity, taken as constant, and f_i is the friction factor of the interface between the two layers. For moving bed:

$$C(y) = C_2 \exp[V_C (y - y_2)/\varepsilon] \tag{27}$$

where y_2 is the bed height.

Doron *et al.* (1987) evaluated V_0 terminal falling velocity of particle from the equation:

$$V_0 = \frac{4}{3} \sqrt{\frac{(\rho_S - \rho_L) dg}{\rho_L C_D}}, \tag{28}$$

where C_D is the drag coefficient given by Bird *et al.* (1960) as:

$$C_D = 18.5 \, \mathrm{Re}_p^{0.6} (2 < \mathrm{Re}_p < 500) \tag{29}$$

and

$$C_D = 0.44 \qquad (500 < \mathrm{Re}_p < 2 \times 10^5). \tag{30}$$

V_C is evaluated in terms of V_0 and C_1 using the relation (Richardson & Zaki 1954):

$$\frac{V_C}{V_0} = (1 - C_1)^n \tag{31}$$

where

$$n = 4.45\ \mathrm{Re}_p^{-0.1}(1 < \mathrm{Re}_p < 500),\ \text{and} \tag{32}$$

$$n = 2.39 \qquad (\mathrm{Re}_p > 500). \tag{33}$$

In the above paper Re_p is based on the velocity V_C, whereas is should have been based on V_0. The viscosity is taken as that of the carrier fluid. The mean concentration C_1 in the upper layer is then obtained by integration:

$$C_1 = \frac{C_2 D^2}{2A_1} \int_0^{\pi/2} \exp\left[-\left(\frac{V_C D^2}{2\varepsilon}(\cos\theta - \cos\gamma)\right)\right] \sin^2\gamma\, d\gamma. \tag{34}$$

In Eq. (31), a mean value of C_1 must be assumed and if necessary an interactive process must be followed using the integrated value obtained from Eq. (34).

The fluid shear stresses were calculated using the relations for turbulent flow:

$$\tau_1 = \frac{f_1 \rho_1 V_1^2}{2} \tag{35}$$

and

$$\tau_2 = \frac{f_2 \rho_2 V_2^2}{2}. \tag{36}$$

Using

$$f_1 \quad \text{or} \quad f_2 = 0.046\ \mathrm{Re}^{-0.2} \tag{37}$$

based on Re_1 or Re_2 respectively

$$\tau_i = \frac{f_1 \rho_1 (V_1 - V_2)^2}{2} \tag{38}$$

where f_i is given by the Colebrook equation (1939):

$$\frac{1}{\sqrt{2f_i}} = -0.86 \ln\left[\frac{d/D}{3.7} + \frac{2.51}{\mathrm{Re}_1\sqrt{2f_i}}\right]. \tag{39}$$

The solutions to all the above equations involve extremely complex iterations, as the result of the interactions between the equations and the fact that velocities V_1, V_2 and concentrations C_1 are not known ab-initio.

The calculation of the normal force term $\sum F$ which determines the coulombic friction at the pipe walls was regarded as consisting of two components:
(i) The pseudo-hydrostatic component (Wilson 1970)

$$\sum F_{HS} = \frac{(\rho_S - \rho_L) g C_2 D^2}{2}(\sin\theta - \theta\cos\theta). \tag{40}$$

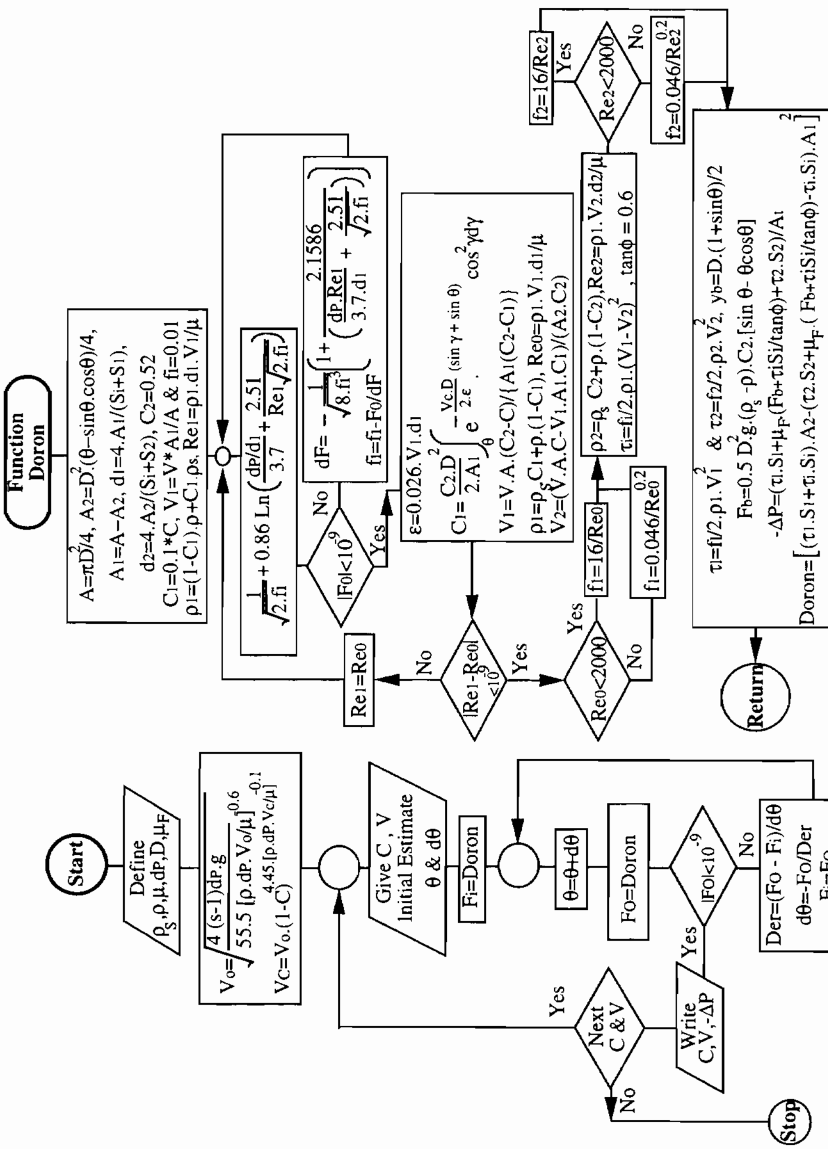

Fig.4. Flow chart for Doron *et al.* (1987) model.

(ii) The additional normal force due to transmission of stress from the interface through the bed (Bagnold 1954, 1957)

$$F_{\text{Bag}} = \frac{\tau_i S_i}{\tan\phi} \tag{41}$$

where ϕ is the angle of internal friction. The value of $\tan\phi$ varies from 0.35 to 0.75 according to the particle characteristics and type of flow.

The total force due to solids friction per unit length of pipe:

$$\mu_F \sum F = \mu_F \left\{ \sum F_{HS} + F_{\text{Bag}} \right\}. \tag{42}$$

It is noteworthy that in their experimental work no internal parameters of the flowing suspension have been measured. The only measure of concentration is that obtained by diverting the flow into a receiver. As stated by Doron & Barnea (1995), "If the two phases are distributed uniformly with no slip between them, the slurry concentration can be extracted". Doron & Barnea (1992, 1993, 1995) have modified their initial two layer model incorporating the presence of a stationary bed. The details of the computational procedure are given in Fig.4.

RESULTS AND DISCUSSION

The predicted pressure gradients for the experimental system (38 mm pipe) used by Richardson *et al.* (1996) have been expressed as pressure gradient versus mixture velocity with delivered concentration C as parameter for gravel particles (sizes 3.5, 5.7 and 8.1 mm, $\mu_F = 0.435$) using the Shook/Wilson model and the Richardson *et al.* model. The results are shown in Figs. 5, 6 and 7 from which it is seen that the Wilson model gives a slightly lower prediction, particularly at the higher concentrations of the smallest particles. A comparison with the experimental results, shown as points, shows a satisfactory agreement with both models, the Richardson *et al.* model being a little closer. For the 3.5 mm particles, the Doron model has also been used and it is seen in Fig. 5 to yield a significant over-prediction.

In Fig. 8, experimental data for large sand particles (2.4 mm) in a 263 mm pipe (Shook *et al.* 1986) have been compared with the predicted values for both the Richardson *et al.* (1996) model and the modified Wilson model. The reported experimental results are for two values of in-line concentrations (0.13 and 0.22, volumetric); it is somewhat surprising that the experimental curve is concave towards the velocity axis. In the paper of Shook *et al.* (1986), no comparison is made between experimental and predicted results. Figure 8 shows that both models give a satisfactory interpolation, but the Richardson *et al.* (1996) model shows a slightly better fit at the lower concentration than the modified Wilson model. In both models, the concentration distribution has been assumed to be as suggested by Shook *et al.* (1986), (i.e., C_2 is taken 0.5 and 0.6 for the Richardson *et al.* model and modified Wilson model respectively, and C_1 is calculated from Eq. (16)).

For the 3.5 min particles at a concentration C of 0.25, the calculated values of the individual components of the force (expressed as N/m of pipe) are shown as a function of mixture velocity ($\tau_1 S_1$ and $\tau_i S_i$ in Fig. 9, $\tau_2 S_2$ and $\mu_F \sum F$ in Fig. 10) using

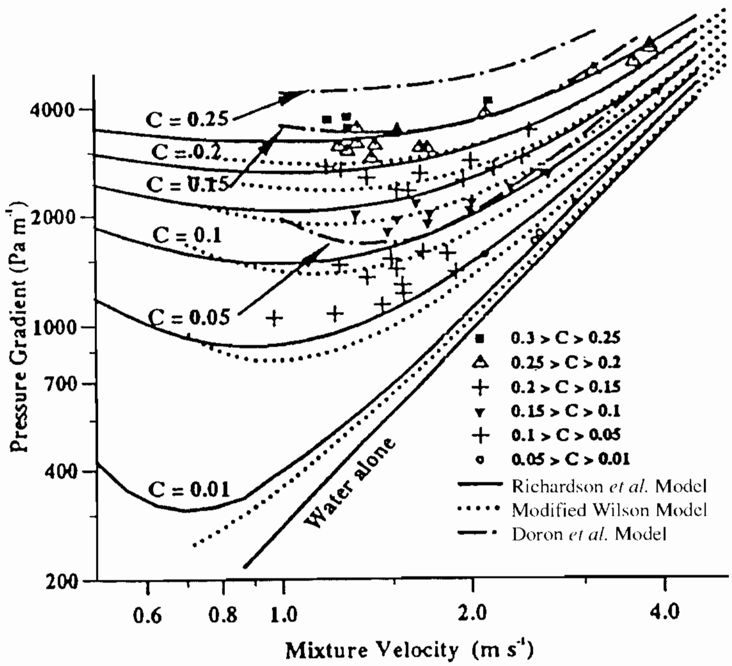

Fig.5. Comparison of different models with experimental data 3.5 mm gravel particles in 38 mm pipe (Pirie 1990).

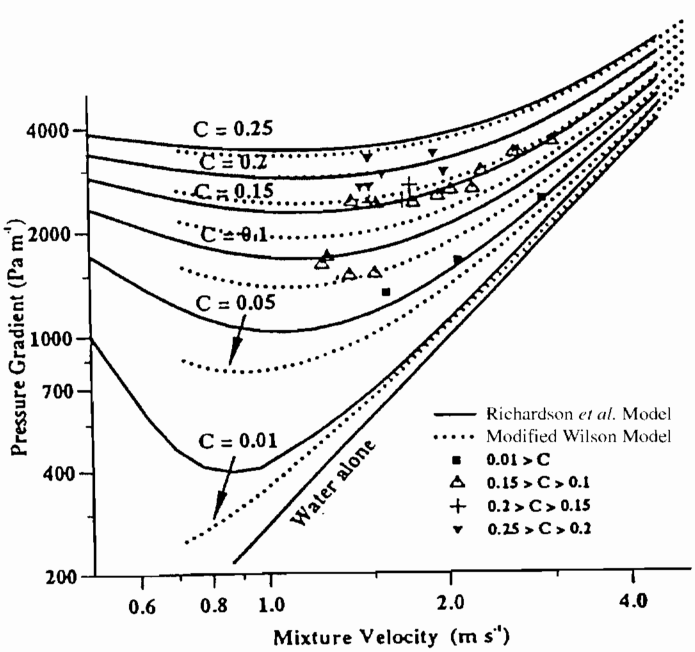

Fig.6. Comparison of different models with experimental data 5.7 mm gravel particles in 38 mm pipe (Pirie 1990).

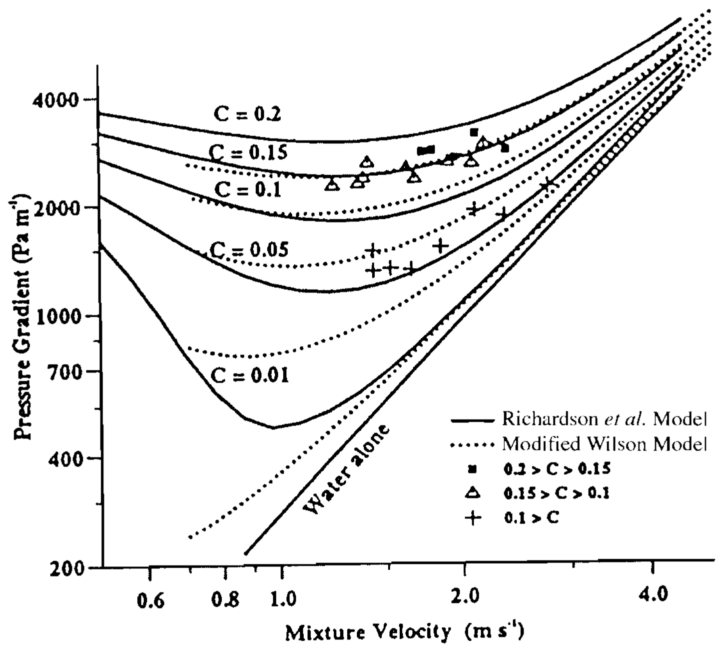

Fig.7. Comparison of different models with experimental data 8.1 mm gravel particles in 38 mm pipe (Pirie 1990).

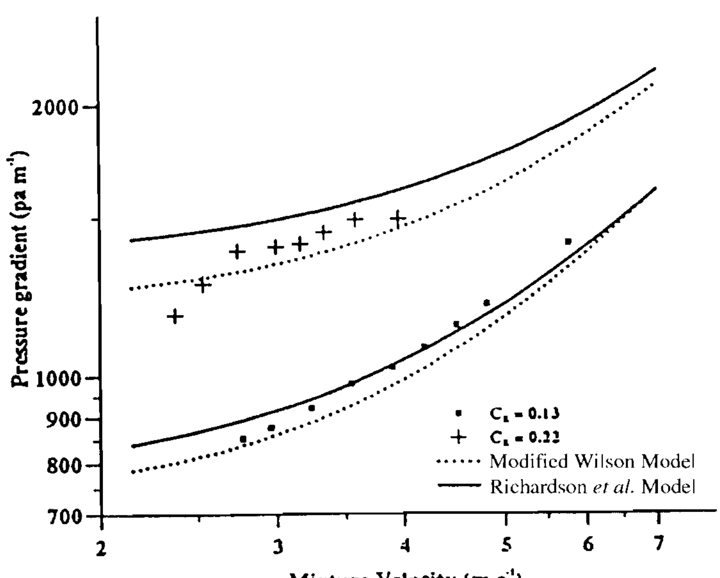

Fig.8. Comparison of different models with experimental data 2.4 mm sand particles in 263 mm pipe (Shook *et al.* 1986).

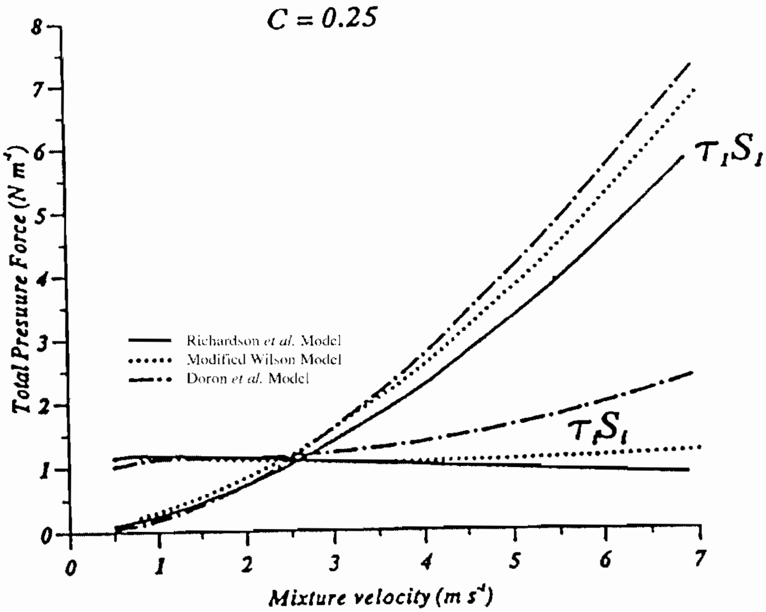

Fig.9. Force components comparison with different models for 3.5 mm gravel particles in 38 mm pipe for $C = 0.25$.

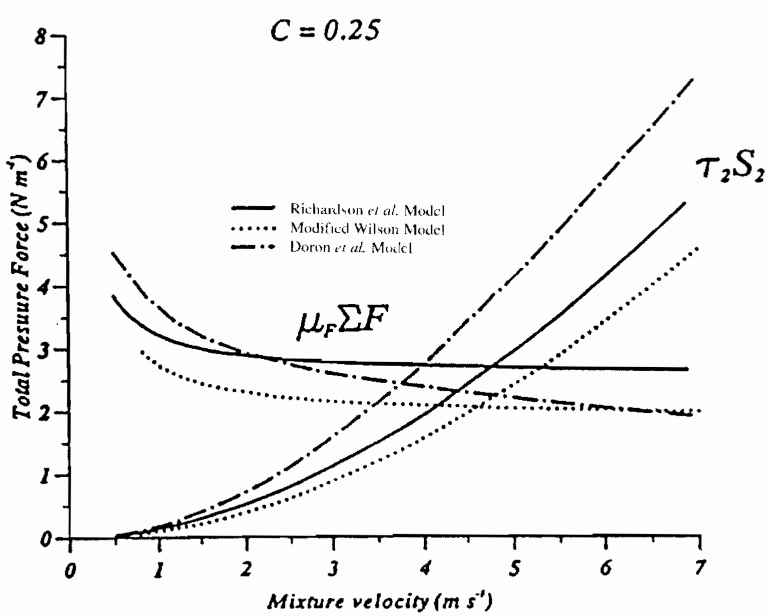

Fig.10. Force component comparison with different models for 3.5 mm gravel particles in 38 mm pipe for $C = 0.25$.

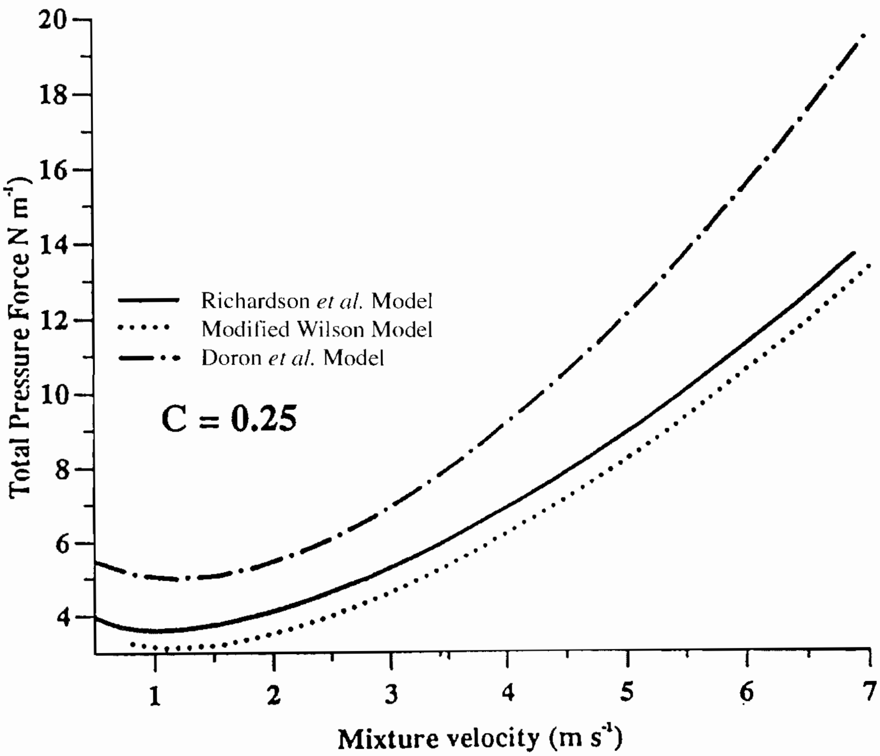

Fig.11. Compasrison of total force for all models for 3.5 mm gravel particles in 38 mm pipe for $C = 0.25$.

all three models. There is fair agreement, especially considering the different approaches used in the three models. Generally, the results predicted by the Doron model are significantly the highest. It should be noted that in all cases the coulombic friction decreases as velocity increases. This is because at constant delivered concentration C, the in-line concentration C_x decreases as velocity increases. In Fig. 11, the total force is plotted and this shows that the Doron model gives values that are about 40% higher than the others and, by comparison with the experimental results, this appears to be serious over-prediction. It is of interest to note that all three models predict a shallow minimum in the pressure gradient as velocity changes.

Symbols

A	Cross-sectional area	L^2
Ar	Archimedes Number $\frac{4}{3}\frac{d^3\rho_L(\rho_S-\rho_L)g}{\mu^2}$	—
C	Concentration, fractional volumetric (delivered value without suffix)	—
C_D	Drag coefficient on particle $\left(\frac{2\tau}{\rho_L V^2}\right)$	—
D	Pipe diameter on hydraulic mean diameter	L

d	Particle diameter	L
$\sum F$	Sum of normal forces at pipe wall due to particles in unit length of pipe	MT^{-2}
f	Pipe fluid friction factor $\left(\frac{2\tau}{\rho_L V^2}\right)$	—
F_i	Friction factor at the interface between the two layers as a surface of equivalent roughness d/D	—
g	Acceleration due to gravity	LT^{-2}
k	Pipe roughness	L
n	Parameter in relation to hindered settling	—
P	Pressure	$ML^{-1}T^{-2}$
Re	Reynolds number (pipe without suffix)	—
S	Part-perimeter on chord	L
V	Velocity (average value-no subscript)	LT^{-1}
x	Distance in direction of flow	L
ε	Turbulent mixing coefficient	—
ρ	Fluid or suspension density	ML^{-3}
ρ_L	Liquid density	ML^{-3}
μ_F	Coefficient of coulombic (solid–solid) friction between particles and pipe wall	—
θ	Half angle substended at centre of pipe by surface of bed	—
μ_L	Liquid viscosity	$ML^{-1}T^{-1}$
τ	Shear stress	$ML^{-1}T^{-2}$
ϕ	Angle of internal friction in solids	—

Suffixes

1	Upper layer
2	Lower layer
i	Interface between layers
x	In-line value
C	Value at concentration C
c	Contact load value
L	Carrier liquid
p	With respect to particle in fluid
o	Terminal falling condition

REFERENCES

Bagnold, R.A. 1954. Experiments on a gravity-free dispersion of large solid spheres in a Newtonian fluid under shear. Proceedings of the Royal Society. **A225:** 49–63.

Bagnold, R.A. 1957. The flow of cohesionless grains in fluids. Philosophical Transactions of the Royal Society. **A249:** 235–297.

Bird, R. B., Stewart, W.E. & Lightfoot, E.N. 1960. Transport Phenomena, Chapter 6. Wiley & Sons, New York, NY, USA.

Churchill, S.W. 1977. Friction factor equation spans all fluid flow requires. Chemical Engineering **84(24):** 90–91.

Colebrook, C.F. 1939. Turbulent flow in pipes with particular reference to the transition region between smooth and rough pipe laws. Journal of the Institution of Civil Engineers **11:** 133–156.

Doron, P., Granica, D. & Barnea, D. 1987. Slurry flow in horizontal pipes—experimental and modelling. International Journal of Multiphase Flow **13(4):** 535–547.

Doron, P. & Barnea, D. 1992. Effect of the no-slip assumption on the prediction of solid–liquid flow characteristics. International Journal of Multiphase Flow **18(4):** 617–622.

Doron, P., Granica, D. & Barnea, D. 1993. A three layer model for solid–liquid flow in pipes. International Journal of Multiphase Flow **19(6):** 1029–1043.

Doron, P. & Barnea, D. 1995. Pressure drop and limit deposit velocity for solid–liquid flow in pipes. Chemical Engineering Science **50(10):** 1595–1604.

Gillies, R.G., Shook, C.A. & Wilson, K.C. 1991. Improved two layer model for horizontal slurry pipeline flow. Canadian Journal of Chemical Engineering **69:** 173–178.

Newitt, D.M., Richardson, J.F., Abbott, M. & Turtle, R.B. 1955. Hydraulic conveying of solids in horizontal pipes. Transactions of the Institution of Civil Engineers **33:** 93–113.

Nikuradse, J. 1932. Gestzmassigkeit der turbulenten stromung in glatten rohen. Forschings Arbeit Ingenieur **No. 356**.

Pirie, R.L., Davies, T., Khan, A.R. & Richardson, J.F. 1988. Measurement of liquid velocity in multiphase flow by salt injection method, 2nd International Conference on Flow Measurement. BHRA, London, Paper F3: 187–193.

Pirie, R.L. 1990. Transport of coarse particles in water and shear thinning suspensions in horizontal pipes. Ph.D. Thesis. University of Wales, U.K.

Richardson, J.F. & Zaki, W.N. 1954. Sedimentation and Fluidization, Part 1. Transactions of the Institution of Civil Engineers. **32:** 35-53

Richardson, J.F., Khan, A.R. & Blackwell, R.L. 1996. Hydraulic conveying in small diameter horizontal pipes. Czechoslovakian Journal of Hydrology and Hydrodynamics **44(5):** 262–290.

Shook, C.A., Geller, L., Gillies, R.G., Husband, W.H.W. & Small, M. 1986. Experiments with coarse particles in a 250 mm pipeline. Hydrotransport 10. Proceedings of the 10th International Conference on the Hydraulic Transport of Solids in Pipes. BHRA Fluid Engineering, Cranfield, U.K. Paper **F4:** 219–227.

Shook, C.A. & Roco, M.C. 1991. Surry Flow; Principle and Practice, Chapter 6, The Two Layer Model. Butterworth-Heinemann, Oxford, U.K.

Taylor, G. 1954. The dispersion of matter in turbulent flow through a pipe. Proceedings of the Royal Society **A223:** 446–468.

Wilson, K.C. 1970. Stationary deposits and sliding beds in pipes transporting solids. Proceedings of the 1st International Conference on the Hydraulic Transport of Solids in Pipes. BHRA Fluid Engineering, Cranfield, U.K. Paper **C3:** 28–40.

Wilson, K.C., Streat, M. & Bantin, R.A. 1972. Slip model correlation of dense two phase flow. Proceedings of the 2nd International Conference on the Hydraulic Transport of Solids in Pipes. BHRA Fluid Engineering, Cranfield, U.K. Paper **B1:** 1–10.

Wilson, K.C. 1975. Co-ordinates for the limit of deposition in pipeline flow. Proceedings of the 3rd International Conference on the Hydraulic Transport of Solids in Pipes. BHRA Fluid Engineering, Cranfield, U.K. Paper **E1:** 1–13.

Wilson, K.C. 1988. Evaluation of interfacial fraction for pipeline transport models. Proceedings of the 11th International Conference on the Hydraulic Transport of Solids in Pipes. BHRA Fluid Engineering, Cranfield, U.K. Paper **B4:** 107–116.

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مقارنة النماذج الرياضية لسريان السوائل المحملة بالجزئيات كبيرة الحجم

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خلاصة

تعتمد النماذج الرياضية لحساب فقدان الضغط في السوائل المحملة بالجزئيات على اتـــزان القوى في السوائل وذلك يتطلب معرفة أو فرض توزيع الجزيئات في مساحة السريان وأكـــثر النماذج استخداماً لحساب فقدان الضغط نموذج الطبقة الثنائية حيث يفترض أن الجزيئات فـــي الطبقة العليا في حالة سريان معلق والجزيئات في الطبقة السفلي تتأثر بالاحتكاك مع الســـطح الأنبوبي وقد تم تطوير النموذج الرياضي مع مقارنة النتائج والقياسات المعمليـــة فـــي أنبـــوب أفقي ذو قطر يساوي 38 مم وتشمل المقارنة على ثلاثـــة صـــور مـــن النمـــوذج الريـــاضي وبالإضافة إلى ذلك تمت مقارنة نتائج السريان في أنبوب ذو قطـــر يســـاوي 263 مـــم مـــع نموذجين رياضيين.