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# An application of absorbing Markov analysis to the student flow in an academic institution

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## ABSTRACT

This paper presents an application of Markov analysis of student flow in a higher educational institution. Historical data of a random sample of 250 students of the Faculty of Science at Kuwait University was investigated. Results indicated that: (i) a freshman student has about 0.401 probability of graduating, (ii) freshman, sophomore, junior students stay on average 3, 2.3, 2.2 semesters at their respective levels before they pass on to the next level of study, while senior students stay longer, an average of 3.7 semesters, (iii) a high percentage of 39% of incoming freshman students withdraw from their study, and (iv) the probability of progression to a higher level and graduating increases as students move on to a higher level in the system.

**Keywords:** Fundamental matrix; Markov analysis; Recurrent state; Transition probabilities; Transient state.

## INTRODUCTION

Higher educational institutions in the state of Kuwait have encountered more challenges in recent years than perhaps any other segment of Kuwaiti society. The rapid expansion of programs, departments and colleges in Kuwait University in terms of both size and quality has had a profound impact on the entire society. The economic and social needs associated with higher educational institutions have caused many segments of Kuwaiti public to observe the performance of these organizations with renewed and increased interest.

The higher educational process also continues to demand a greater proportion of the country's financial resources. This cost is being borne solely by the government. During the past couple of decades, inspiriting numbers of high school graduates have placed tremendous pressures on the efficiencies of these institutions. Kuwait University consists of 11 faculties in which there are more than eighteen thousand students studying in different areas.

One result of these events is an increased interest in the performance of students in Kuwait University. Much of this attention centers on the time required for students to graduate. The purpose of this paper is to investigate the flow of students in the Faculty of Science at Kuwait University via Markov Analysis. Markov Analysis has been used in various social settings (Kemeny & Snell 1960, Leibman 1972, McNamara 1974, Merdith 1967, Bessent and Bessent 1980, Kwak *et al.* 1986, Ling & Guozhong 1987, Reid *et al.* 1989, Darmon & Rene 1990, Desai & Gupta 1996, Pandey & Khohkhajaikat 1996.) Applications in educational systems may be found in the May-June of 1972 special issue of Operations Research. Other applications of particular relevance to the present research are the study by Bessent and Bessent (1980) and the study by Kwak *et al.* (1986). Bessent and Bessent (1980) applied Markov Analysis to examine the student flow in a university department. The study was concerned with the progression of doctoral students to complete so as to determine if the number of current admissions was creating an undesirable future dissertation overload for supervising professors. The study by Kwak *et al.* (1986) was concerned with forecasting student enrollment variations for an academic institution. Both studies have reported valuable insights as a result of using Markov Analysis.

The outline of this paper is as follows. The next section describes the nature of the problem and more precisely sets the questions to be answered. The third section presents the conceptual framework that was suggested to analyze the matter. The fourth section analyzes the data and derives the required probabilities and matrices. Section 5 presents and discusses the results, and the final section presents conclusions and gives directions for future research.

## CONTEXT OF THE PROBLEM

The Faculty of Science follows the traditional two semester calendar. A semester consists of sixteen weeks and the academic year includes two semesters, fall (September-January), and winter (February-May). A typical B.Sc.program requires 126 credit hours and can be completed in 8 semesters of study i.e. 4 years, assuming that the student enrolls for every semester. There is also an optional 8 weeks summer course available. The official policy outlining the program of study at Faculty of Science requires that students attend continuously. Realistically, however, many students do not attend for 8 consecutive semesters due to lack of intellectual or academic desire necessary to study. A student may also be academically dismissed for one semester until they are reinstated to the University system for a final opportunity. In addition, students may face difficulties adjusting to the university environment and regulations. Finally, preliminary examination of Faculty of Science student progression has suggested that the average time a student spends in the program exceeds five years.

In light of these and other social and economic factors, a study of student flow at Faculty of Science in Kuwait University was undertaken to understand and explain the nature of student progression, to evaluate the options available to admission policy and decision makers, and to recommend appropriate adjustments to existing policies and procedures pertaining to admissions, transfer, leave of absence, and student advising and counseling services. This paper is designed to answer the following questions:

(i) At any given time, what is the probability that a student who has been admitted will graduate from Faculty of Science?

(ii) At any given time, what is the probability that a student who has been admitted will withdraw from Faculty of Science?

(iii) At any given time, what is the probability that a student who has been admitted will transfer to other colleges of Kuwait University?

(iv) What is the average length of time a student will spend in the program at each stage level?

## THE CONCEPTUAL FRAMEWORK

We consider a discrete time stochastic process with index set $Z^+ = \{0, 1, 2, ...\}$; that is, we have a sequence $\{X_n : n \in Z^+\}$ of random variables. As usual the subscript $n$ in $X_n$ stands for the time and $X_n$ denotes the state of the process at time n. The stochastic processes that we consider here satisfy the Markov property. The future of the process is independent of the past and depends only on the present state. In a probabilistic setting this means:

$$\begin{aligned} &P(X_{n+1} = j | X_0 = x_0, ..., x_{n-1}, X_n = i) \\ &= P(X_{n+1} = j | X_n = i) \\ &= P_{ij}. \end{aligned}$$

A stochastic process with this property is called a discrete-time Markov chain (DTMC) (Ross 1996). The quantity $P_{ij}$ determines the probability of moving from state $i$ to state $j$ in just one transition and all these quantities define the matrix of one-step transition probabilities $P$:

$$P = \begin{pmatrix} P_{11} & P_{12} & \cdot & \cdot & \cdot & P_{1k} \\ P_{21} & P_{22} & \cdot & \cdot & \cdot & P_{2k} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ P_{k1} & P_{k2} & \cdot & \cdot & \cdot & P_{kk} \end{pmatrix} = (P_{ij})$$

where the finite set $I_k = \{1, 2,...,k\}$ is called the state space of the Markov chain. The entries $P_{ij}$ of the matrix P must satisfy:

(i) $P_{ij} \geq 0$, (ii) $\Sigma_j P_{ij} = 1$, and $i, j \in I_k$.

## Transient, Recurrent, and Absorbing states

A state $i$ is said to be transient if it can be reached only finitely many times, i.e. $X_n =$ i for only finitely many $n$, and is said to be recurrent if it can be reached infinitely often. A recurrent state is called absorbing if there is no way to escape it once it is entered. A class $C$ of states is called recurrent if all its states are recurrent and closed if $P_{ij} = 0$ for all $i \in C, j \notin C$. If all nontransient states in the Markov chain are absorbing, the chain is called an absorbing Markov chain.

## Canonical form of the transition matrix

Based on the classifications of the states of being transient or recurrent (absorbing and non-absorbing), the transition matrix P can be partitioned into its canonical form:

$$P = \left(\begin{array}{c|c} I & 0 \\ \hline R & Q \end{array}\right)$$

If T denotes the set of transient states, then: $Q = (P_{ij} : i, j \in T)$. That is Q is the restriction of the matrix P to the states of T and

$$R = (P_{kl} : k \in T, 1 \notin T). \tag{1}$$

In addition, I stands for the identity matrix corresponding to the absorbing states and 0 is the zero-matrix.

## The Fundamental matrix

It is known that if the state space ( or the set T of transient states) of an absorbing Markov chain is finite, then the square matrix [I - Q] is invertible (Resnick 1994).

Let $N = (I - Q)^{-1}$, which is called the fundamental matrix and plays a vital role in analyzing Markov chains. Define an indicator random variable $I_{(X_n=j)}$ which takes value 1 if $X_n = j$ and 0 if $X_n \neq j$. It is known that for two transient states $i$ and $j$, the ij$^{th}$ entry $N_{ij}$ of the matrix N is:

$$N_{ij} = E_i \sum_{n=0}^{\infty} I_{(X_n=j)} = \sum_{n=0}^{\infty} P_{ij}^{(n)} \tag{2}$$

where $P_{ij}^{(n)}$ is the n-step transition probabilities. As such $N_{ij}$ stands for the mean number of visits to the state $j$ having started at state $i$.

Define $\tau = inf\{n \geq 0 : X_n \notin T\}$ to be exit time of T (the time to leave T). For an absorbing Markov chain, $\tau$ is the time of absorption. Let $M_i = E_i\tau$ denote the mean time of absorption starting at state $i$. Let M stands for the vector whose components are $M_i$, i $= 1, 2, ..., k$. It is known that:

$$M = N\xi$$

where $\xi$ is the vector that all of its components are 1; that is:

$$M_i = \sum_{j=1}^{k} N_{ij}$$

See Kemeny & Snell (1960, pg.51).

Another important quantity is defined as follows: for $i \in T$ and $j \notin T$, let:

$$u_{ij} = P(X_\tau = j | X_0 = i)$$

denote the probability that the chain starting in a transient state $i$ ends up in an absorbing state $j$. If U denotes the matrix whose entries are $u_{ij}$, it is known that:

$$U = NR$$

where N and R are given in Equations (1) and (2) respectively. See Resnick (1994 ,pg.105]. Finally, if $r_i$ denotes the number of times the chain remains in a non absorbing state $i$ once it is entered (including the entering step) then:

$$E_i(r_i) = \sum_{k=1}^{\infty} kP_i(r_i = k) = \sum_{k=1}^{\infty} kP_{ii}^{k-1}(1 - P_{ii}) = \frac{1}{1 - P_{ii}}.$$

## DATA ANALYSIS AND TRANSITION PROBABILITIES

Data was obtained from student's records of Kuwait University's student master file. The data sample covered the semesters from the fall semester 1996 - 1997 to the spring semester 2004 - 2005.

Historical data of students from the Faculty of Science were investigated. Individual student records containing academic status and grade reports of a random sample of 250 students were examined. These were students who had either graduated from or dropped out of the Faculty of Science. A student who stayed in the faculty for only one semester provided one piece of transition data from the state F (freshman) to either NR (not registered) or O (dropped out) after that semester. A student who stayed two or more semesters provided one piece of transition data for each semester that he / she enrolled; graduated from or dropped out of the faculty. A transition from one state to another was considered to take place at the end of the semester.

For the purpose of this paper, a Markov chain of 8 states was identified. A student in the Faculty of Science should complete 128 credit hours in less than 14 semesters to graduated. Each state was classified as follows:

F : Freshman, a student who successfully completed less than 31 credit hours.

So : Sophomore, a student who successfully completed between 31-60 credit hours.

J : Junior, a student who successfully completed between 61-90 credit hours.

Se : Senior, a student who successfully completed more than 91 credit hours.

G : Graduated from the Faculty of Science.

O : Dropped out or academically dismissed from Kuwait University.

NR : Not registered, perhaps to take care of personal problems, but eventually will return to the system.

T : Transferred between Faculty of Science and other Faculties in Kuwait University.

These states are classified in accordance with Faculty of Science curriculum.

Table (1) shows the frequency transition data. In this study, 380 pieces of transition data from (F) to (F) were gathered, 144 pieces of transition data from (F) to (So) have been gathered, etc. To elaborate, after a semester was finished, a freshman student, say, who successfully competed 15 credit hours provided one piece of transition data - from (F) to (F). Thus, the results of random sample of 250 individual students provided a total of 1237 pieces of transition data.

**TABLE 1:** Frequency transition of 250 random students from Faculty of Science

| | *F* | *So* | *J* | *Se* | *NR* | *G* | *O* | *T* | *Total* |
|---|---|---|---|---|---|---|---|---|---|
| *F* | 380 | 144 | 0 | 0 | 30 | 0 | 34 | 17 | 605 |
| *So* | 0 | 152 | 82 | 0 | 9 | 0 | 17 | 29 | 289 |
| *J* | 0 | 0 | 72 | 59 | 1 | 0 | 2 | 3 | 137 |
| *Se* | 0 | 0 | 0 | 117 | 2 | 40 | 2 | 0 | 161 |
| *NR* | 14 | 6 | 4 | 2 | 9 | 0 | 9 | 1 | 45 |
| *G* | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| *O* | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| *T* | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

The row totals from Table 1 were used to construct the corresponding probabilities. The matrix of transition probabilities was obtained by dividing each frequency by the appropriate row total. Table 2 shows the eight-by-eight matrix of transition probabilities P.

**TABLE 2:** Transition probability matrix P

| | *F* | *So* | *J* | *Se* | *NR* | *G* | *O* | *T* |
|---|---|---|---|---|---|---|---|---|
| *F* | 0.628 | 0.238 | 0 | 0 | 0.049 | 0 | 0.056 | 0.028 |
| *So* | 0 | 0.535 | 0.288 | 0 | 0.031 | 0 | 0.059 | 0.102 |
| *J* | 0 | 0 | 0.525 | 0.43 | 0.007 | 0 | 0.014 | 0.021 |
| *Se* | 0 | 0 | 0 | 0.726 | 0.012 | 0.248 | 0.012 | 0 |
| *NR* | 0.311 | 0.133 | 0.088 | 0.044 | 0.2 | 0 | 0.2 | 0.022 |
| *G* | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| *O* | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| *T* | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

Table 2 shows that states G, O and T are absorbing states, while the other states are transient states. The matrix P in its canonical form is:

| | *G* | *O* | *F* | *So* | *J* | *Se* | *NR* | *T* |
|---|---|---|---|---|---|---|---|---|
| *G* | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| *O* | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| *T* | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| *F* | 0 | 0.056 | 0.028 | 0.628 | 0.238 | 0 | 0 | 0.049 |
| *So* | 0 | 0.059 | 0.102 | 0 | 0.535 | 0.288 | 0 | 0.031 |
| *J* | 0 | 0.014 | 0.021 | 0 | 0 | 0.525 | 0.43 | 0.007 |
| *Se* | 0.248 | 0.012 | 0 | 0 | 0 | 0 | 0.726 | 0.012 |
| *NR* | 0 | 0.2 | 0.022 | 0.311 | 0.133 | 0.088 | 0.044 | 0.2 |

which is equivalent to:

$$P = \left(\begin{array}{c|c} I & 0 \\ \hline R & Q \end{array}\right)$$

and from which the matrix (I - Q ) is obtained.

$$I - Q = \begin{pmatrix} 0.372 & -0.238 & 0.000 & 0.000 & -0.049 \\ 0.000 & 0.465 & -0.288 & 0.000 & -0.031 \\ 0.000 & 0.000 & 0.475 & -0.43 & -0.007 \\ 0.000 & 0.000 & 0.000 & 0.274 & -0.012 \\ -0.311 & -0.133 & -0.088 & -0.044 & 0.8 \end{pmatrix}.$$

Hence, the fundamental matrix is given by $N$:

$$N = \begin{pmatrix} 2.916 & 1.570 & 1.003 & 1.617 & 0.272 \\ 0.117 & 2.250 & 1.390 & 2.204 & 0.140 \\ 0.063 & 0.054 & 2.152 & 3.389 & 0.076 \\ 0.051 & 0.043 & 0.038 & 3.719 & 0.061 \\ 1.163 & 0.993 & 0.860 & 1.572 & 1.391 \end{pmatrix}.$$

It follows from the elements of the main diagonal of the matrix N that the student needs, on average, about 3 semesters to finish the requirements of the freshman level. This period of time decreases as the student progresses to higher levels of study with one exception that the senior student stays for a lengthy period of time. This delay in moving to the next state, which is mostly the state of graduation, is due to the fact that the student needs to successfully finish 38 credit hours in order to reach the graduation state. It may also be due to difficulty of some courses at the senior level. This difficulty makes some students repeat courses intentionally in order to improve their grade point average (GPA).

In Kuwait University, students have the opportunity to repeat 10 courses with grades below C (less than 2.00) in order to improve the GPA. Therefore, many students repeat some courses in order to reach the minimum GPA requirement for graduation (2.00, C grade).

The vector $\mathrm{M} = N\xi$ can be now calculated to obtain:

$$M = \begin{pmatrix} 7.379 \\ 6.101 \\ 5.734 \\ 3.912 \\ 5.979 \end{pmatrix}.$$

We see from the vector M that a freshman student stays on average more than 7 semesters in the system before reaching one of the absorbing states, while a sophomore student needs about 6 semesters to reach the next state of absorbtion, etc. The matrix U turned out to be:

$$U = \begin{pmatrix} 0.401 & 0.344 & 0.269 \\ 0.547 & 0.213 & 0.265 \\ 0.841 & 0.093 & 0.054 \\ 0.922 & 0.063 & 0.008 \\ 0.39 & 0.433 & 0.183 \end{pmatrix}.$$

Taking a look at the matrix $U$, we notice that a freshman student has a probability 0.4 of progressing to the graduation state, a probability 0.34 of dropping out of Kuwait University and a probability 0.27 to transfer to other colleges of Kuwait university. We also notice that these probabilities change significantly as the student progresses to the higher levels.

If the non-absorbing states Freshman, Sophomore, Junior, Senior and Non-Registered states are assigned respectively the numbers 4, 5, 6, 7 and 8, then the mean number of times the chain remains in each state once entered (including the entering step) are calculated by the equation:

$$E_i(r_i) = \frac{1}{1 - P_{ii}}, \quad i = 3, 4, 5, 6, 7$$

where : $E_4(r_4) = \frac{1}{1-0.628} = 2.69; E_5(r_5) = \frac{1}{1-0.535} = 2.15; E_6(r_6) = \frac{1}{1-0.525} = 2.11; E_7(r_7) = \frac{1}{1-0.726} = 3.65$; and $E_8(r_8) = \frac{1}{1-0.2} = 1.25$.

One important fact to mention is that the mean number of changes of state in an absorbing chain can be calculated by setting $P_{ii} = 0$ for every transient state in the transition matrix P and then dividing each row by its row sum to obtain $P^*$. The $i^{th}$ component of the new vector $M^*$ gives the mean number of changes of the state for the original process. It is worth mentioning that $M^*$ differs in its meaning from M. For instance, the $i^{th}$ component in the two vectors may differ slightly if the repetition of the state $i$ is rare in the system. However, the components of the two vectors corresponding to some state may differ significantly if the repetition of that state occurs frequently.

Now, let $P^*$ be

$P^* =$

| | G | O | T | F | So | J | Se | NR |
|---|---|---|---|---|---|---|---|---|
| G | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| O | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| T | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| F | 0 | 0.151 | 0.075 | 0 | 0.642 | 0 | 0 | 0.132 |
| So | 0 | 0.123 | 0.213 | 0 | 0 | 0.600 | 0 | 0.065 |
| J | 0 | 0.030 | 0.044 | 0 | 0 | 0 | 0.911 | 0.015 |
| Se | 0.912 | 0.044 | 0 | 0 | 0 | 0 | 0 | 0.044 |
| NR | 0 | 0.251 | 0.028 | 0.390 | 0.167 | 0.110 | 0.055 | 0 |

$$P^* = \left( \begin{array}{c|c} I^* & 0 \\ \hline R^* & Q^* \end{array} \right)$$

from which we obtain $(I - Q)^*$ and the corresponding $N^*$.

$$(I-Q)^* = \begin{pmatrix} 1.000 & -0.642 & 0.000 & 0.000 & -0.132 \\ 0.000 & 1.000 & -0.600 & 0.000 & -0.065 \\ 0.000 & 0.000 & 1.000 & -0.911 & -0.015 \\ 0.000 & 0.000 & 0.000 & 1.000 & -0.044 \\ -0.390 & -0.167 & -0.110 & -0.055 & 1.000 \end{pmatrix}$$

$$N^* = \begin{pmatrix} 1.084 & 0.732 & 0.463 & 0.410 & 0.216 \\ 0.042 & 1.045 & 0.639 & 0.576 & 0.108 \\ 0.024 & 0.025 & 1.022 & 0.928 & 0.061 \\ 0.019 & 0.020 & 0.018 & 1.013 & 0.049 \\ 0.431 & 0.462 & 0.399 & 0.302 & 1.106 \end{pmatrix}$$

The new vector $M^*$ is now given by:

$$M^* = N^*\xi = \begin{pmatrix} 2.91 \\ 2.41 \\ 2.06 \\ 1.12 \\ 2.70 \end{pmatrix}.$$

As we mentioned earlier, the components of the vector $M^*$ represent the mean number of changes of the state for the original process. We notice that the $4^{th}$ component of the vector $M^*$ is too small compared with the $4^{th}$ component of the vector M, which This again confirms the remarkable repetitions of the senior level for the reasons pointed out in the comments on the main diagonal of the matrix N.

## RESULTS AND DISCUSSION

The transition probabilities for the student flow (shown in matrix $P$) may be interpreted as the probability that a student from a given state will be in another state at the next time period. For instance, it was found that the incoming freshman student had probabilities 0.628 of remaining a freshman, 0.049 of not registering, 0.056 of dropping out of the system, and 0.238 of advancing to the sophomore state.

The transition probabilities revealed that: (a) The failure rate among senior students is significantly high. This might require an investigative effort at the course levels so as to pinpoint the courses or group of courses which have high failure rate. A parallel investigation of the admission policies might be warranted. (b) About 60% of students who are not registered for some reasons come back to the system, about 20% of them drop out, and the rest (20%) remains non-registered.

The transition probabilities were also used to calculate the mean number of times (semesters) that a student in the program will be in a given state. These are the quantities $E_i(r_i)$, for $i = 4, 5, 6, 7, 8$.Senior students were seem to have on average the highest stay times of about 4 semesters. This confirms the fact presented in the past paragraph.

The matrix U showed that about 40% of students get absorbed in state G (graduated), 34% of freshman students get absorbed in the state O ( dropped out) and about 26% of students get absorbed in the state T (transferred). As anticipated, the percentages of students graduating from Faculty of Science increase as students advanced to higher levels. One observation to mention is that about 43.3% of non-registered students get absorbed in state O, about 18% of them get absorbed in state T, while the rest graduate (39%).

The number of times (semesters) a student may be expected to be in a given state b discovered before graduation or leaving the system was calculated in vector $M$. It is noted that the $4^{th}$ component of $M^*$ is too small compared with the corresponding component of $M$. This supports the observation that the students stay longer in the senior level, and suggests that extensive investigative efforts should be directed towards this issue. There are also differences between the components of $M$ and the corresponding components of $M^*$ which is due to the repetition in the corresponding states.

The minimum requirement for high school graduates for admission to the Faculty of Science is 70% or a GPA of 2.5. Most of the students with high GPAs are admitted to Colleges of Medical Sciences or Engineering, and usually the Faculty of Science is last choice. The core courses offered at the Faculty of Science such as calculus or physics are challenging, and many students face difficulty in passing these courses, which in turn increases the time period they spend in college.

Also we observed that the central acceptance policy of the Faculty of Science is better than accepting student to a direct program as it decreases the probability of dropping out at later stages. Once a freshman level student discovers that he is unable to continue in Faculty of Science, he can either transfer to another college or drop out. Course counseling for students at the time of admission will help them to choose the right courses according to the major sheet which suits their interest and capability. Also guidance can be offered to the students regarding an easier minor subject for their minor.

After admission, if a student feels that he is unable to pursue with his major, it is advisable to guide him to transfer to different college rather than dropping out in a late stage from the KU system. As a new policy to help the students, the number of repetitions that a student can make for a course so as to improve his grade has been increased to 10. This in turn increases the time period that a senior level student spends in the system, but gives more chance to graduate rather than drop out.

Also since the language of instruction for all the courses at Faculty of Science is English, many students find it difficult with language as well as the course materials. Thus, it is important to guide students for to take the four levels of English courses at an early stage.

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# تطبيق تحليل ماركوف لتدفق الطلاب لكلية العلوم بجامعة الكويت

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## خلاصة

يتناول البحث عملية تدفق الطلاب لكلية العلوم بجامعة الكويت ويقدم معالجة للعملية باستخدام تحليل ماركوف. استخدمت في التحليل عينة من 250 طالبا سحبت عينة عشوائية من بين طلاب كلية العلوم. وخلص البحث إلى بعض النتائج والتي فيها أن طالب المستوى الأول بالكلية ينتقل للمستوى التالي باحتمال يقدر بالقيمة 0,401، وكذلك يعطي البحث متوسط عدد الفصول التي يقضيها الطالب في كل مستوى من مستويات الدراسة. بالإضافة إلى ذلك يقدر البحث نسبة الطلاب الذين يتركون الكلية لأسباب مختلفة بنسبة 39٪. وأخيرا وكما هو متوقع فإن طلاب المستويات المتقدمة ينتقلون للمستويات التالية بمعدل أسرع من طلاب المستويات الأولى.