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Scheduling in stochastic bicriteria single machine systems with job-dependent learning effects

H. M. SOROUSH* AND F. O. AMIN**

**Department of Statistics and Operations Research, Kuwait University, P. O. Box 5969, Safat 13060, Kuwait; e-mail: h.soroush@ku.edu.kw*
Corresponding author
***6/46 Hiropi Street, Post Code 6021, Newtown Wellington, New Zealand.*
e-mail: fatma.o.a. sheikh@gmail.com

ABSTRACT

A stochastic bicriteria single machine scheduling problem with job-dependent learning effects in which the normal processing times of jobs (i.e., processing times without any learning effects) are random variables was studied. The job-dependent learning effects show that the random actual processing times are unique functions of the positions of jobs in a sequence. The goal was to derive the optimal sequence that minimizes the expected value of a general quadratic function of each pair of criteria consisting of the makespan, total completion time, total lateness, total waiting cost, total waiting time, total absolute differences in completion times, and the sum of earliness, tardiness and common due date penalty. The resultant problems were formulated as quadratic assignment problems that could be solved exactly or heuristically, and proved that their special cases with linear cost functions are solvable in polynomial time. Computational results on problems with quadratic assignment formulations indicated that near-optimal solutions can be obtained with attractive CPU times.

Keywords: Bicriteria; learning effect; scheduling; single machine; stochastic

INTRODUCTION

In classical scheduling problems, processing times are usually assumed to be known constants independent of the positions of jobs in a sequence. However, in many real world environments, especially in labor intensive systems, the actual processing time of a job is shorter if it is scheduled later in a sequence, due to the phenomenon known as the theory of "learning effect" (Badiru, 1992). In general, this theory states that the time required to process a single unit decreases continuously with the processing of additional units; thus, the unit costs decline due to the decline in processing times (Yelle, 1979). The impact of learning on productivity in manufacturing was first discovered by Wright (1936) in the aircraft industry, and was later observed in manufacturing and service organizations. Recently, there has been a growing interest in incorporating learning into scheduling problems (Biskup, 1999, 2008; Cheng & Wang, 2000; Wang & Li, 2011).

Most of the studies on scheduling with learning deal with the deterministic single machine scheduling problem involving one criterion and a linear objective function. For example, Biskup (1999) examined a single machine problem where the learning effect is defined in its popular form of the log-linear curve, a job-independent and position-based learning effect model, in which the actual processing time of a job is a function of the job position. Cheng & Wang (2000) introduced a volume-dependent processing time function to model the learning effects on processing times. Many studies have incorporated the learning model of Biskup (1999) into the single machine scheduling problem to optimize some performance criteria (Mosheiov, 2001; Zhao *et al.*, 2004; Eren & Guner, 2007; Wu *et al.*, 2007). Mosheiov & Sidney (2003) extended the Biskup's (1999) learning model to a job-dependent one. Mosheiov & Sidney (2005) defined learning effects by non-increasing job-dependent learning curves. Soroush (2012) studied a single machine problem with job-dependent past-sequence-dependent setup times and job-dependent position-based learning effects where the setup time and the actual processing time of a job are respectively defined as unique functions of the actual processing times of already processed jobs and the position of the job in the sequence. Lately, some researchers have examined single machine scheduling using other learning effect models. For example, Wang *et al.* (2009) analyze a case with exponential time-dependent learning. Wang & Li (2011) study a case with position-dependent and time-dependent learning. Huang *et al.* (2010) consider exponential learning and time-dependent job deterioration. Cheng *et al.* (2010) address a case with learning and job deterioration. To learn more about scheduling problems with various learning models, the reader is referred to, e.g., Biskup (2008), Cheng *et al.* (2011), Wang *et al.* (2009), and Zhang & Yan (2010).

In real world environments, scheduling decisions are made with respect to (w.r.t.) multiple criteria rather than a single criterion. These criteria are often conflicting and no single schedule would simultaneously optimize all criteria (Hoogeveen, 2005; T'Kindt & Billaut, 2001, 2002). Most of the literature on multiple criteria scheduling deals with the deterministic bicriteria single machine systems with linear objective functions. These studies can be divided into three groups. The first group finds the optimal sequence by minimizing a linear composite function of two criteria (Mani *et al.*, 2009; Mazdeh *et al.*, 2011; Shabtay *et al.*, 2010; Soroush, 2013a; and Yedidsion *et al.*, 2009). The second group minimizes a primary linear function w.r.t. one criterion subject to the constraint that a secondary linear function w.r.t. another criterion is attained for some specified value (Angel *et al.*, 2005; Chen & Sheen, 2007; Erenay *et al.*, 2010; Liu, 2010; Shabtay & Steiner, 2011; and Wang & Wang, 2012). The third group determines a set of *Pareto-optimal, efficient,* or *non-dominated* sequences (Gawiejnowicz *et al.*, 2006; Koksalan & Keha, 2003; Molaee *et al.*, 2010; and Steiner & Stephenson, 2007). A Pareto-optimal sequence is such that it is not possible to find another sequence with a better value in at least one criterion without worsening the value of at least one other criterion.

In comparison to deterministic single machine scheduling with learning effect and single criterion, the amount of literature on its bicriteria counterpart is very limited. For example, Mosheiov (2001) solves a single machine bicriteria scheduling problem to minimize simultaneously the total completion and variation of completion times. Lee *et al.* (2004) minimize a linear combination of the total completion time and the maximum tardiness. Mani *et al.* (2009) minimize the total completion time and total absolute differences in completion times. Lee *et al.* (2009) study a single machine scheduling problem with learning and release times to minimize the sum of makespan and total completion time.

Another important issue in real world scheduling systems is the stochasticity of job attributes (e.g., processing times, setup times) since these attributes are subject to random variability (Baker & Trietsch, 2009; Soroush & Alqallaf, 2009; Soroush, 2010a; and Sotskov & Lai, 2012). It is important to incorporate variations of job attributes into scheduling decisions because schedulers encounter such deviations. The significance of research in stochastic scheduling is also emphasized by the interest in synchronous manufacturing, which recognizes that variations in job attributes disrupt schedules (Umble & Srikanth, 1995).

There is a limited amount of literature on the stochastic bicriteria single machine scheduling problem. These studies do not consider learning effects and, either implicitly or explicitly, use linear objective functions of two criteria to derive the optimal sequences. For example, Forst (1995) addresses a bicriteria problem with random processing times and a common random due date to minimize the expected value of a linear function of the total weighted tardiness and total weighted flow time. Soroush & Fredendall (1994) investigate a problem with random processing times and deterministic due dates to minimize the expected value of a weighted linear function of job earliness and tardiness. Soroush (2007) studies a problem with random processing times and deterministic due date to minimize the expected value of a weighted linear function of the number of early and tardy jobs.

Recently, Soroush (2011, 2013b, 2013c) has studied some stochastic bicriteria single machine scheduling problems with linear/nonlinear cost functions. In particular, Soroush (2011) examines a problem with random processing times and sequence-dependent setup times. Soroush (2013b) utilizes two quadratic cost functions of various regular and non-regular performance criteria. The first function includes both the linear and quadratic terms of two regular criteria, while the second function possesses the linear and quadratic terms of a regular criterion and the linear term of a non-regular criterion. (We remark that quadratic functions have been used in deterministic scheduling problems with single criterion but without learning (Alidaee, 1993; Baker & Scudder, 1990; Lu & Sun, 2011; Soroush, 2010b; Valente & Goncalves, 2009; and Wei & Wang, 2010). Soroush (2013c) considers job attributes such as processing times, setup times, and reliabilities/un-reliabilities are sequence-dependent or position-dependent random variables and the learning effects are job-dependent and position-based. The objective is to derive the optimal sequences that minimize the expected values of linear, exponential, and fractional cost functions of different pairs of criteria.

In this paper, we extend the stochastic bicriteria single machine scheduling problem of Soroush (2013b) to a stochastic single machine scheduling problem with job-dependent and position-based learning effects wherein the normal processing times of jobs (i.e., processing times without any learning effects) are random variables. The job-dependent and position-based learning effects show that the random actual processing times are unique functions of the positions of jobs in a sequence. The aim is to find the optimal sequence that minimizes the expected value of a quadratic function of each pair of criteria consisting of the makespan, total completion time, total lateness, total waiting cost, total waiting time, total absolute differences in completion times, and the sum of earliness, tardiness and common due date penalty. The quadratic function allows the cost to grow nonlinearly with the criteria, and lets the scheduler to utilize not only the means but also the variances of criteria. Furthermore, this cost (or disutility) function can capture the behavior of decreasing risk averse or decreasing risk prone schedulers (Keeney & Raiffa, 1976). To the best of our knowledge, there are no prior studies on stochastic bicriteria scheduling dealing with learning effects and quadratic objective functions.

The organization of the rest of this paper is as follows. The stochastic bicriteria single machine scheduling problem with learning effect and quadratic cost function is formulated in the next section. The third section fully explores the problem w.r.t. various pairs of criteria, and introduces exact solution approaches. Polynomial time solutions are also presented for the special cases with linear cost functions. The fourth section contains some computational results. Finally, we give a summary and some concluding remarks.

PROBLEM FORMULATION

A set of n independent jobs are available at time zero for processing, without preemption and idle time insertions, on a continuously available single machine. Let p_i be the probabilistic normal processing time (i.e., the random processing time without any learning effects) of job $i, i = 1, \ldots, n$, and μ_i and ν_i be the mean and variance of p_i, respectively. Moreover, let $p_{[\,i\,]}$ and $p^A_{[\,i\,]}$ denote, respectively, the probabilistic normal and the probabilistic actual processing time of job $[\,i\,], [\,i\,] = 1, \ldots, n$, appearing in the ith position, $i = 1, \ldots, n$, of a sequence $S = ([1], \ldots, [\,i\,], \ldots [n]) \in \Psi$ where Ψ is the set of all sequences. The normal processing time of a job is incurred if the job is scheduled first in any S. The actual processing times of the following jobs in S are stochastically smaller than their normal processing times because of the learning phenomenon. Using the job-dependent and log-linear learning effect model of Mosheiov & Sidney (2003), $p^A_{[i]}, i = 1, \ldots, n$, is defined as

$$p^A_{[\,i\,]} = i^{a[\,i\,]} p_{[\,i\,]}, \tag{1}$$

where $a_{[i]} \leq 0$ is the learning index of job $[\,i\,]$. Let the learning model (1) be denoted by LE_g. Also, let $t_i, w_i, \gamma_i, d_i, L_i = t_i - d_i, E_i = max\{d_i - t_i, 0\}$, and $T_i = max\{t_i - d_i, 0\}$ represent, respectively, the completion time, waiting time, unit delay cost, due date, lateness, earliness time, and tardiness time of job i. Then, the makespan $MSP = max_{i=1,\ldots,n}\{t_i\}$, the total completion time $TCT = \sum_{i=1}^n t_i$, the total lateness $TL = \sum_{i=1}^n L_i$, the total waiting cost $TWC = \sum_{i=1}^n \gamma_i w_i$, the total waiting time $TWT = \sum_{i=1}^n w_i$, the total absolute differences in completion times $TADC = \sum_{i=1}^n \sum_{j=i}^n |\,t_j - t_i|$, and the sum of earliness, tardiness and common due-date penalty $ETCP = \sum_{i=1}^n (\pi E_i + \rho T_i + \xi d)$ where d is an unrestricted common due date (i.e., d is very large or it is a decision variable) and π, ρ and ξ are the unit earliness, tardiness and due date penalty.

We study a stochastic bicriteria single machine scheduling problem utilizing the learning effect model (1) and the general quadratic cost function of Soroush (2013b) given by

$$g(C_1, C_2) = \alpha C_1^2 + \beta C_1 + \delta C_2^2 + \theta C_2, \beta, \theta > 0, \tag{2}$$

where the criteria C_1 and C_2 are stochastic since they are functions of processing times. Note that if $\alpha, \delta > 0 (\alpha, \delta < 0, C_1 \leq -\beta/2\alpha, C_2 \leq -\theta/2\delta)$, then $g(C_1, C_2)$ is non-decreasing and convex (concave), and models the behavior of a *decreasing risk averse (decreasing risk prone*) scheduler w.r.t. both C_1 and C_2 (Keeney & Raiffa, 1976). If $\alpha > 0, \delta < 0, C_2 \leq -\theta/2\delta, g(C_1, C_2)$ is non-decreasing, convex w.r.t. C_1, concave w.r.t. C_2, and models the behavior of a scheduler who is *decreasing risk averse* w.r.t. C_1 and *decreasing risk prone* w.r.t.

C_2. If $\alpha < 0, \delta > 0, C_1 \leq -\beta/2\alpha, g(C_1, C_2)$ is non-decreasing, concave w.r.t. C_1, convex w.r.t. C_2, and models the behavior of a scheduler who is *decreasing risk prone* w.r.t. C_1 and *decreasing risk averse* w.r.t. C_2. In addition, we examine the special case with linear cost function $g(C_1, C_2) = \beta C_1 + \theta C_2, \beta, \theta > 0$, which models the behavior of a *risk neutral* scheduler w.r.t. C_1 and C_2.

Using the three-field notation of Graham *et al.* (1979), the proposed stochastic scheduling problem with learning is denoted by $1/LE_g/G(C_1, C_2)$ where $G(C_1, C_2) = E[g(C_1, C_2)]$, the expected value of $g(C_1, C_2)$, using (2), is defined as

$$G(C_1, C_2) = \alpha E[C_1^2] + \beta E[C_1] + \delta E[C_2^2] + \theta E[C_2], \beta, \theta > 0. \tag{3}$$

Thus, $G(C_1, C_2)$ is a function of the first and second moments of C_1 and C_2. In $1/LE_g/G(C_1, C_2)$, the goal is to find the optimal sequence $S^* = arg\min_{S\in\Psi}\{G_S(C_1, C_2)\}$ w.r.t. each pair of criteria consisting of *MSP, TCT, TL, TWC, TWT, TADC,* and *ETCP*.

THE STOCHASTIC BICRITERIA SCHEDULING PROBLEMS WITH LEARNING

The $1/LE_g/G(MSP, C_2)$ problem with C_2 = *TCT, TL, TWC, TWT, TADC* and *ETCP*

We first examine the problem $1/LE_g/G(MSP, C_2)$ where $C_2 = TCT$. Here, the goal of a scheduler is to find the sequence that minimizes the expected cost w.r.t. the completion time of the entire set of jobs and each individual job. Since the completion time $t_{[i]}$ of job $[i], i = 1, \ldots, n$, using (1), is given by

$$t_{[i]} = \sum_{k=1}^{i} k^{a_{[k]}} p_{[k]}, \tag{4}$$

then

$$MSP = t_{[n]} = \sum_{i=1}^{n} i^{a_{[i]}} p_{[i]}, \tag{5}$$

and

$$TCT = \sum_{i=1}^{n} \sum_{k=1}^{i} k^{a_{[k]}} p_{[k]} = \sum_{i=1}^{n} (n - i + 1)\, i^{a_{[i]}} p_{[i]}. \tag{6}$$

We now present the following lemma. (The proofs to all lemmas and corollaries are given in an appendix available at http://db.tt/06DjBBPm.)

Lemma 1.

(i) The optimal sequence for $1/LE_g/G(MSP, TCT)$ is the solution to a quadratic assignment problem (QAP) with the objective function:

$$Minimize \sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}x_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1,k\neq i}^{n}\sum_{\ell=j+1}^{n} u_{ijk\ell}x_{ij}x_{k\ell}, \tag{7}$$

where

$$q_{ij} = \left[[\beta + \theta(n-j+1)]\mu_i + [\alpha + \delta(n-j+1)^2]\left(\mu_i^2 + \nu_i\right)j^{a_i}\right]j^{a_i};$$

and

$$u_{ijk\ell} = \begin{cases} 2[\alpha + \delta(n-j+1)(n-\ell+1)]j^{a_i}\ell^{a_k}\mu_i\mu_k; \\ 0, \ otherwise. \end{cases}$$

(ii) *The optimal sequence for $1/LE_g, \alpha = \delta = 0/G(MSP, TCT)$ (i.e., $1/LE_g/G(MSP, TCT)$ when $g(C_1, C_2) = \beta C_1 + \theta C_2$) is the solution to a linear assignment problem (AP) whose costs are given by $q_{ij} = [\beta + \theta(n-j+1)]j^{a_i}\mu_i; i, j = 1, \ldots, n$.*

In general, QAP is defined as

$$Minimize \sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}x_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1,k\neq i}^{n}\sum_{\ell=1,\ell\neq j}^{n} u_{ijk\ell}x_{ij}x_{k\ell},$$

subject to: $\sum_{i=1}^{n} x_{ij} = 1, j = 1, \ldots, n, \sum_{j=1}^{n} x_{ij} = 1, i = 1, \ldots, n, x_{ij} = 0, 1; i, j = 1, \ldots, n.$

In $1/LE_g/G(C_1, C_2), x_{ij} = 1$ if job $i, i = 1, \ldots, n,$ is assigned to position $j, j = 1, \ldots, n,$ of a sequence and $x_{ij} = 0$ otherwise; q_{ij} is the cost for assigning job i to position j, and $u_{ijk\ell}$ is the interaction cost for assigning job i to position$j$ and job k to position $\ell; i, j, k, \ell = 1, \ldots, n, i \neq k, j < \ell$. Since QAP is NP-hard, some researchers have presented branch-and-bound (B & B) based exact methods for small QAP, and lower bound based heuristics for moderately large QAP (Adams & Johnson, 1994; Adams *et al.*, 2007; James *et al.*, 2009; Xia, 2010; Zhang *et al.* 2010). In addition, AP is defined as Minimize $\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}x_{ij}$ subject to $\sum_{i=1}^{n} x_{ij} = 1, \ j = 1, \ldots, n, \sum_{j=1}^{n} x_{ij} = 1,$ $i = 1, \ldots n, x_{ij} = 0, 1, i, j = 1, \ldots, n,$ which is solvable exactly in $O(n^3)$time (Papadimitriou & Steiglitz, 1982).

The objective of a scheduler in $1/LE_g/G(MSP, TL)$ is to minimize the expected cost w.r.t. the completion time of the entire set of jobs and the total deviations of job completion times from their due dates. The later criteria, given as $TL = TCT - n\bar{d}$ where $\bar{d} = \sum_{i=1}^{n} d_{[\,i\,]}/n$, is important in just-in-time (JIT) production systems where both job earliness and tardiness are undesirable.

Lemma 2.

(i) The optimal sequence for $1/LE_g/G(MSP, TL)$ is the solution to QAP with objective function (7) where

$$q_{ij} = \left[[\beta + (\theta - 2\delta n\bar{d})(n-j+1)]\mu_i + [\alpha + \delta(n-j+1)^2]\,(\mu_i^2 + \nu_i)j^{a_i}\right]j^{a_i}; i,j = 1,\ldots,n;$$

and

$$u_{ijk\ell} = \begin{cases} 2[\alpha + \delta(n-j+1)(n-\ell+1)]\,j^{a_i}\ell^{a_k}\mu_i\mu_k;\; i,j,k = 1,...,n,\; k \neq i,\; \ell = j+1,...,n; \\ 0,\; \textit{otherwise.} \end{cases}$$

(ii) *The optimal sequence for $1/LE_g, \alpha = \delta = 0/G(MSP, TL)$ is the solution to AP whose costs are given by $q_{ij} = [\beta + \theta(n-j+1)]\,j^{a_i}\mu_i; i,j = 1,\ldots,n.$*

In $1/LE_g/G(MSP, TWC)$, a scheduler's aim is to find the sequence that minimizes the expected cost w.r.t. both the completion time of the entire set of jobs and the waiting cost of each individual job.

Since $w_{[i]} = \sum_{k=1}^{i-1} k^{a_{[k]}}p_{[k]}$, then

$$TWC = \sum_{i=1}^{n} \gamma_{[\,i\,]}w_{[\,i\,]} = \sum_{i=1}^{n} i^{a_{[\,i\,]}}p_{[\,i\,]}\sum_{k=i+1}^{n}\gamma_{[k]}. \tag{8}$$

Substituting (5) and (8) into (3), we obtain

$$\begin{aligned} G(MSP,TWC) \;&= \sum_{i=1}^{n}\left[(\beta + \theta\sum_{k=i+1}^{n}\gamma_{[k]})E[p_{[i]}] + [\alpha + \delta(\sum_{k=i+1}^{n}\gamma_{[k]})^2]i^{a_{[i]}}E[p_i^2]\right]i^{a_{[i]}} \\ &+2\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\alpha + \delta\sum_{k=i+1}^{n}\gamma_{[k]}\sum_{\ell=j+1}^{n}\gamma_{[\ell]})\,i^{a_{[i]}}j^{a_{[j]}}E[p_{[i]}]E[p_{[j]}]. \end{aligned}$$

Since $\mu_{[i]} = E[p_{[i]}]$ and $E[p_{[i]}^2] = \mu_{[i]}^2 + \nu_{[i]}$, then

$$\begin{aligned} G(MSP,TWC) \;&= \sum_{i=1}^{n}\left[(\beta + \theta\sum_{k=i+1}^{n}\gamma_{[k]})\mu_{[i]} + [\alpha + \delta(\sum_{k=i+1}^{n}\gamma_{[k]})^2](\mu_{[i]}^2 + \nu_{[i]})i^{a_{[i]}}\right]i^{a_{[i]}} \\ &+2\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\alpha + \delta\sum_{k=i+1}^{n}\gamma_{[k]}\sum_{\ell=j+1}^{n}\gamma_{[\ell]})\,i^{a_{[i]}}j^{a_{[j]}}\mu_{[i]}\mu_{[j]}. \end{aligned} \tag{9}$$

For general unit delay costs (or weights) $\gamma_i, i = 1, \ldots, n$, it is difficult to minimize (9). However, this can be done if $\gamma_{[i]} = \tau^i, \tau > 1 (0 < \tau < 1), i = 1, \ldots, n$, that is, if the weight for job $[i], [i] = 1, \ldots, n$, increases (decreases) nonlinearly with the job position $i, i = 1, \ldots, n$. The use of this weight function can be justified, e.g., in the scheduling problem of boarding different classes of passengers into an aircraft, and in the production scheduling of items involving in the ABC inventory system.

Lemma 3.

(i) *The optimal sequence for* $1/LE_g, \gamma_{[i]} = \tau^i, 0 < \tau \neq 1/G(MSP, TWC)$ *is the solution to QAP with objective function (7) where*

$$q_{ij} = \left[[\beta + \frac{\theta(\tau^{j+1} - \tau^{n+1})}{1 - \tau}]\mu_i + [\alpha + \frac{\delta(\tau^{j+1} - \tau^{n+1})^2}{(1 - \tau)^2}] (\mu_i^2 + \nu_i) j^{a_i} \right] j^{a_i}, i, j = 1, \ldots, n;$$

and

$$u_{ijk\ell} = \begin{cases} 2[\alpha + \dfrac{\delta(\tau^{j+1} - \tau^{n+1})(\tau^{\ell+1} - \tau^{n+1})}{(1 - \tau)^2}] j^{a_i} \ell^{a_k} \mu_i \mu_k; \ i, j, k = 1, ..., n, \ k \neq i, \ \ell = j + 1, ..., n; \\ 0, \ otherwise. \end{cases}$$

(ii) *The optimal sequence for* $1/LE_g, \gamma_{[i]} = \tau^i, 0 < \tau \neq 1, \alpha = \delta = 0/G(MSP, TWC)$ *is the solution to AP with costs* $q_{ij} = [\beta + \theta(\tau^{j+1} - \tau^{n+1})/(1 - \tau)] j^{a_i} \mu_i, i, j = 1, \ldots, n$.

Note that when $\gamma_i = 1, i = 1, \ldots, n, 1/LE_g/G(MSP, TWC)$ reduces to $1/LE_g/G(MSP, TWT)$ (i.e., $1/LE_g, \gamma_i = 1/G(MSP, TWC)$). The following corollary solves this special case.

Corollary 1.

(i) The optimal sequence for $1/LE_g/G(MSP, TWT)$ *is the solution to QAP with objective function (7) where*

$$q_{ij} = \left[[\beta + \theta(n - j)]\mu_i + [\alpha + \delta(n - j)^2] (\mu_i^2 + \nu_i) j^{a_i} \right] j^{a_i}; i.j = 1, \ldots, n,$$

and

$$u_{ijk\ell} = \begin{cases} 2[\alpha + \delta(n - j)(n - \ell)] j^{a_i} \ell^{a_k} \mu_i \mu_k; \ i, j, k = 1, ..., n, \ k \neq i, \ \ell = j + 1, ..., n; \\ 0, \ otherwise. \end{cases}$$

(ii) The optimal sequence for $1/LE_g, \alpha = \delta = 0/G(MSP, TWT)$ *is the solution to AP whose costs are given by* $q_{ij} = [\beta + \theta(n - j)] j^{a_i} \mu_i; i, j = 1, \ldots, n$.

In $1/LE_g/G(MSP, TADC)$, the goal of a scheduler is to minimize the expected cost w.r.t. the completion time of the entire set of jobs and the total absolute differences in completion times. This is essential in reducing inventory costs and the variations in job completion times so that, e.g., the finished jobs can be delivered together in batches.

For the deterministic single criterion problem $1//TADC$, Kanet (1981) introduced $TADC = \sum_{i=1}^{n}(i-1)(n-i+1)p_{[i]}$ as a measure of variation in completion times, and showed that the optimal sequence is V-shaped (i.e., a subset of jobs placed in the longest processing time (LPT) order is followed by the remaining jobs in the shortest processing times (SPT) order). However, in $1/LE/G(MSP, TADC)$, the optimal sequence is not V-shaped because, using (1),

we have

$$TADC = \sum_{i=1}^{n}(i-1)(n-i+1)i^{a_{[i]}}p_{[i]}, \tag{10}$$

which is influenced by MSP and learning effects.

Lemma 4.

(i) *The optimal sequence for* $1/LE_g/G(MSP, TADC)$ *is the solution to QAP with objective function (7) where*

$$q_{ij} = \left[[\beta + \theta(j-1)(n-j+1)]\mu_i + [\alpha + \delta(j-1)^2(n-j+1)^2](\mu_i^2 + \nu_i)j^{a_i}\right]j^{a_i}; i,j = 1, \ldots, n;$$

and

$$u_{ijk\ell} = \begin{cases} 2[\alpha + \delta(j-1)(\ell-1)(n-j+1)(n-\ell+1)]j^{a_i}\ell^{a_k}\mu_i\mu_k; \; i,j,k = 1,...,n, \; k \neq i, \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \ell = j+1,...,n; \\ 0, \; otherwise. \end{cases}$$

(ii) *The optimal sequence for* $1/LE_g, \alpha = \delta = 0/G(MSP, TADC)$ *is the solution to AP whose costs are* $q_{ij} = [\beta + \theta(j-1)(n-j+1)]j^{a_i}\mu_i; i,j = 1, \ldots, n.$

Finally, the objective of a scheduler in $1/LE_g/G(MSP, ETCP)$ is to jointly minimize the expected cost w.r.t. the completion time of the entire set of jobs and the sum of earliness, tardiness and common due-date penalty $ETCP = \sum_{i=1}^{n}(\pi E_i + \rho T_i + \xi d)$ where d is an unrestricted common due date, and π, ρ and ξ are the unit earliness, tardiness, and due date penalty, respectively. The later criteria is also significant in JIT production systems where job earliness and tardiness costs as well as the costs of assigning due dates need to be minimized.

Panwalkar *et al.* (1982) gave the following useful results for the deterministic single criterion problem $1//ETCP$. (i) The optimal sequence is V-shaped, that is, early jobs are arranged in LPT order and tardy jobs are arranged in SPT order; (ii) the optimal due date is the completion time of the $k-$th job in the optimal sequence where k is the smallest integer greater than or equal to $(n\rho - n\xi)/(\pi + \rho)(i.e., k = \lceil (n\rho - n\xi)/(\pi + \rho) \rceil)$; and (iii) the positional weight of a job when scheduled in position $r, r = 1, \ldots, n$, in the sequence is given by

$$\lambda_r = min\{n\xi + (r-1)\pi, (n+1-r)\rho\}. \tag{11}$$

In $1/LE/G(MSP, ETCP)$, the optimal sequence is not also V-shaped since, using (1), we have

$$ETCP = \sum_{i=1}^{n} (\pi E_i + \rho T_i + \xi d) = \sum_{i=1}^{n} \lambda_i i^{a_{[i]}} p_{[i]}, \tag{12}$$

which is affected by MSP and learning effects.

Lemma 5.

(i) The optimal sequence for $1/LE_g/G(MSP, ETCP)$ is the solution to QAP with objective function (7) where

$$q_{ij} = \left[(\beta + \theta\lambda_j)\mu_i + (\alpha + \delta\lambda_j)\left(\mu_i^2 + \nu_i\right)j^{a_i}\right]j^{a_i}; i,j = 1, \ldots, n;$$

and

$$u_{ijk\ell} = \begin{cases} 2(\alpha + \delta\lambda_j\lambda_\ell) j^{a_i} \ell^{a_k} \mu_i \mu_k; \;\; i,j,k = 1, ..., n, \;\; k \neq i, \;\; \ell = j+1, ..., n; \\ 0, \;\; otherwise; \end{cases}$$

where λ_j is defined by (11).

(ii) The optimal sequence for $1/LE_g, \alpha = \delta = 0/G(MSP, ETCP)$ is the solution to AP whose costs are given by $q_{ij} = (\beta + \theta\lambda_j) j^{a_i} \mu_i; i,j = 1, \ldots, n$.

Lemmas 1-5 indicate that scheduling decisions in the proposed stochastic bicriteria scheduling problem can be affected by the stochasticity of job attributes (i.e., their means and variances), the job-dependent learning effects, the convexity, concavity, or linearity of the cost functions (i.e., the decreasing risk averse, decreasing risk prone, or risk neutral behavior of schedulers), and the two criteria. The results also show that, unlike the deterministic single criterion problem which yields the same optimal sequence w.r.t. TCT, TL, and TWT, the optimal sequences for the proposed stochastic bicriteria problem w.r.t pairs of criteria (MSP, TCT), (MSP, TL), and (MSP, TWT) can be different.

The $1/LE_g/G(TCT,C_2)$ problem with $C_2 = TL, TWC, TWT, TADC$ and $ETCP$

A scheduler's goal in the problem $1/LE_g/G(TCT, C_2)$ where $C_2 = TL$ is to find the sequence that minimizes the expected cost w.r.t. the completion time of each job and the deviation of each job's completion time from its due date.

Lemma 6.

(i) The optimal sequence for $1/LE_g/G(TCT, TL)$ is the solution to QAP with objective function (7) where

$$q_{ij} = \left[(n-j+1)[(\beta+\theta-2\delta n\bar{d})\mu_i + (\alpha+\delta)(n-j+1)^2(\mu_i^2+\nu_i)j^{a_i}]\right]j^{a_i};\, i,j = 1,\ldots,n;$$

and

$$u_{ijk\ell} = \begin{cases} 2(\alpha+\delta)(n-j+1)(n-\ell+1)]j^{a_i}\ell^{a_k}\mu_i\mu_k; & i,j,k = 1,\ldots,n,\ k \neq i,\ \ell = j+1,\ldots,n; \\ 0, & \textit{otherwise}. \end{cases}$$

(ii) The optimal sequence for $1/LE_g, \alpha = \delta = 0/G(TCT, TL)$ is the solution to AP whose costs are given by $q_{ij} = (\beta+\theta)(n-j+1)]j^{a_i}\mu_i;\, i,j = 1,\ldots,n.$

In $1/LE_g/G(TCT, TWC)$, a scheduler's aim is to minimize the expected cost w.r.t. the completion time and the waiting cost of each job. Substituting (6) and (8) into (3), we obtain

$$\begin{aligned} &G(TCT,TWC) = \\ &\sum_{i=1}^{n}\left[[\beta(n-i+1)+\theta\sum_{k=i+1}^{n}\gamma_{[k]}]\mu_{[i]} + [\alpha(n-i+1)^2+\delta(\sum_{k=i+1}^{n}\gamma_{[k]})^2](\mu_{[i]}^2+\nu_{[i]})i^{a_{[i]}}\right]i^{a_{[i]}} \\ &+2\sum_{i=1}^{n}\sum_{j=i+1}^{n}[\alpha(n-i+1)(n-j+1)+\delta\sum_{k=i+1}^{n}\gamma_{[k]}\sum_{\ell=j+1}^{n}\gamma_{[\ell]}]\, i^{a_{[i]}}j^{a_{[j]}}\mu_{[i]}\mu_{[j]}. \end{aligned} \tag{13}$$

For general $\gamma_i, i = 1,\ldots,n$, it is hard to minimize (13); however, S^* can be derived when $\gamma_{[i]} = \tau^i, 0 < \tau \neq 1, i = 1,\ldots,n.$

Lemma 7.

(i) The optimal sequence for $1/LE_g, \gamma_{[i]} = \tau^i, 0 < \tau \neq 1/G(TCT, TWC)$ is the solution to QAP with objective function (7) where

$$q_{ij} = \left[[\beta(n-j+1) + \frac{\theta(\tau^{j+1} - \tau^{n+1})}{1-\tau}]\mu_i + [\alpha(n-j+1)^2 + \frac{\delta(\tau^{j+1} - \tau^{n+1})^2}{(1-\tau)^2}]\,(\mu_i^2 + \nu_i)j^{a_i} \right] j^{a_i}, \quad i,j = 1, ..., n;$$

and

$$u_{ijk\ell} = \begin{cases} 2[\alpha(n-j+1)(n-\ell+1) + \dfrac{\delta(\tau^{j+1} - \tau^{n+1})(\tau^{\ell+1} - \tau^{n+1})}{(1-\tau)^2}] j^{a_i}\ell^{a_k}\mu_i\mu_k; \\ \qquad i,j,k = 1, ..., n, \ k, \ \ell = j+1, ..., n; \\ 0, \ \textit{otherwise}. \end{cases}$$

(ii) The optimal sequence for $1/LE_g, \gamma_{[i]} = \tau^i, 0 < \tau \neq 1, \alpha = \delta = 0/G(TCT, TWC)$ *is the solution to AP with costs* $q_{ij} = [\beta(n-j+1) + \theta(\tau^{j+1} - \tau^{n+1})\,/(1-\tau)] j^{a_i}\mu_i; i,j = 1, \ldots, n.$

Corollary 2.

(i) The optimal sequence for $1/LE_g/G(TCT, TWT)$ *(i.e.,* $1/LE_g, \gamma_i = 1/G(TCT, TWC)$*) is the solution to QAP with objective function (7) where*

$$q_{ij} = \left[[\beta(n-j+1) + \theta(n-j)]\mu_i + [\alpha(n-j+1)^2 + \delta(n-j)^2]\,(\mu_i^2 + \nu_i)j^{a_i} \right] j^{a_i}; i,j = 1, \ldots, n,$$

and

$$u_{ijk\ell} = \begin{cases} 2[\alpha(n-j+1)(n-\ell+1) + \delta(n-j)(n-\ell)] j^{a_i}\ell^{a_k}\mu_i\mu_k; \ i,j,k = 1, ..., n, \ k \neq i, \\ \qquad \ell = j+1, ..., n; \\ 0, \ \textit{otherwise}. \end{cases}$$

(ii) The optimal sequence for $1/LE_g, \alpha = \delta = 0/G(TCT, TWT)$ *(i.e.,* $1/LE_g, \alpha = \delta = 0, \gamma_i = 1/G(TCT, TWC)$*) is the solution to AP with costs* $q_{ij} = [\beta(n-j+1) + \theta(n-j)] j^{a_i}\mu_i; i,j = 1, \ldots, n.$

The next two lemmas derive the optimal sequences for $1/LE_g/G(TCT, TADC)$ and $1/LE_g/G(TCT, ETCP)$. That is, when a scheduler's goal is to minimize the expected cost w.r.t. the completion time of each job and either the deviation of the job's completion time from its due-date or the total earliness, tardiness and common due-date penalty for the job.

Lemma 8.

(i) The optimal sequence for $1/LE_g/G(TCT, TADC)$ *is the solution to QAP with objective function (7) where*

$$q_{ij} = \left[[\beta+\theta(j-1)](n-j+1)\mu_i + [\alpha+\delta(j-1)^2]\,(n-j+1)^2(\mu_i^2+\nu_i)j^{a_i}\right]j^{a_i}; i,j = 1,\ldots,n;$$

and

$$u_{ijk\ell} = \begin{cases} 2[\alpha+\delta(j-1)(\ell-1)]\,(n-j+1)(n-\ell+1)j^{a_i}\ell^{a_k}\mu_i\mu_k;\ i,j,k = 1,...,n,\ k\neq i,\ \ell = j+1,...,n; \\ 0,\ \textit{otherwise}. \end{cases}$$

(ii) The optimal sequence for $1/LE_g, \alpha = \delta = 0/G(TCT, TADC)$ *is the solution to AP whose costs are* $q_{ij} = [\beta+\theta(j-1)](n-j+1)j^{a_i}\mu_i; i,j = 1,\ldots,n.$

Lemma 9.

(i) The optimal sequence for $1/LE_g/G(TCT, ETCP)$ *is the solution to QAP with objective function (7) where*

$$q_{ij} = \left[[\beta(n-j+1)+\theta\lambda_j]\mu_i + [\alpha(n-j+1)^2+\delta\lambda_j^2]\,(\mu_i^2+\nu_i)j^{a_i}\right]j^{a_i}; i,j = 1,\ldots,n;$$

and

$$u_{ijk\ell} = \begin{cases} 2[\alpha(n-j+1)(n-\ell+1)\ +\delta\lambda_j\lambda_\ell]j^{a_i}\ell^{a_k}\mu_i\mu_k;\ i,j,k = 1,...,n,\ k\neq i,\ \ell = j+1,...,n; \\ 0,\ \textit{otherwise}. \end{cases}$$

(ii) The optimal sequence for $1/LE_g, \alpha = \delta = 0/G(TCT, ETCP)$ *is the solution to AP whose costs are* $q_{ij} = [\beta(n-j+1)+\theta\lambda_j]j^{a_i}\mu_i; i,j = 1,\ldots,n.$

Based on Lemmas 6-9, we observe that scheduling decisions can be affected by the stochasticity of job attributes, the job-dependent learning effects, the characteristics of the cost functions, and the two criteria. The results also indicate that, on contrary to the deterministic problem w.r.t. single criterion *TCT, TL*, and *TWT*, the optimal sequences for the proposed bicriteria problem w.r.t pairs of criteria (*TCT,TL)* and (*TCT,TWT*) can be different.

The $1/LE_g/G(TL,C_2)$ problem with C_2 = *TWC, TWT, TADC* and *ETCP*

The following lemmas and corollary (Lemmas 10-12 and Corollary 3) solve the four problems $1/LE_g, \gamma_{[i]} = \tau^i, 0 < \tau \neq 1/G(TL, TWC); 1/LE_g/G(TL, TWT)$; $1/LE_g/G(TL, TADC)$; and $1/LE_g/G(TL, ETCP)$. That is, they derive the optimal sequences when a scheduler is interested in minimizing the expected costs w.r.t. the total deviations of job completion times from due dates and the total waiting cost, the total waiting time, the total absolute deviations of completion times, or the total earliness, tardiness and common due-date penalty.

Lemma 10.

(i) The optimal sequence for $1/LE_g, \gamma_{[i]} = \tau^i, 0 < \tau \neq 1/G(TL, TWC)$ *is the solution to QAP with objective function (7) where*

$$q_{ij} = \left[[(\beta - 2\alpha n\bar{d})(n-j+1) + \frac{\theta(\tau^{j+1} - \tau^{n+1})}{1-\tau}]\mu_i + [\alpha(n-j+1)^2 + \frac{\delta(\tau^{j+1} - \tau^{n+1})^2}{(1-\tau)^2}]\,(\mu_i^2 + \nu_i) j^{a_i} \right] j^{a_i}, \quad i,j = 1,...,n;$$

and

$$u_{ijk\ell} = \begin{cases} 2[\alpha(n-j+1)(n-\ell+1) + \frac{\delta(\tau^{j+1} - \tau^{n+1})(\tau^{\ell+1} - \tau^{n+1})}{(1-\tau)^2}]\, j^{a_i} \ell^{a_k} \mu_i \mu_k; \\ \qquad i,j,k = 1,...,n,\ k \neq i,\ \ell = j+1,...,n; \\ 0, \ \textit{otherwise}. \end{cases}$$

(ii) *The optimal sequence for* $1/LE_g, \gamma_{[i]} = \tau^i, 0 < \tau \neq 1, \alpha = \delta = 0/G(TCT, TWC)$ *(see Lemma 7(ii)) is also optimal for* $1/LE_g, \gamma_{[i]} = \tau^i, 0 < \tau \neq 1, \alpha = \delta = 0/G(TL, TWC)$.

Corollary 3.

(i) *The optimal sequence for* $1/LE_g/G(TL, TWT)$ $(i.e., 1/LE_g, \gamma_i = 1/G(TL, TWC))$ *is the solution to QAP with objective function (7) where*

$$q_{ij} = \left[[(\beta - 2\alpha n\bar{d})(n-j+1) + \theta(n-j)]\mu_i + [\alpha(n-j+1)^2 + \delta(n-j)^2]\,(\mu_i^2 + \nu_i) j^{a_i} \right] j^{a_i}; \quad i,j = 1,...,n;$$

and

$$u_{ijk\ell} = \begin{cases} 2[\alpha(n-j+1)(n-\ell+1) + \delta(n-j)(n-\ell)]\, j^{a_i} \ell^{a_k} \mu_i \mu_k; \ i,j,k = 1,...,n,\ k \neq i, \\ \qquad \ell = j+1,...,n; \\ 0, \ \textit{otherwise}. \end{cases}$$

(ii) *The optimal sequence for* $1/LE_g, \alpha = \delta = 0/G(TCT, TWT)$ *(see Corollary 2(ii)) is also optimal for* $1/LE_g, \alpha = \delta = 0/G(TL, TWT)$ $(i.e., 1/LE_g, \alpha = \delta = 0, \gamma_i = 1/G(TL, TWC))$.

Lemma 11.

(i) *The optimal sequence for* $1/LE_g/G(TL, TADC)$ *is the solution to QAP with objective function (7) where*

$$q_{ij} = \left[[\beta - 2\alpha n\bar{d} + \theta(j-1)](n-j+1)\mu_i + [\alpha + \delta(j-1)^2]\,(n-j+1)^2(\mu_i^2 + \nu_i) j^{a_i} \right] j^{a_i}; \quad i,j = 1,...,n;$$

and

$$u_{ijk\ell} = \begin{cases} 2[\alpha + \delta(j-1)(\ell-1)]\,(n-j+1)(n-\ell+1)j^{a_i}\ell^{a_k}\mu_i\mu_k;\ i,j,k = 1,\ldots,n,\ k \neq i,\ \ell = j+1,\ldots,n; \\ 0,\ \textit{otherwise}. \end{cases}$$

(ii) The optimal sequence for $1/LE_g, \alpha = \delta = 0/G(TCT, TADC)$ *(see Lemma 8(ii)) is also optimal for* $1/LE_g, \alpha = \delta = 0/G(TL, TADC)$.

Lemma 12.

(i) The optimal sequence for $1/LE_g/G(TL,ETCP)$ *is the solution to QAP with objective function (7) where*

$$q_{ij} = \Big[[(\beta - 2\alpha n\bar{d})(n-j+1) + \theta\lambda_j]\mu_i + [\alpha(n-j+1)^2 + \delta\lambda_j^2]\,(\mu_i^2 + \nu_i)j^{a_i}\Big]j^{a_i}; \quad i,j = 1,\ldots,n;$$

and

$$u_{ijk\ell} = \begin{cases} 2[\alpha(n-j+1)(n-\ell+1) + \delta\lambda_j\lambda_\ell]j^{a_i}\ell^{a_k}\mu_i\mu_k;\ i,j,k = 1,\ldots,n,\ k \neq i,\ \ell = j+1,\ldots,n; \\ 0,\ \textit{otherwise}. \end{cases}$$

(ii) The optimal sequence for $1/LE_g, \alpha = \delta = 0/G(TCT,ETCP)$ *(see Lemma 9(ii)) is also optimal for* $1/LE_g, \alpha = \delta = 0/G(TL,ETCP)$.

The $1/LE_g/G(TWC,TADC)$; $1/LE_g/G(TWC,ETCP)$; and $1/LE_g/G(TADC,ETCP)$ problems

A scheduler's objectives in the problems $1/LE_g/G(TWC,TADC)$; $1/LE_g/G(TWC,ETCP)$; and $1/LE_g/G(TADC,ETCP)$ are to minimize the expected costs w.r.t. the total waiting cost and either the total absolute deviations of completion times or the total earliness, tardiness and common due-date penalty.

For $1/LE_g/G(TWC,TADC)$, substituting (8) and (10) into (3), we obtain

$$G(TWC,TADC) = \sum_{i=1}^{n}\Bigg[[\theta(i-1)(n-i+1) + \beta\sum_{k=i+1}^{n}\gamma_{[k]}]\mu_{[i]} + [\delta(i-1)^2(n-i+1)^2 + \alpha(\sum_{k=i+1}^{n}\gamma_{[k]})^2](\mu_{[i]}^2 + \nu_{[i]})i^{a_{[i]}}\Bigg]i^{a_{[i]}} \tag{14}$$
$$+ 2\sum_{i=1}^{n}\sum_{j=i+1}^{n}[\delta(i-1)(j-1)(n-i+1)(n-j+1) + \alpha\sum_{k=i+1}^{n}\gamma_{[k]}\sum_{\ell=j+1}^{n}\gamma_{[\ell]}]\,i^{a_{[i]}}j^{a_{[j]}}\mu_{[i]}\mu_{[j]}.$$

For general $\gamma_i, i = 1,\ldots,n$, it is hard to minimize (14); however, if $\gamma_{[i]} = \tau^i$, $0 < \tau \neq 1, i = 1,\ldots,n$, one can use the next two lemmas to optimally solve $1/LE_g/G(TWC,TADC)$ and $1/LE_g/G(TWC, ETCP)$.

Lemma 13.

(i) *The optimal sequence for $1/LE_g,\gamma_{[i]}=\tau^i,0<\tau\neq 1/G(TWC,TADC)$ is the solution to QAP with objective function (7) where*

$$q_{ij} = \left[[\theta(j-1)(n-j+1) + \frac{\beta(\tau^{j+1}-\tau^{n+1})}{1-\tau}]\mu_i + [\delta(j-1)^2(n-j+1)^2 + \frac{\alpha(\tau^{j+1}-\tau^{n+1})^2}{(1-\tau)^2}](\mu_i^2+\nu_i)j^{a_i} \right] j^{a_i},\ \ i,j=1,...,n;$$

and

$$u_{ijk\ell} = \begin{cases} 2[\delta(j-1)(\ell-1)(n-j+1)(n-\ell+1) + \frac{\alpha(\tau^{j+1}-\tau^{n+1})(\tau^{\ell+1}-\tau^{n+1})}{(1-\tau)^2}]j^{a_i}\ell^{a_k}\mu_i\mu_k; \\ \qquad i,j,k=1,...,n,\ k,\ \ell=j+1,...,n; \\ 0,\ \textit{otherwise}. \end{cases}$$

(ii) *The optimal sequence for $1/LE_g,\gamma_{[i]}=\tau^i,0<\tau\neq 1,\alpha=\delta=0/G(TWC,TADC)$ is the solution to AP with costs $q_{ij} = [\theta(j-1)(n-j+1)+\beta(\tau^{j+1}-\tau^{n+1})/(1-\tau)]j^{a_i}\mu_i; i,j=1,\ldots,n.$*

For the problem $1/LE_g/G(TWC,ETCP)$ we obtain

$$G(TWC,TADC) = \sum_{i=1}^{n}\left[[\theta\lambda_i + \beta\sum_{k=i+1}^{n}\gamma_{[k]}]\mu_{[i]} + [\delta\lambda_i^2 + \alpha(\sum_{k=i+1}^{n}\gamma_{[k]})^2](\mu_{[i]}^2+\nu_{[i]})i^{a_{[i]}} \right] i^{a_{[i]}} + 2\sum_{i=1}^{n}\sum_{j=i+1}^{n}[\delta\lambda_i\lambda_j + \alpha\sum_{k=i+1}^{n}\gamma_{[k]}\sum_{k=j+1}^{n}\gamma_{[k]}]\, i^{a_{[i]}}j^{a_{[j]}}\mu_{[i]}\mu_{[j]}. \tag{15}$$

Lemma 14.

(i) *The optimal sequence for $1/LE_g,\gamma_{[i]}=\tau^i,0<\tau\neq 1/G(TWC,ETCP)$ is the solution to QAP with objective function (7) where*

$$q_{ij} = \left[[\theta\lambda_j + \frac{\beta(\tau^{j+1}-\tau^{n+1})}{1-\tau}]\mu_i + [\delta\lambda_j^2 + \frac{\alpha(\tau^{j+1}-\tau^{n+1})^2}{(1-\tau)^2}](\mu_i^2+\nu_i)j^{a_i} \right] j^{a_i},\ \ i,j=1,...,n;$$

and

$$u_{ijk\ell} = \begin{cases} 2[\delta\lambda_j\lambda_\ell + \dfrac{\alpha(\tau^{j+1} - \tau^{n+1})(\tau^{\ell+1} - \tau^{n+1})}{(1-\tau)^2}] j^{a_i}\ell^{a_k}\mu_i\mu_k; \; i,j,k = 1,\ldots,n, \; k \neq i, \; \ell = j+1,\ldots,n; \\ 0, \; \textit{otherwise.} \end{cases}$$

(ii) The optimal sequence for $1/LE_g, \gamma_{[i]} = \tau^i, 0<\tau \neq 1, \alpha = \delta = 0/G(TWC,ETCP)$ *is the solution to AP with costs* $q_{ij} = [\theta\lambda_j + \beta(\tau^{j+1} - \tau^{n+1})/(1-\tau)] j^{a_i}\mu_i; i,j = 1,\ldots,n.$

Finally, the following lemma solves $1/LE_g/G(TADC,ETCP)$.

Lemma 15.

(i) The optimal sequence for $1/LE_g/G(TADC,ETCP)$ *is the solution to QAP with objective function (7) where*

$$q_{ij} = \Big[[\beta(j-1)(n-j+1) + \theta\lambda_j]\mu_i + [\alpha(j-1)^2(n-j+1)^2 + \delta\lambda_j^2]\,(\mu_i^2 + \nu_i)j^{a_i}\Big]j^{a_i}; \quad i,j = 1,\ldots,n;$$

and

$$u_{ijk\ell} = \begin{cases} 2[\alpha(j-1)(\ell-1)(n-j+1)(n-\ell+1) + \delta\lambda_j\lambda_\ell] j^{a_i}\ell^{a_k}\mu_i\mu_k; \; i,j,k = 1,\ldots,n, \; k \neq i, \\ \qquad \ell = j+1,\ldots,n; \\ 0, \; \textit{otherwise.} \end{cases}$$

(ii) The optimal sequence for $1/LE_g, \alpha = \delta = 0/G(TCT,ETCP)$ *is the solution to AP whose costs are given by* $q_{ij} = [\beta(j-1)(n-j+1) + \theta\lambda_j] j^{a_i}\mu_i;$ $i,j = 1,\ldots,n.$

Lemmas 10-15 also indicate that scheduling decisions can be affected by the stochasticity of job attributes, the job-dependent learning effects, the characteristics of the cost functions, and the two criteria. The results further show that, contrary to the deterministic problem w.r.t. single criterion *TCT, TL*, and *TWT*, the optimal sequences for the proposed stochastic problem w.r.t pairs of criteria involving *TCT, TL*, and *TWT* can be different.

COMPUTATIONAL RESULTS

We carry out some computational experiments on the exact and heuristic solution methods for the QAP formulations of $1/\mathrm{LE}_g/G(C_1, C_2)$. For comparison, we solve $1/LE_g/G(MSP,TCT)$; $1/LE_g/G(MSP,TADC)$; $1/LE_g/G(TCT,TADC)$; and $1/LE_g/G(TADC,ETCP)$. The job learning indices $a_i < 0$, are sampled from U[-0.862, -0.014] (i.e., between 55% and 99% learning rates). The coefficients β and θ of $g(C_1, C_2)$ are sampled from U[1,10] and coefficients α

and δ from U[-5,5] generating convex and concave cost functions to model the behavior of decreasing risk averse and decreasing risk prone schedulers. The means μ_i and variances $\nu_i, i = 1, \ldots, n$, of normal processing times are respectively generated from $U[1,20]$ and $U[0.5,10]$.

A computer with Intel core 2 due, 2.24 GHz processor and with 1.99 GB RAM is used to run the experiments. For each problem size, ten instances are generated and solved exactly using the B&B algorithm for QAP of Cplex, and heuristically using the modified version of the integer linear programming (ILP) model of Soroush (2013b).

The experimental results on the problems $1/LE_g/G(MSP,TCT)$; $1/LE_g/G(MSP,TADC)$; $1/LE_g/G(TCT,TADC)$; and $1/LE_g/G(TADC,ETCP)$ are shown in Table 1. For each problem size and type, the table displays the number of heuristic solutions for the ten instances that turned out to be optimal, the average percentage of gaps between the objective values of the optimal and heuristic solutions (i.e., 100[(optimal objective value - heuristic objective value)/ heuristic objective value]%), and the average CPU times (in seconds) for the exact and heuristic methods. The results indicate that at least 60% of the heuristic solutions for the problems with 5 to 12 jobs are optimal. In the remaining problems, where the heuristic could not find the optimal sequences, the percentages of gaps between objective values of the exact and heuristic solutions are at most 4.82%. As expected, the heuristic's CPU time is much lower than that of the exact method. We did not optimally solve problems with more than 12 jobs since their solutions could not be found within our time limit of 4,000 CPU seconds. However, the heuristic was used to approximate the solutions for such problems with attractive CPU times.

The results also show that the pairs of criteria *(MSP,TCT)*, *(MSP,TADC)*, *(TCT,TADC)* and *(TADC,ETCP)* did not affect the difficulty of solving the problem exactly or approximately. Given the NP-hard nature of the problem, the heuristic performs well in providing optimal or near-optimal sequences. Our results also indicate that the optimal sequences are affected by the stochasticity of job attributes, the job-dependent learning effects, the convexity (or concavity) of the cost functions, and the two criteria.

Table 1. Computational results on problems $1/LE_g/G(MSP, TCT)$; $1/LE_g/G(MSP, TADC)$; $1/LE_g/G(TCT, TADC)$; and $1/LE_g/G(TADC, ETCP)$

	1/LEg/G(MSP,TCT)				1/LEg/G(MSP,TADC)				1/LEg/G(TCT,TADC)				1/LEg/G(TADC,ETCP)			
	No. of heu.	Avg. perc.	CPU time (sec)		No. of heu.	Avg. perc.	CPU time (sec)		No. of heu.	Avg. perc.	CPU time (sec)		No. of heu.	Avg. perc.	CPU time (sec)	
Prob. size	opt.	gap	Exact	Heuristic	opt.	gap	Exact	Heuristic	opt.	gap	Exact	Heuristic	opt.	gap	Exact	Heuristic
5	9	1.18	0.09	0.08	9	1.65	0.10	0.09	9	1.23	0.10	0.09	9	1.23	0.10	0.09
6	9	1.65	0.12	0.11	8	2.29	0.12	0.11	8	1.56	0.12	0.11	8	1.56	0.11	0.10
7	8	2.69	0.55	0.15	8	2.15	0.59	0.16	9	1.08	0.59	0.15	9	1.08	0.57	0.15
8	7	2.08	0.84	0.22	8	1.96	0.89	0.25	7	2.75	0.93	0.24	7	2.75	0.90	0.26
9	8	3.41	10.01	0.32	7	3.21	9.68	0.31	8	2.42	10.39	0.35	8	2.42	10.47	0.33
10	6	4.46	48.43	0.49	6	3.85	49.57	0.41	6	4.75	49.13	0.47	6	4.75	48.55	0.49
11	7	2.98	334.27	0.50	7	4.54	333.69	0.47	7	3.08	335.59	0.54	7	3.08	336.18	0.51
12	6	4.73	3896.25	1.61	7	3.25	3881.98	1.55	8	2.57	3962.65	1.68	8	2.57	3941.20	1.60
13	na	na	*	1.81	na	na	*	1.67	na	na	*	1.86	na	na	*	1.78
14	na	na	*	2.60	na	na	*	2.78	na	na	*	2.78	na	na	*	2.61
15	na	na	*	5.36	na	na	*	5.46	na	na	*	6.15	na	na	*	5.98
16	na	na	*	9.79	na	na	*	8.96	na	na	*	10.14	na	na	*	10.40
17	na	na	*	14.70	na	na	*	13.65	na	na	*	15.36	na	na	*	16.77
18	na	na	*	38.05	na	na	*	37.14	na	na	*	38.95	na	na	*	39.39
19	na	na	*	67.21	na	na	*	66.92	na	na	*	68.10	na	na	*	67.13
20	na	na	*	124.87	na	na	*	123.77	na	na	*	125.25	na	na	*	124.75
25	na	na	*	796.28	na	na	*	791.82	na	na	*	797.64	na	na	*	795.27
30	na	na	*	2449.18	na	na	*	2504.32	na	na	*	2485.21	na	na	*	2478.53

*The time exceeded the time limit of 4,000 CPU seconds.

CONCLUSION

In this paper, we have addressed a stochastic bicriteria single machine scheduling problem with job-dependent and log-linear learning effects wherein the normal processing times of jobs (i.e., processing times without any learning effects) are random variables. The goal is to determine the optimal sequence that minimizes the expected value of a general quadratic cost function of each pair of criteria consisting of the makespan, total completion time, total lateness, total waiting cost, total waiting time, total absolute differences in completion times, and the sum of earliness, tardiness and common due date penalty. The resultant problems have been formulated as quadratic assignment problems that can be solved exactly or approximately by using the relevant branch-and-bound methods. We have also formulated special cases with linear cost functions as linear assignment problems that are solvable in polynomial time. The proposed models demonstrate that scheduling decisions can be affected by the stochasticity of job attributes (i.e., their means and variances), the job-dependent learning effects, the convexity, concavity, or linearity of the cost functions (i.e., the decreasing risk averse, decreasing risk prone, or risk neutral behavior of schedulers), and the two criteria. Our computational experiments, on some problems formulated as quadratic assignment models, show that the heuristic performs well in producing optimal or near-optimal sequences. We have solved problems with up to 12 jobs optimally, and up to 30 jobs approximately within our CPU time limit of 4,000 seconds. Given the NP-hard nature of the problem, the CPU time of the heuristic is attractive, and the amount of gap between the objective values of the exact and heuristic solutions is small. The problem studied here is general in the sense that its special cases reduce to some new stochastic and deterministic single criterion/bicriteria single machine models with/without learning effects. Furthermore, the proposed approaches can be modified to solve the stochastic bicriteria scheduling problem with both job-dependent deterioration and learning effects where the deterioration indices are the negative of those of learning effects. We are currently extending this research to a problem scenario with sequence-dependent setup times. Future research should focus on incorporating other learning models into the problem.

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جدولة عمل نظام ماكينة واحدة باستخدام قاعدة ثنائية عشوائية حيث يعتمد تأثير التعلم على الوظيفة

[*]حسين سوروش و [**]ف. أمين

[*]قسم الإحصاء وبحوث العمليات – كلية العلوم – جامعة الكويت

[**]6/ 46 شارع هيروباي – 6021 – نيوتاون – ولينجتون – نيوزيلاند

خلاصة

ندرس في هذا البحث مسألة الجدولة لماكينة واحدة بهدف الحصول على قاعدة ثنائية تعتمد على التعلم الوظيفي. أزمنة العملية العامة للوظائف في هذه العملية عبارة عن متغيرات عشوائية. توضح تأثيرات التعلم أن أزمنة العملية الفعلية هي دوال وحيدة في مواضع الوظائف في المتتابعة. هدفنا هو اشتقاق متتابعة مثالية تدني القيمة المتوقعة لدالة عامة من الدرجة الثانية للتكلفة لكل زوج من القواعد التي تتكون من الزمن الكلي لأكمال العملية، اجمالي زمن التأخير، تكلفة زمن الانتظار، اجمالي زمن الانتظار، اجمالي الفرق، المطلق في أزمنة اكمال المهمة، مجموع أزمنة التبكير والتأخير. سنعمل على صياغة المسائل العشوائية الناتجة كمسائل تخصيص من الدرجة الثانية والتي يمكن حلها بدقة أو تجريبياً ونبرهن على أن الحالات الخاصة ذات دوال تكلفة خطية يمكن حلها في زمن على شكل كثيرة حدود. النتائج الحسابية على سيناريوهات مسائل التخصيص من الدرجة الثانية توضح أن الحلول القريبة من المثالية يمكن الحصول عليها في أزمنة CPU جذابة.

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صدر العدد الأول في رجب ١٤٠٤هـ - أبريل ١٩٨٤م

* تهدف إلى معالجة المشكلات المعاصرة والقضايا المستجدة من وجهة نظر الشريعة الإسلامية.
* تشمل موضوعاتها معظم علوم الشريعة الإسلامية: من تفسير، وحديث، وفقه، واقتصاد وتربية إسلامية، إلى غير ذلك من تقارير عن المؤتمرات، ومراجعة كتب شرعية معاصرة، وفتاوي شرعية، وتعليقات على قضايا علمية.
* تنوع الباحثون فيها، فكانوا من أعضاء هيئة التدريس في مختلف الجامعات والكليات الإسلامية على رقعة العالمين: العربي والإسلامي.
* تخضع البحوث المقدمة للمجلة إلى عملية فحص وتحكيم حسب الضوابط التي التزمت بها المجلة، ويقوم بها كبار العلماء والمختصين في الشريعة الإسلامية، بهدف الارتقاء بالبحث العلمي الإسلامي الذي يخدم الأمة، ويعمل على رفعة شأنها، نسأل المولى عز وجل مزيداً من التقدم والازدهار.

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